

# Methoden moderner Röntgenphysik: Streuung und Abbildung

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Lecture 3	Vorlesung zum Haupt- oder Masterstudiengang Physik, SoSe 2019 G. Grübel, L. Müller, O. Seeck, L. Frenzel, F. Lehmkuhler, M. Martins, W. Wurth
Location	Lecture hall AP, Physics, Jungiusstraße
Date	Tuesdays                    12:30 - 14:00                    (starting 2.4.) Thursdays                    8:30 - 10:00                    (until 11.7.)

# Methoden moderner Röntgenphysik: Streuung und Abbildung

**Part I:**

## **Basics of X-ray Physics**

by Gerhard Grübel (GG)

- [ 2.4.] Organisation and Introduction
- [ 4.4.] X-ray Scattering Primer
- [ 9.4.] Sources of X-rays, Synchrotron Radiation
- [11.4.] Refraction and Reflection
- [16.4.] Kinematical Scattering Theory (I)
- [18.4.] Kinematical Scattering Theory (II), Applications
- [23.4.] Small Angle Scattering and Soft Matter
- [25.4.] Anomalous Scattering
- [30.4.] Introduction: Coherence I
- [ 2.5.] Coherence II; Applications of Coherent X-ray Beams

# Methoden moderner Röntgenphysik II: Streuung und Abbildung

## Part I:

### **Basics of X-ray Physics**

by Gerhard Grübel (GG)

#### [Introduction](#)

Overview, Introduction to X-ray Scattering

#### [X-ray Scattering Primer](#)

Elements of X-ray Scattering

#### **Sources of X-rays, Synchrotron Radiation**

**Laboratory Sources, Accelerator Bases Sources**



#### [Reflection and Refraction from Interfaces](#)

Snell's Law, Fresnel Equations

#### [Kinematical Diffraction \(I\)](#)

Diffraction from an Atom, a Molecule, from Liquids, Glasses, ...

#### [Kinematical Diffraction \(II\)](#)

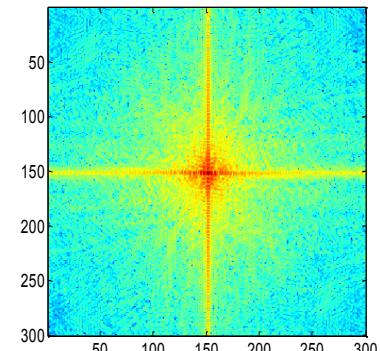
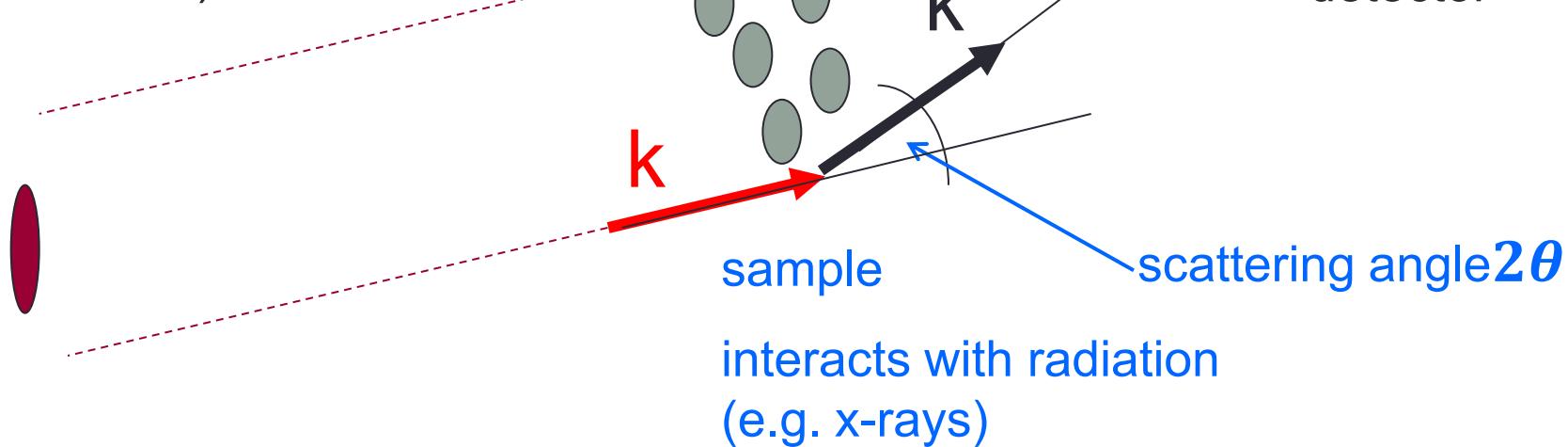
Diffraction from a Crystal, Reciprocal Lattice, Structure Factor, ...

# Set-up for Scattering Experiments

source (visible light, x-rays,...)

source parameters: source size,  $\lambda$ ,  $\frac{\Delta\lambda}{\lambda}$ ...

coherence properties:  
(incoherent, partially coherent,  
coherent)

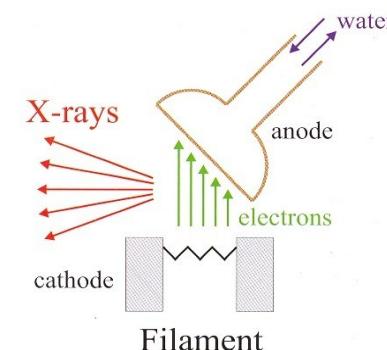


# Source of X-Rays

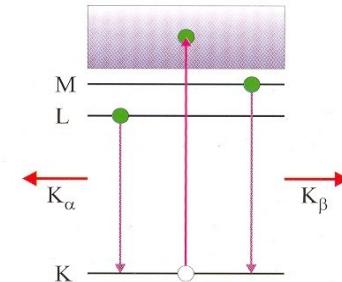
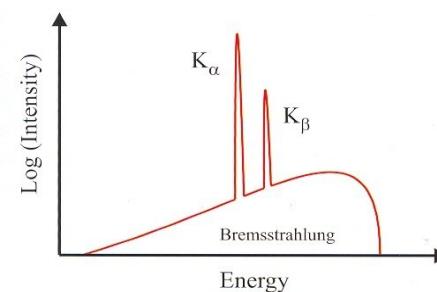
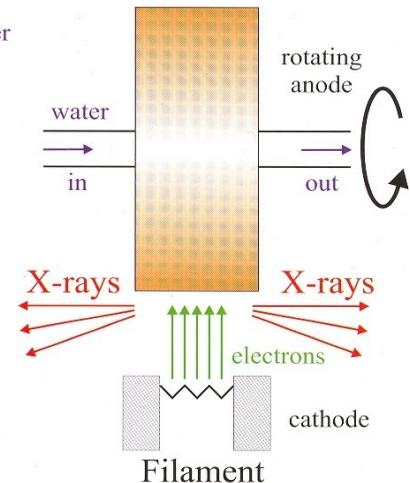
- 1895 Discovered by W.C. Röntgen
- 1912 First diffraction experiment (v. Laue)
- 1912 Coolidge tube (W.D. Coolidge, GE)
- 1946 Radiation from electrons in a synchrotron, GE,  
Physical Review, 71,829 (1947)



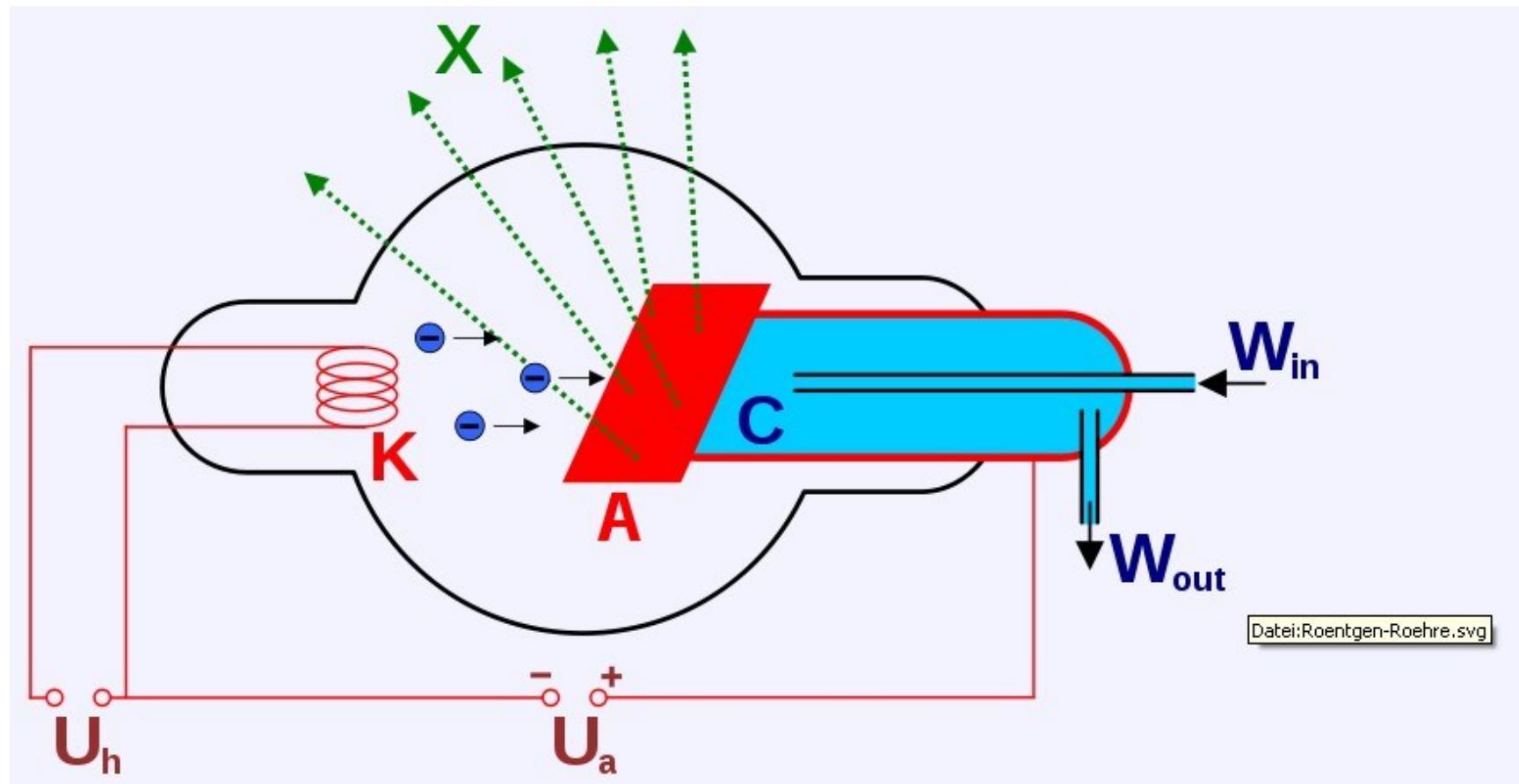
Coolidge Tube



Rotating Anode

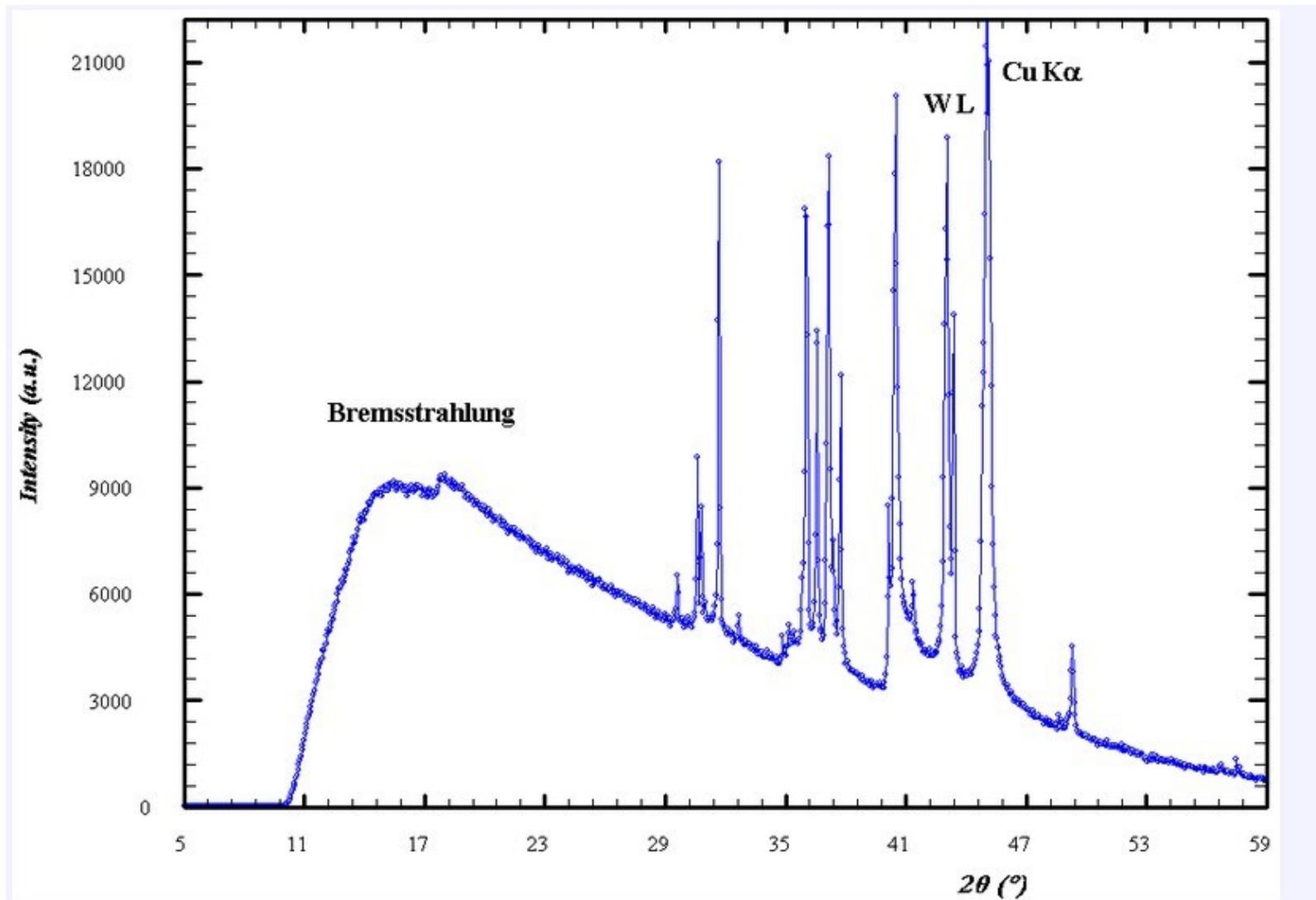


# X-Ray Tube

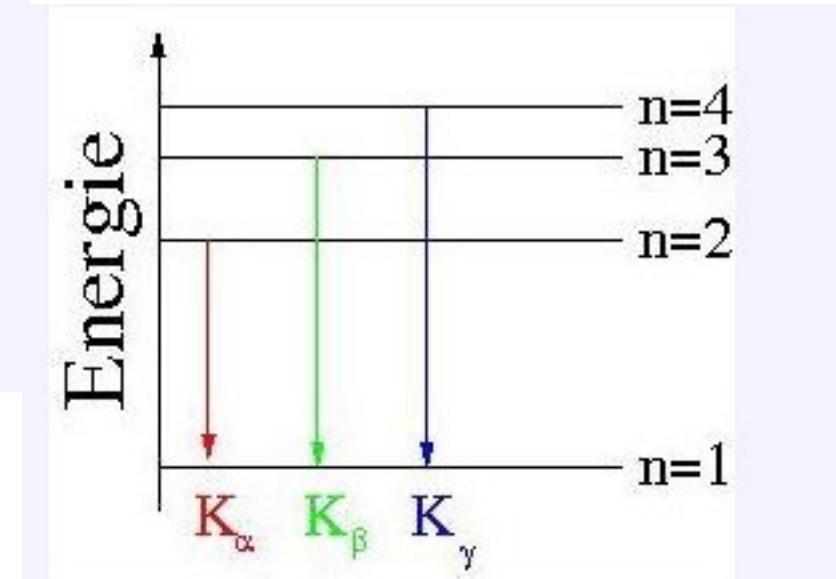
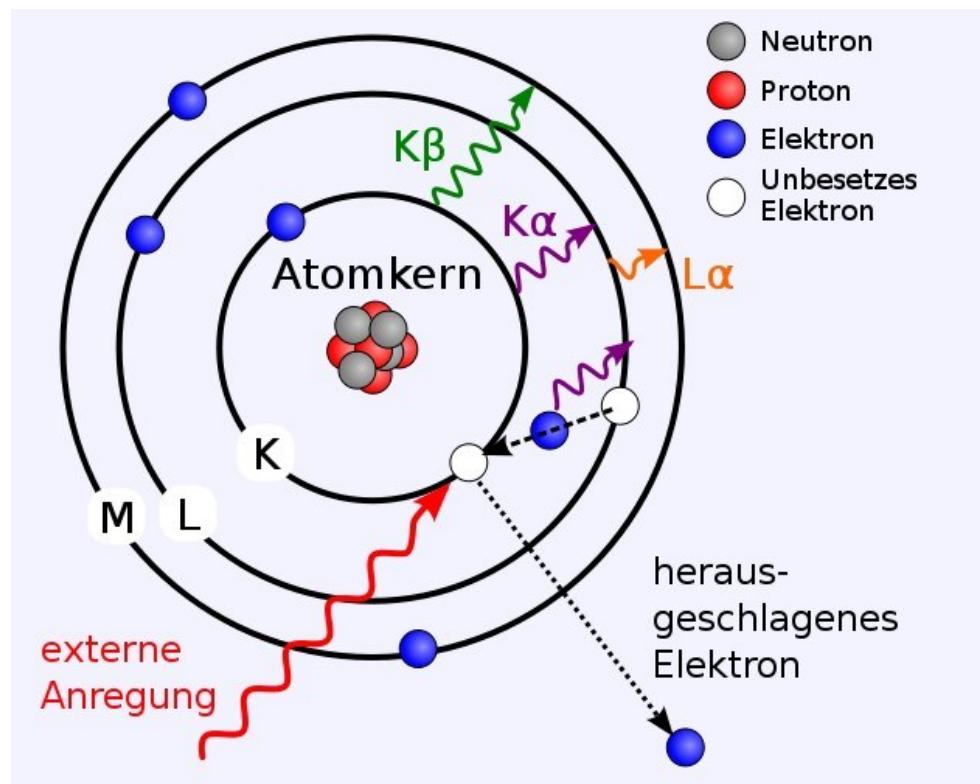


Datei:Roentgen-Röhre.svg

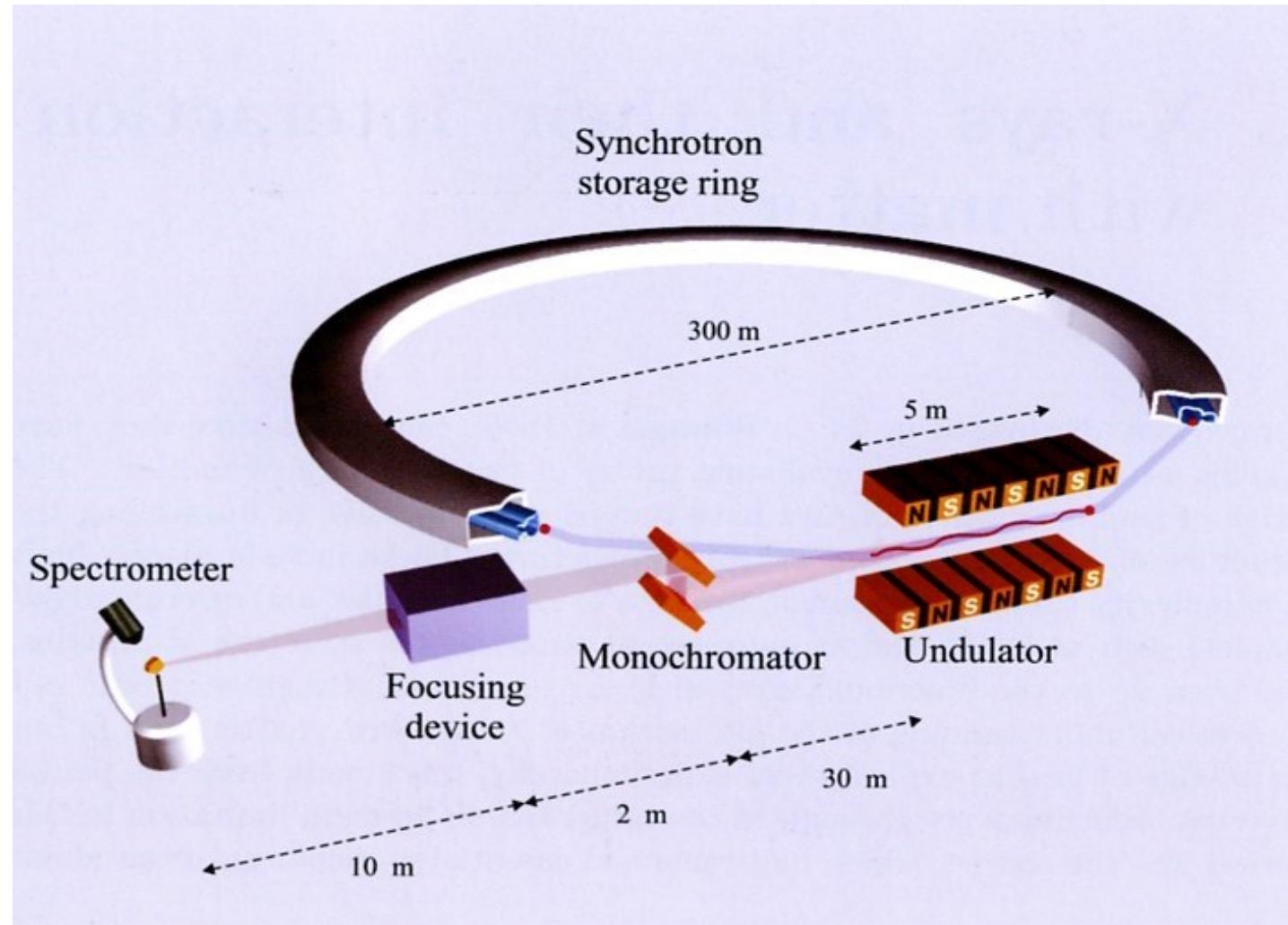
# X-Ray Tube



# X-Ray Tube



# Synchrotron Radiation Storage Ring



# Circular Accelerators

Cyclotron  
Microtron  
Synchrotron  
Storage Ring

# Cyclotron

- Proposed in 1930 by E.O. Lawrence
- Electrons circulate in a homogeneous magnetic field  $B$
- Frequency for one cycle is given by

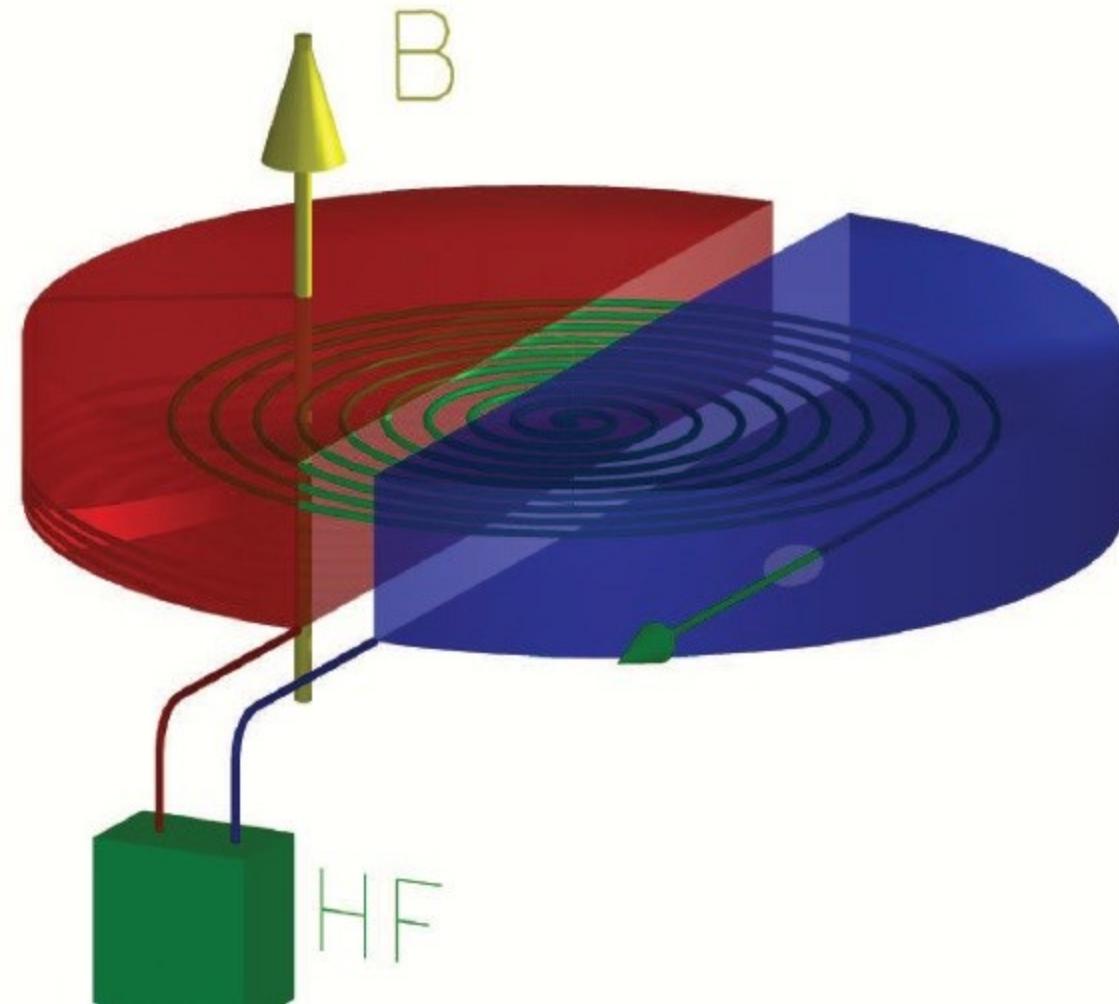
$$\omega_c = \left(\frac{e}{m}\right) B_Z$$

- For non-relativistic electrons  $\omega_c$  is independent of the velocity  $v$   
$$\left(\frac{v}{c} < 0.15\right)$$
- At high energies the mass changes and the frequency of the field needs to be adapted.

Example:  $E_{\text{kin}} = 10 \text{ keV} = eU = m_e \frac{v_e^2}{2} \Rightarrow \frac{v_e}{c} = 0.2!$

- Electrons at 10 keV are already relativistic!

# Cyclotron



# Cyclotron



Zyklotron der  
Uni Bonn

# Microtron

- Acceleration with a linear accelerator
- Circular bend similar to a cyclotron
- Bending radius  $R$  in magnetic field  $B$  for relativistic particles

Lorentz Force = Radial Force

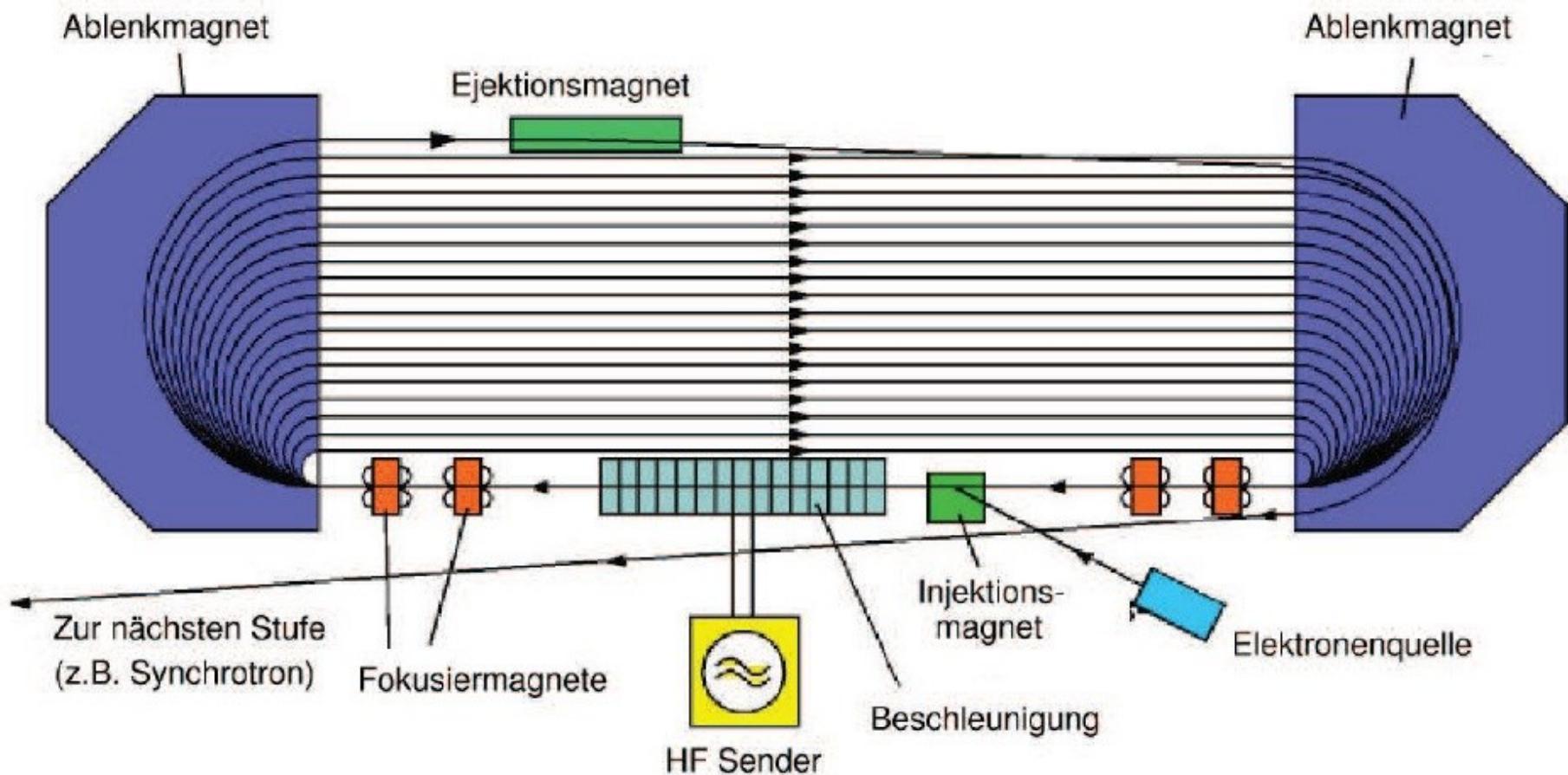
$$evB = m \frac{v^2}{R}$$

$$\Rightarrow R = \frac{mv}{eB} = \frac{vmc^2}{ec^2B} = \left( \frac{v}{ec^2B} \right) E$$

- Such Acceleration that electrons are in phase with RF field
- Energies up to 100 MeV can be reached

Example: BESSY II

# Microtron



# Microtron



# Synchrotron

- For relativistic particles  $v \cong c$  in a B field, the radius is given by

$$R = \frac{E}{ecB}$$

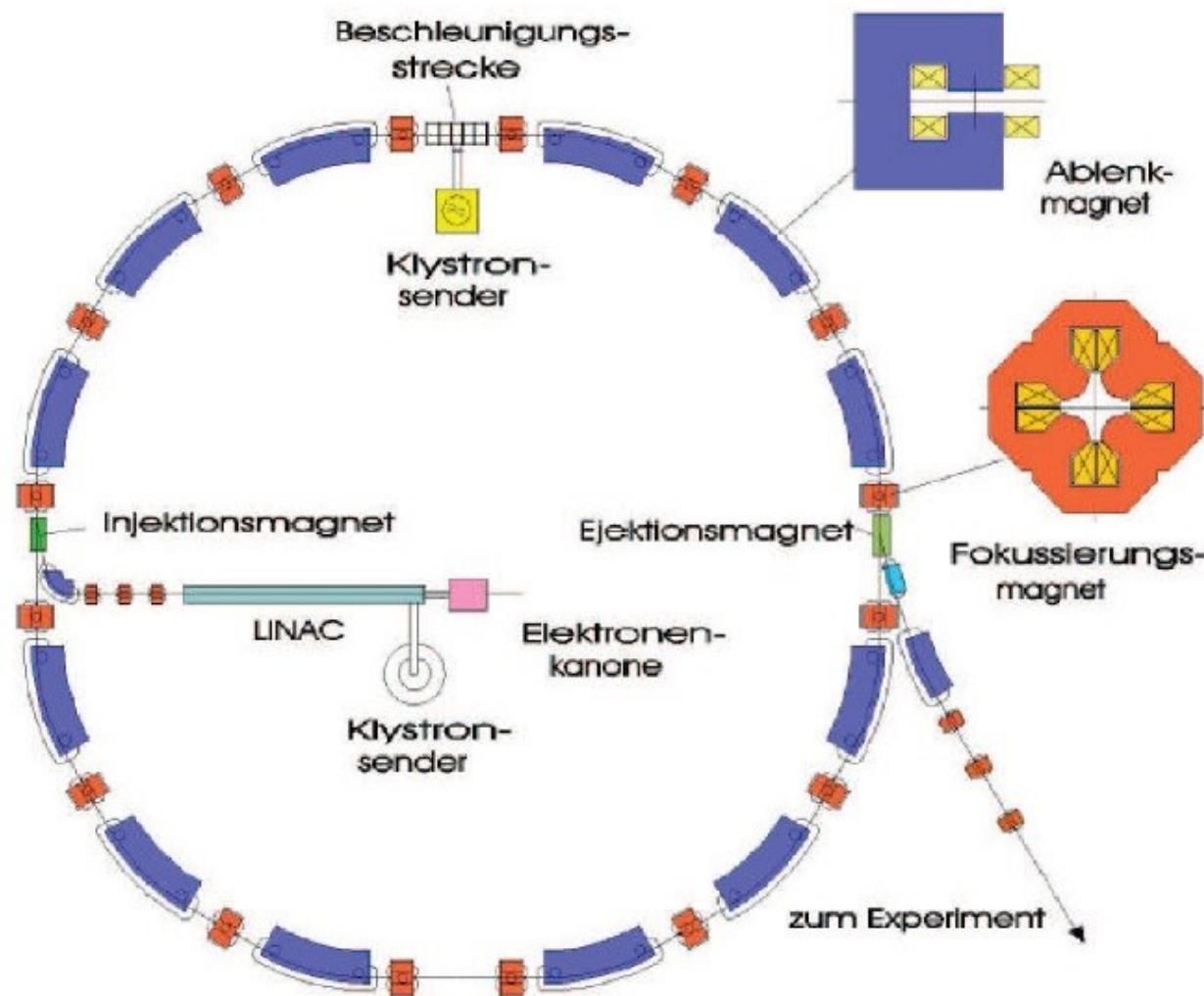
- For  $E > 1$  GeV and  $B = 5T$  :  $R >$  several meter
- Technically difficult
- Enforce trajectory with constant radius

Bends in small , local magnets

$\frac{E}{B} = \text{const.} \Rightarrow$  synchronous ramping of E and B

⇒ Synchrotron

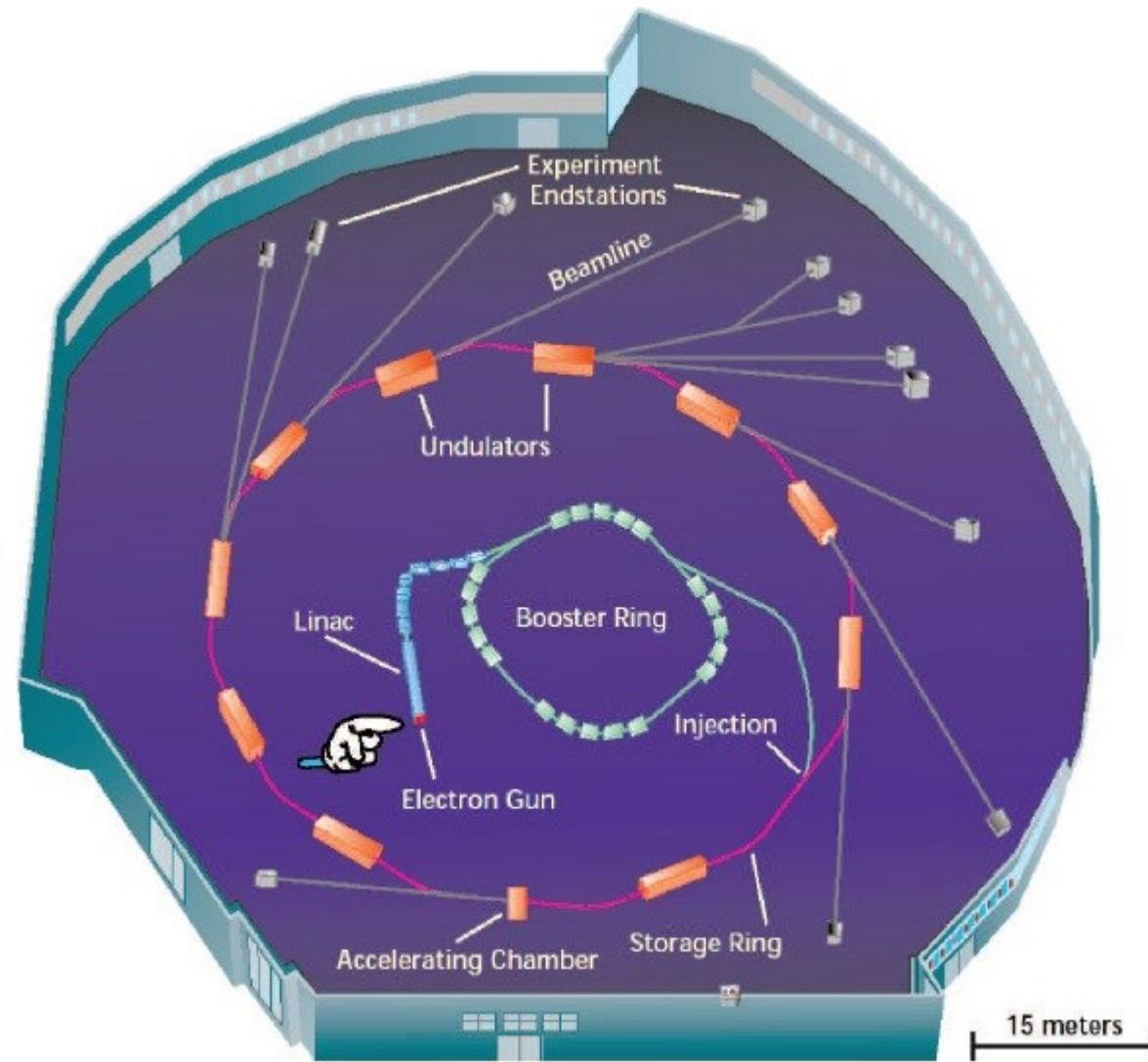
# Synchrotron



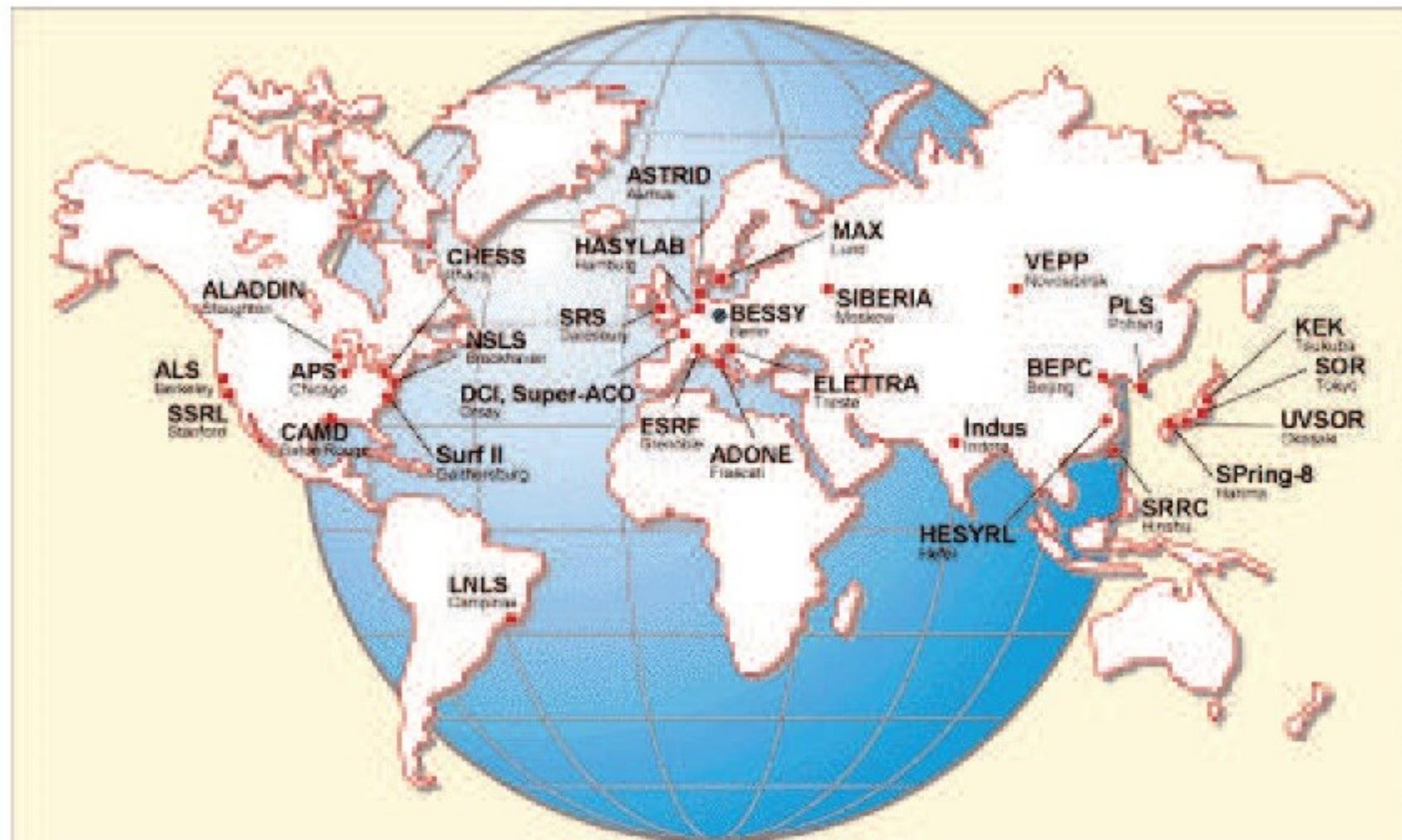
# Synchrotron

- Modern synchrotron radiation sources are built as storage rings
- Synchrotron cannot operate at  $E=0$  since it requires  $B=0$ .
  - ⇒ Use LINAC or Microtron as pre-accelerator
  - Use synchrotron to reach the final energy  $E$
  - Use storage ring to keep electrons at energy
- The storage ring supplies the energy lost by radiation in each turn.
- Typical parameters: Lifetime: up to 30 h  
Current: 100 – 500 mA
- Current losses through interaction with residual gas ⇒ UHV
- Current supplied in bunches.

# Storage Rings



# Storage Rings



# Photon Machines

**The three largest and most powerful synchrotrons in the world**



APS, USA



ESRF, Europe-France



Spring-8, Japan



# Synchrotron Radiation Primer

Radiation of a non-relativistic, accelerated particle:

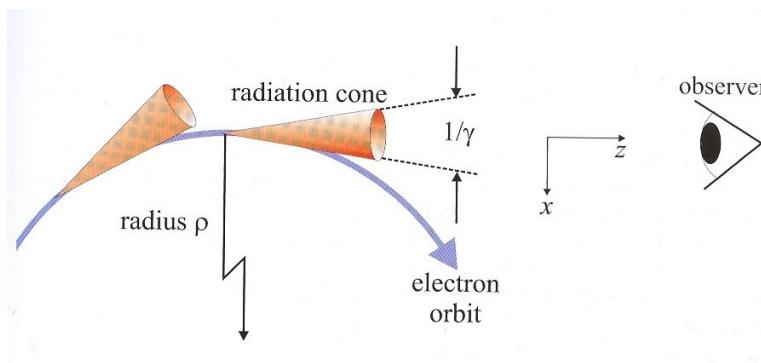
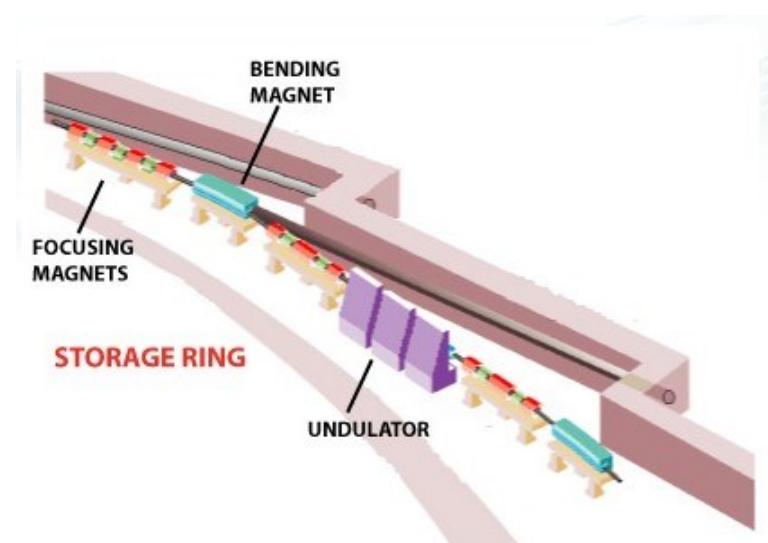
$$P = \left( \frac{e^2}{6\pi\epsilon_0 m_0^2 c^3} \right) \left( \frac{dp}{dt} \right)^2$$

Angular distribution resembles the one of a Hertz dipole:

$$\left( \frac{dP}{d\Omega} \right) = \left( \frac{e^2}{16\pi^2\epsilon_0 m_0^2 c^3} \right) \left( \frac{dp}{dt} \right)^2 \sin^2(\Psi)$$

Radiation is emitted (similar to the dipole) in the direction perpendicular to the acceleration

# Synchrotron Radiation Primer



Energy  $E_e$  of an electron at speed  $v$ :

$$E_e = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma mc^2$$

For 5GeV and  $mc^2=0.511$  MeV get  $\gamma \approx 10^4$

Centrifugal=Lorentz force yields for radius:

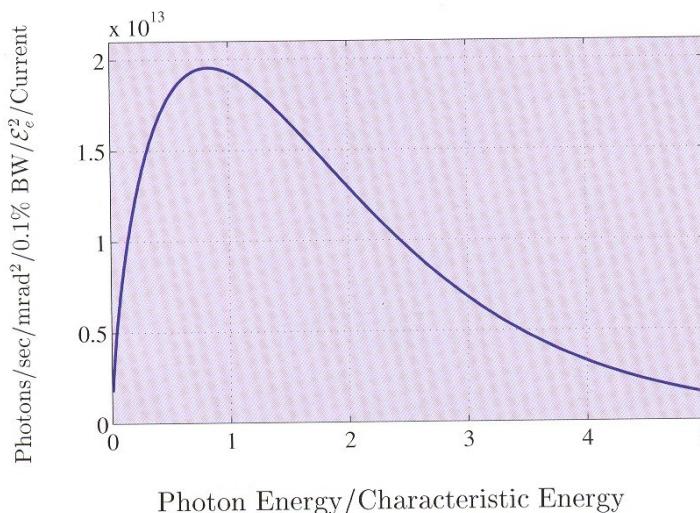
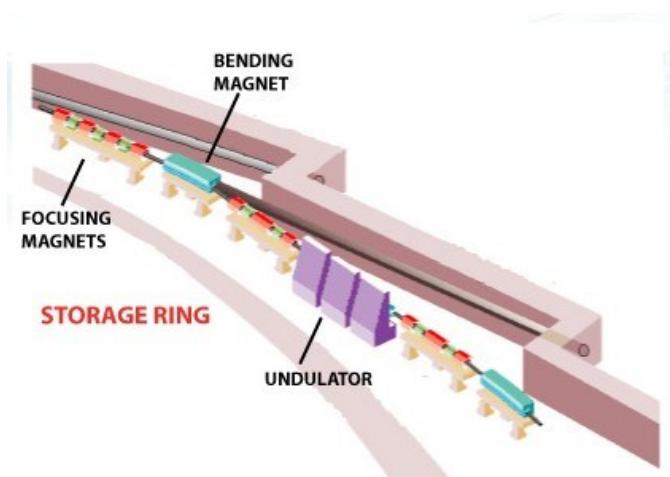
$$\rho = \frac{\gamma mc}{eB} = \frac{3.3 E [\text{GeV}]}{B [\text{T}]} \approx 25 \text{ m}$$

$$E_e = 6 \text{ GeV}, \quad B = 0.8 \text{ T}$$

Opening angle is of order  $\frac{1}{\gamma} \approx 0.1 \text{ mrad}$



# Bending Magnets



Characteristic energy  $\hbar\omega_c$  for bend or wiggler:

$$\hbar\omega_c [\text{keV}] = 0.665 E_e^2 [\text{GeV}] B(\text{T}) \approx 20 \text{ keV}$$

$$\text{Flux} \sim E^2$$

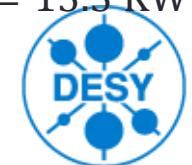
Energy loss by synchrotron radiation per turn:

$$\Delta E [\text{keV}] = \frac{88.5 E^4 [\text{GeV}]}{\rho [\text{m}]}$$

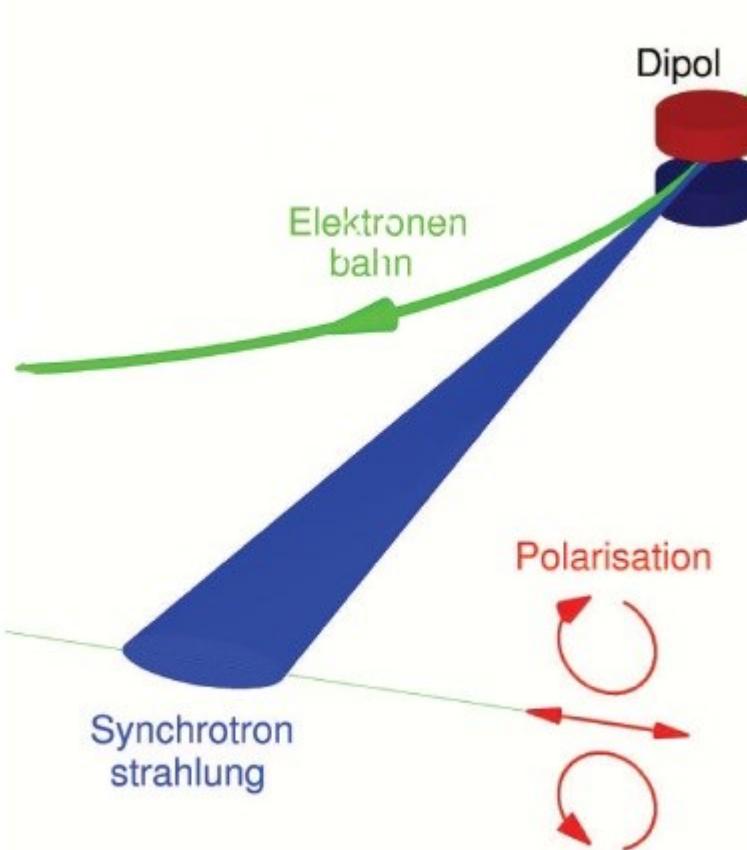
For 1 GeV and  $p = 3.33 \text{ m}$ :  $\Delta E = 26.6 \text{ keV/turn}$

$$\text{For } I = 500 \text{ mA} \equiv 0.5 \frac{\text{C}}{\text{s}} = 0.5 \times 6.25 \times 10^{18} \frac{\text{e}^-}{\text{s}}$$

$$\begin{aligned} \rightarrow P &= 0.5 \times 6.25 \times 10^{18} \frac{\text{e}^-}{\text{s} \times 26.6 \text{ keV}} \\ &= 8.3125 \times 10^{22} \times 1.6 \times 10^{-19} = 13.3 \frac{\text{kJ}}{\text{s}} = 13.3 \text{ KW} \end{aligned}$$



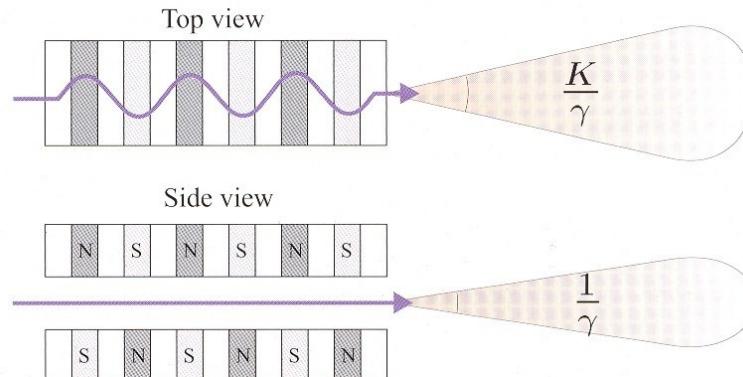
# Polarization



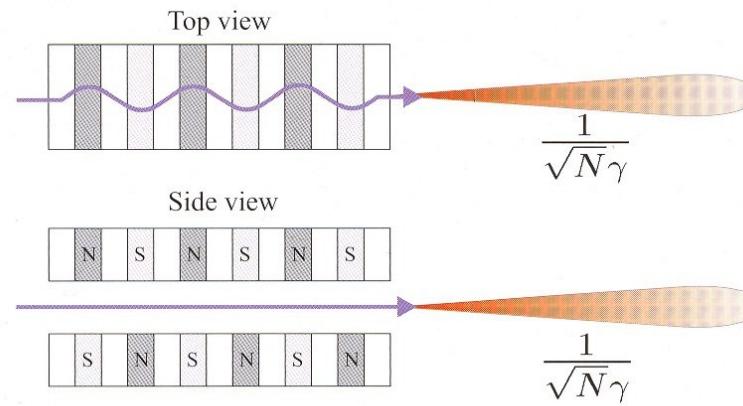
- Synchrotron radiation is polarized linearly in the plane of the orbit
- Above and below the orbital plane of the polarization is circular
- Important applications for magnetic x-ray scattering

# Insertion Devices (Wiggler and Undulators)

(a) Wiggler



(b) Undulator



## Wiggler:

$$P[\text{kW}] = 0.633 E_e^2 [ \text{GeV} ] B^2 [\text{T}] L[\text{m}] I[\text{A}]$$

$$\text{Flux} \sim E^2 \times N$$

N: number poles

## Undulator:

$$k = eB / mc \quad k_u = 0.934 \lambda_u [\text{cm}] \quad B_0 [\text{T}]$$

with  $\lambda_u$  undulator period

undulator fundamental:

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{k^2}{2} + \gamma \theta \right)$$

~~on axis~~

$$\text{Flux} \sim E^2 \times N^2$$

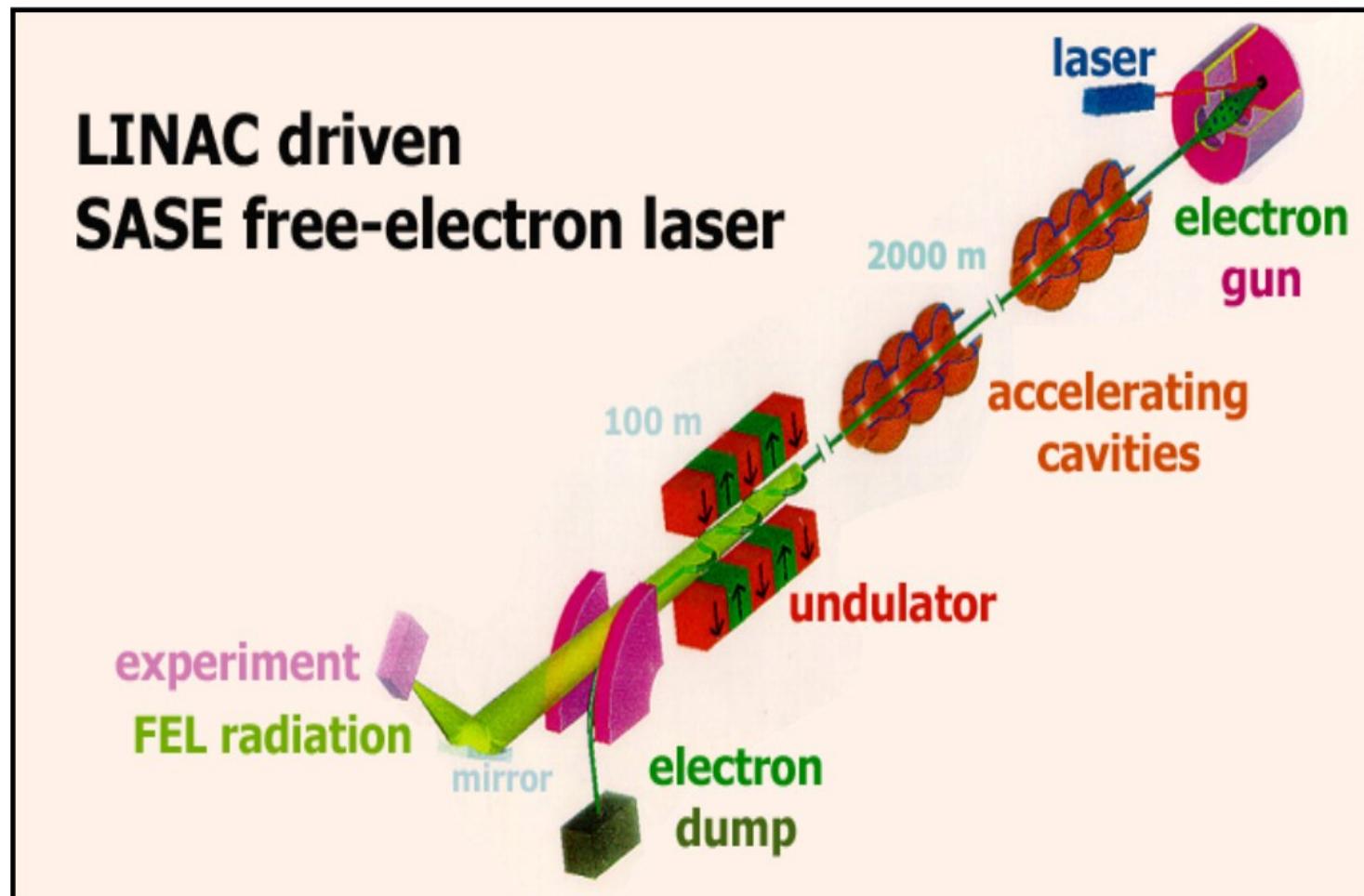
bandwidth:  $\frac{\Delta\lambda}{\lambda} \sim \frac{1}{nN}$



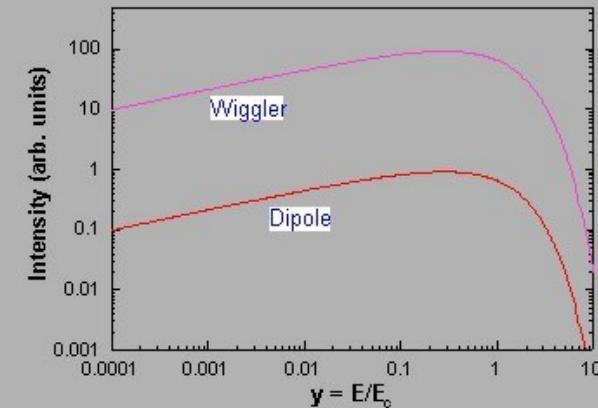
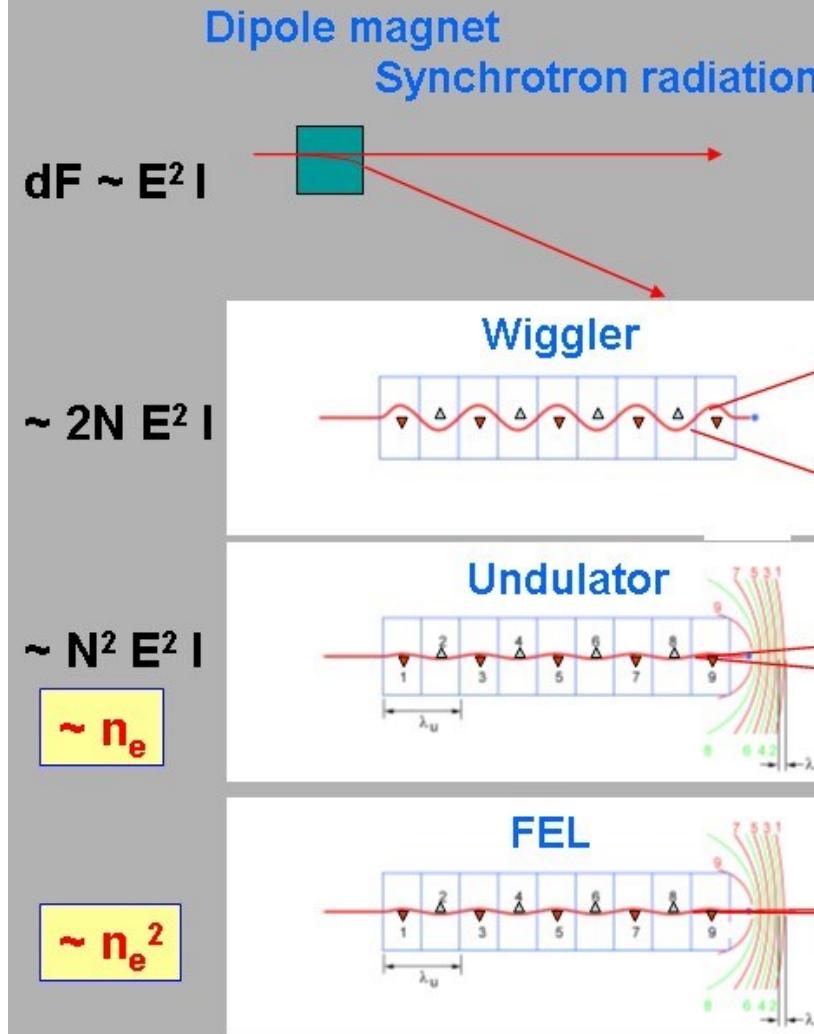
# Towards Diffraction Limited Light Sources: MAX IV (Lund)



# Free Electron Lasers (FELs)



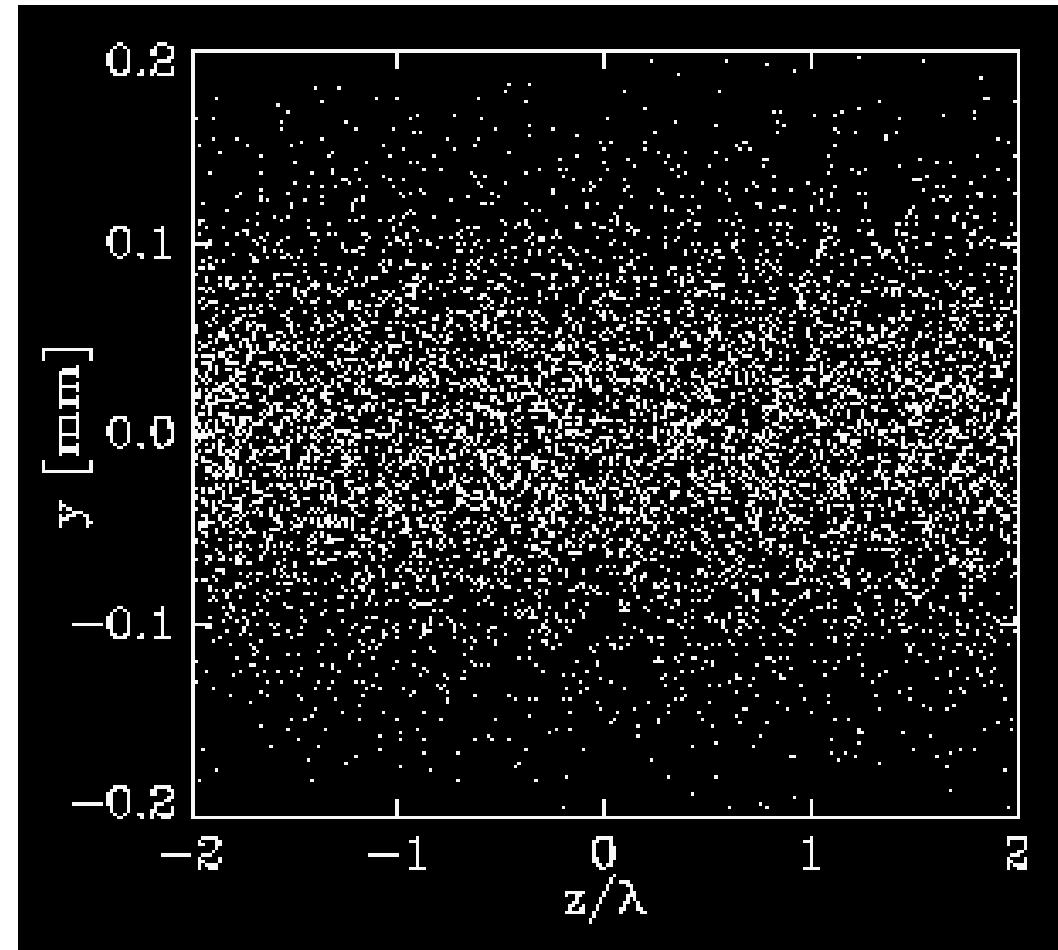
# Synchrotron and FEL Sources



The radiation emitted by a single electron in subsequent oscillations in an undulator is in phase. Radiation from different electrons is NOT (positional disorder in bunch).

"Phasing" is achieved via positional order in the bunch (micro-bunching) with a period equal to the x-ray wavelength.

# Electron Bunching

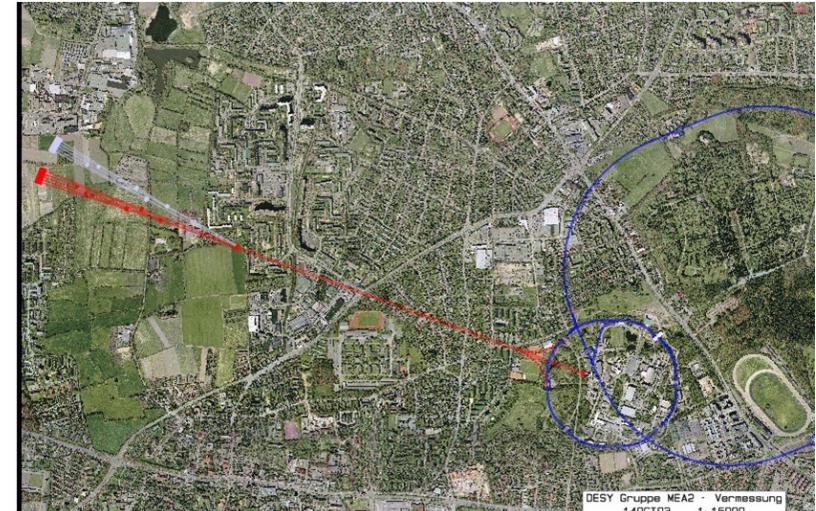


GENESIS – simulation for TTF parameters

Courtesy Sven Reiche  
(UCLA)



# VUV and X-Ray FELs



# Brilliance

$$B = \frac{\text{photons}}{\text{s mm}^2 \text{ mrad}^2 0.1\% \text{ BW}}$$

