Methoden moderner Röntgenphysik: Streuung und Abbildung

Lecture 14	Vorlesung zum Haupt- oder Masterstudiengang Physik, SoSe 2018
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Location	Lecture hall AP, Physics, Jungiusstraße
Date	Tuesday 13:00 - 14:30 (starting 3.4.)
	Thursday 8:30 - 10:00 (until 12.7.)

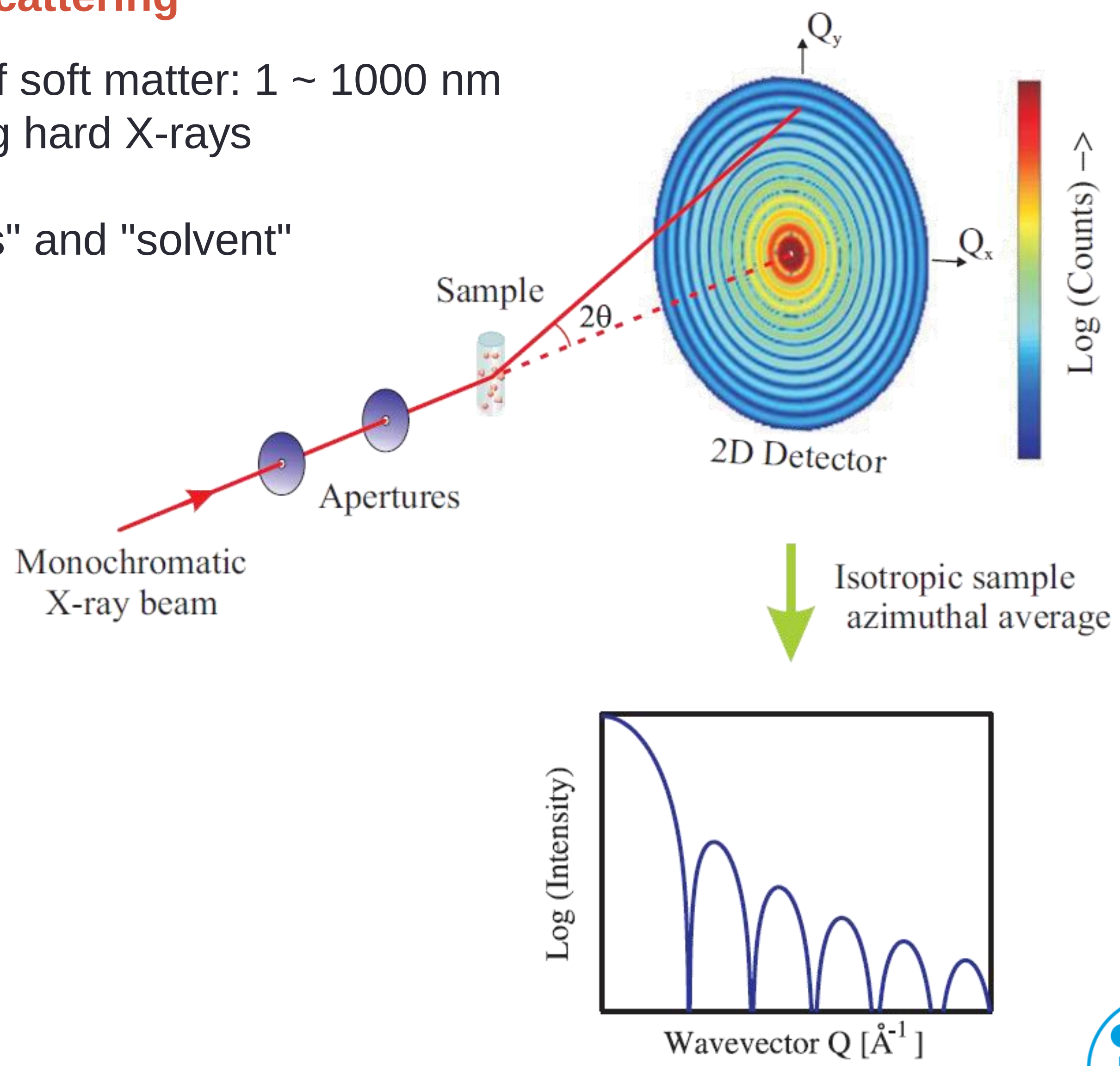
Soft Matter – Timeline

- Di 29.05.2018 Soft Matter studies I: Methods & experiments
  Definitions, complex liquids, colloids, storage ring and FEL experiments, setups, liquid jets, **…**
- Do 30.05.2018 Soft Matter studies II: Structure
  SAXS & WAXS applications, X-ray cross correlations, **…**
- Di 05.06.2018 Soft Matter studies III: Dynamics
  XPCS applications, diffusion, dynamical heterogeneities, ...
- Do 07.06.2018 cancelled!
- Di 12.06.2018 Case study I: Glass transition **at DESY campus!**
  Supercooled liquids, glasses vs. crystals, glass transition concepts, structure-dynamics relations, ...
  + DESY photon science site visit
- Do 14.06.2018 Case study II: Water
  Phase diagram, anomalies, crystalline and glassy forms, FEL studies, ...

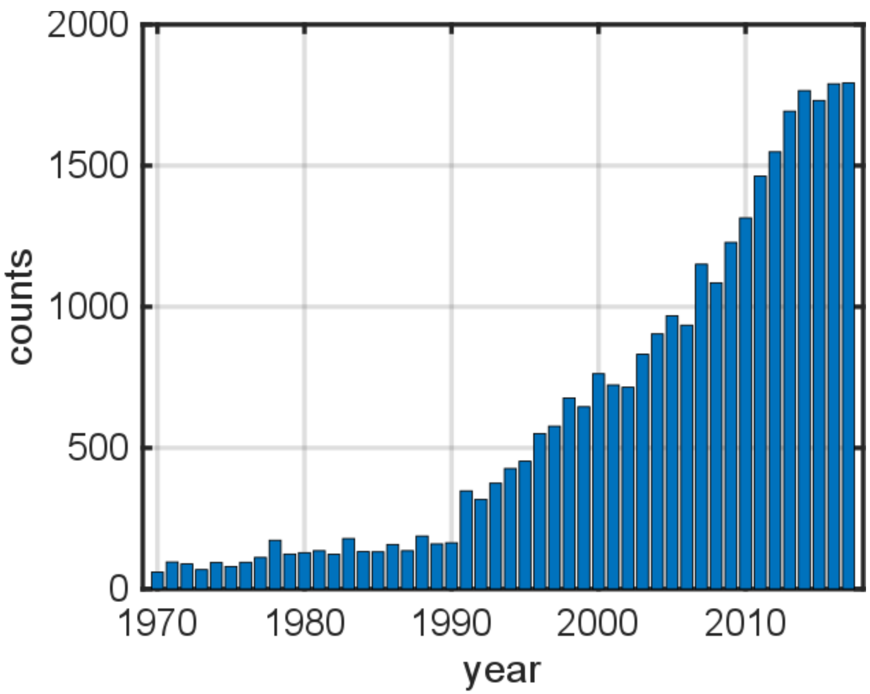
Small-angle X-ray scattering

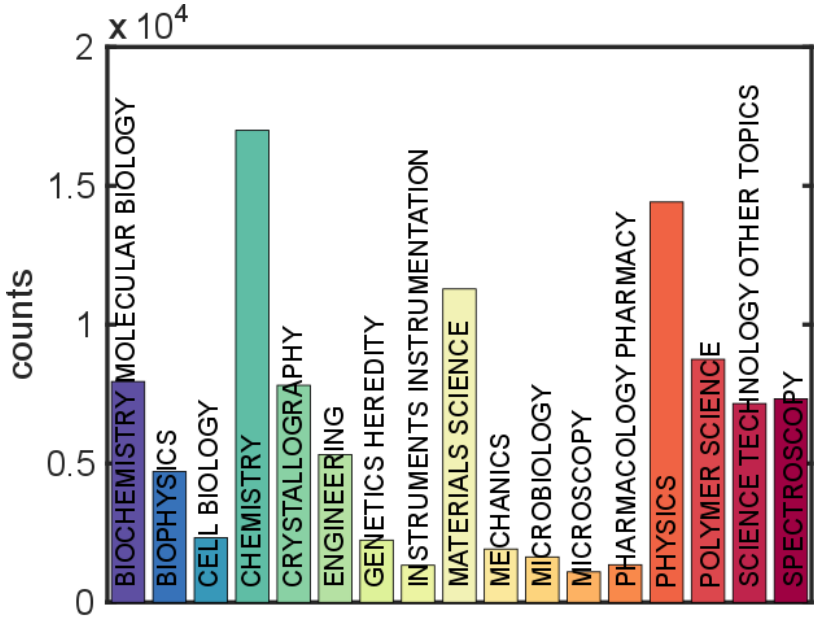
Typical dimensions of soft matter: 1 ~ 1000 nm
→ Small angles using hard X-rays

Soft matter: "particles" and "solvent"



Small-angle X-ray scattering





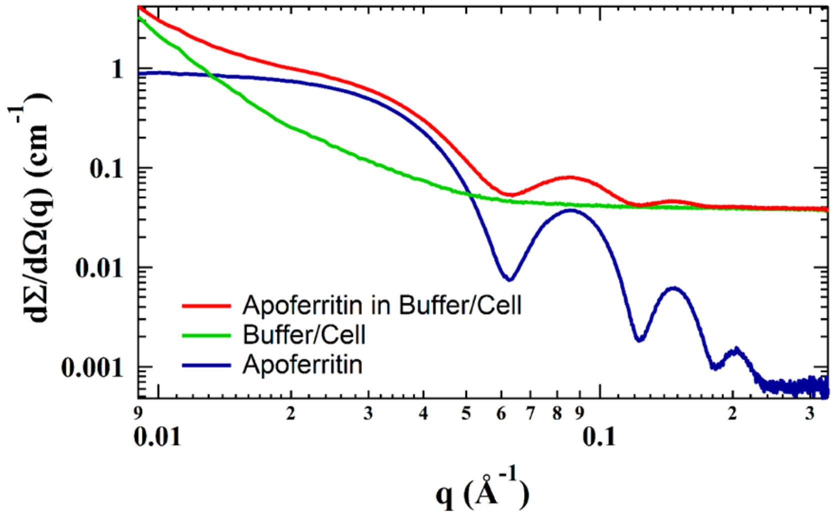
Web of knowledge topic search: "Small angle X-ray scattering"

SAXS – Analysis methods: Formfactor

Lecture 7: $I_{SAXS}(Q) = \left(\rho_{sI,p} - \rho_{sI,0}\right)^2 \left| \int_{V_p} e^{iQr} dV_p \right|^2$ for particle (p) in solvent (0)

Diluted case: Formfactors

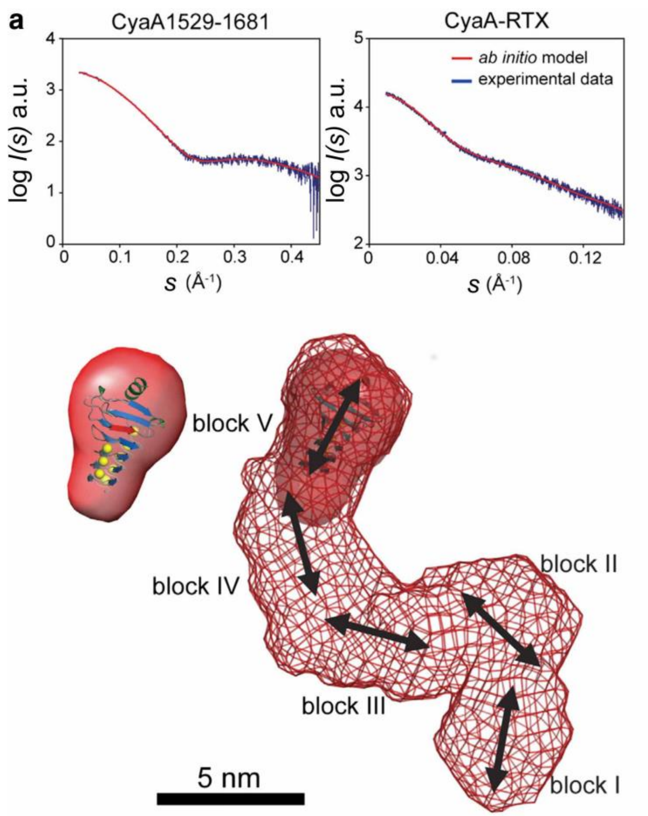
- Spheres: $F(q) = 3\,\frac{\sin(qR) - qR\cos(qR)}{(qR)^3}$
- In general difficult to calculate → numerical approaches
- Soft Matter: Polydispersity & (solvent) background
- $I_c = \frac{1}{I_0} \frac{\frac{I_{raw}}{t_e} - \frac{I_{dark}}{t_{dark}}}{I_{qe}} \cdot \frac{D_p^2}{p^2} \cdot \frac{D_p}{D_0} \Rightarrow I_{particle} = \frac{I_{c,s}}{d_s T_s} - \frac{I_{c,b}}{d_b T_b}$



Chem. Rev. 116, 11128 (2016)

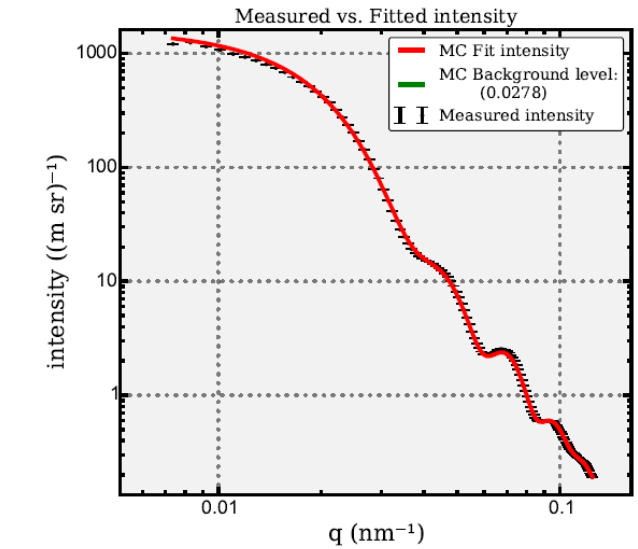
SAXS – Analysis methods: Formfactor

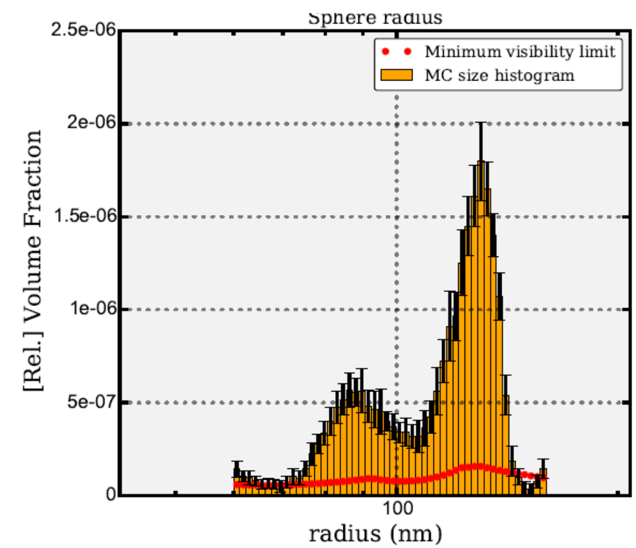
Ab initio methods (use "dummy" bead models) → BioSAXS



doi:10.1042/ETLS20170138

Monte-Carlo methods





SAXS – Analysis methods: Structure factors

From lecture 5 (Kinematical Diffraction I): Structure factor of a liquid (or glass)

$$S(q) = 1 + \rho_0 \int_0^\infty \frac{4\pi r}{q} [g(r) - 1] \sin(qr) \mathrm{d}r$$

With the radial pair correlation function $g(r)$. This relates to the potential of mean force between two particles $U_{MF}(r)$

$$g(r) = \exp\left(-\frac{U_{MF}(r)}{k_B T}\right)$$

For very dilute systems $U_{MF}(r)$ equals the interaction potential $U(r)$.

Relation of $S(q)$ or $g(r)$ and $U(r)$ → **Ornstein-Zernike equation** relating total correlations $h(r) \equiv g(r) - 1$ to direct two-particle correlations $c(r)$ and indirect correlations $c(|\mathbf{r} - \mathbf{r}'|)$

$$h(r) = c(r) + \rho_0 \int c(|\mathbf{r} - \mathbf{r}'|) h(|\mathbf{r}'|) \mathrm{d}\mathbf{r}'$$

SAXS – Analysis methods: Structure factors

$$h(r) = c(r) + \rho_0 \int c(|\mathbf{r} - \mathbf{r}'|) h(|\mathbf{r}'|) \mathrm{d}\mathbf{r}'$$

$c(r)$ short range part

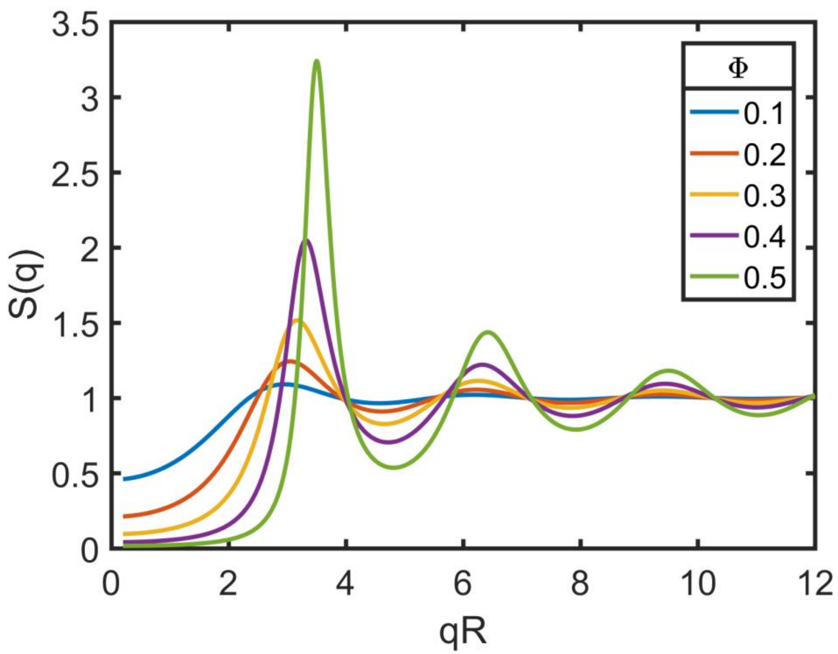
Can be solved using so-called "closure relations".
Percus-Yevick closure:

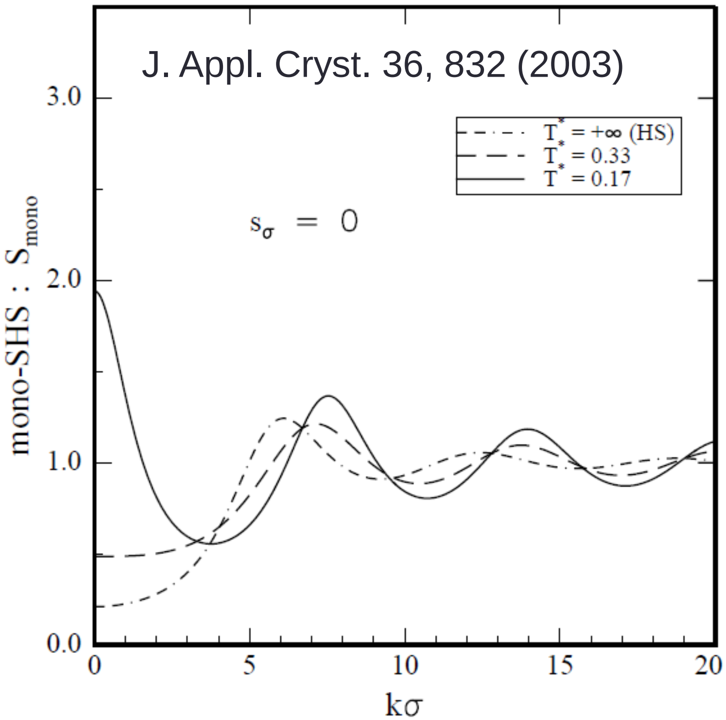
$$c(r) = g(r) \left[1 - \exp\left(\frac{U(r)}{k_B T} \right) \right]$$

→ solves the hard-sphere potential $U_{HS}(r) = \begin{cases} \infty, & r \leq 2R \\ 0, & r > 2R \end{cases}$ analytically.

→ Mean-spherical approximation closure relation $c(r) = -\frac{U_{ES}(r)}{k_B T}$ solves electrostatic interactions (DLVO) [→ Lecture 13]

Structure factors – hard spheres





Hard spheres

- Volume fraction as only parameter
- Does not include crystallisation/glass transition!
- I.e. typically breaks down close to $\Phi \approx 0.5$

Sticky hard spheres

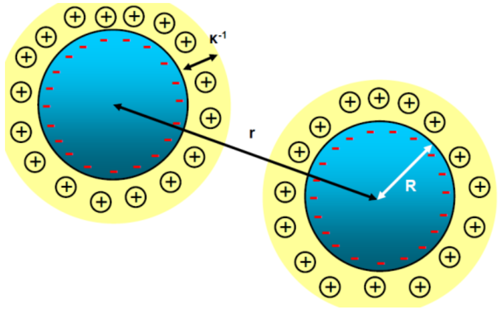
$$\frac{U_{SHS}(r)}{k_B T} = \begin{cases} \infty, & r < \sigma \\ \ln\left(\frac{12\tau\Delta}{\sigma + \Delta}\right), & \sigma \leq r \leq \sigma + \Delta \\ 0, & \sigma + \Delta < r \end{cases}$$

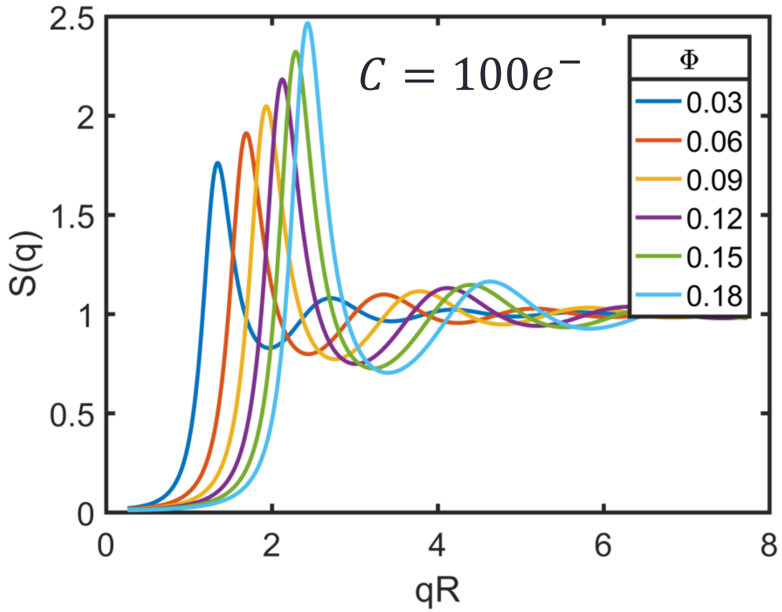
Structure factors – RMSA

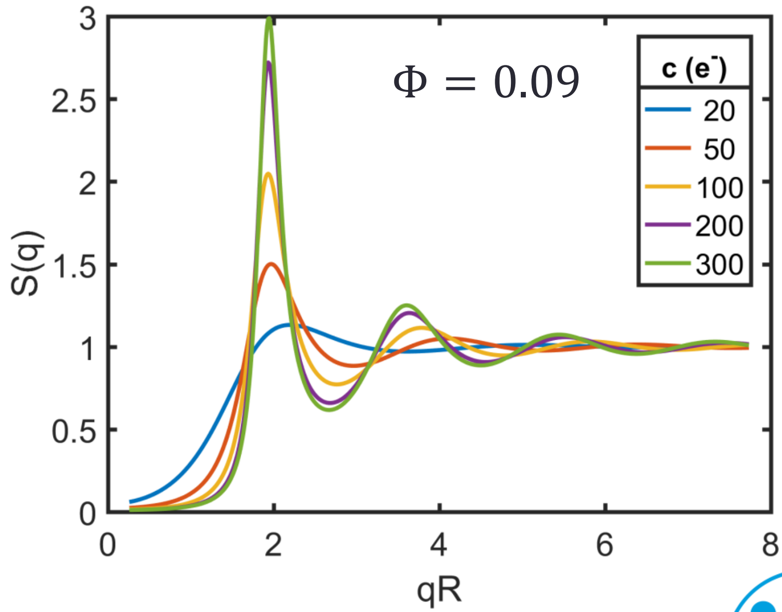
Charge stabilized systems → rescaled mean spherical approximation (RMSA)

Structure factor as function of Φ, charge, screening

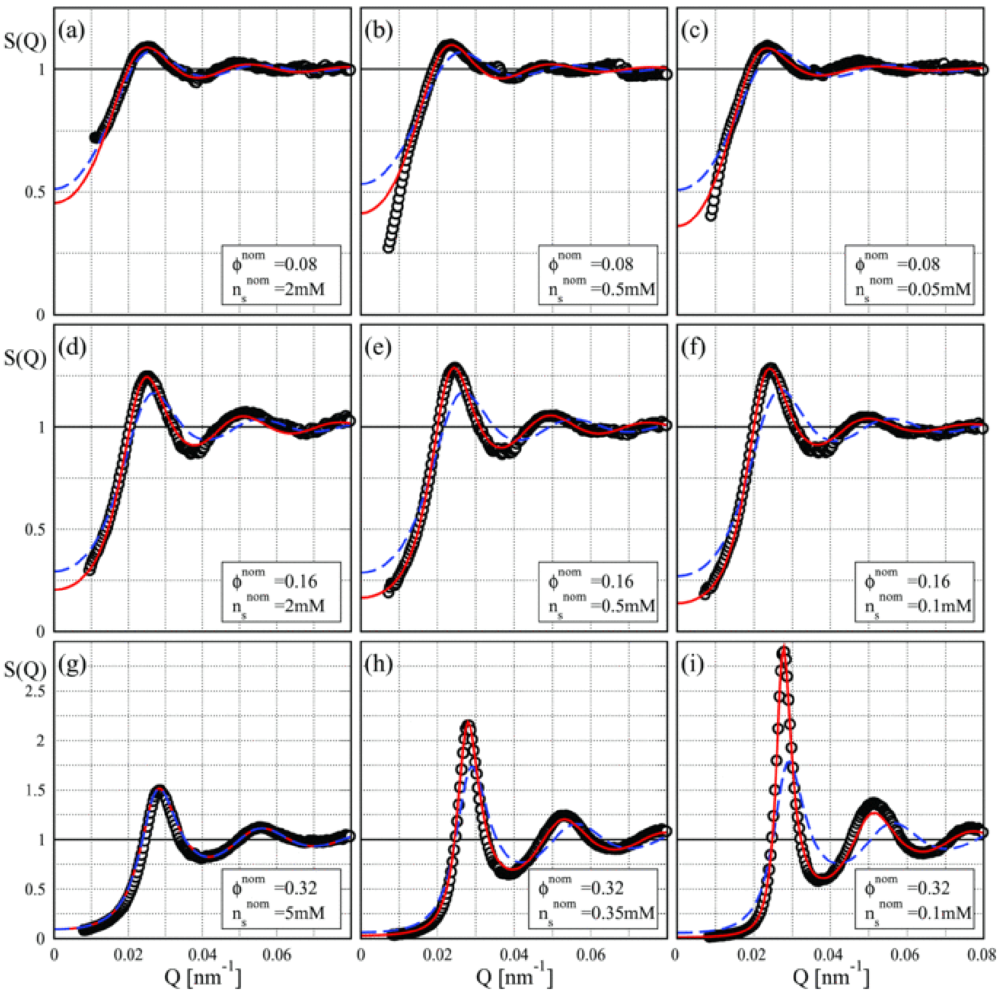
High screening → hard spheres

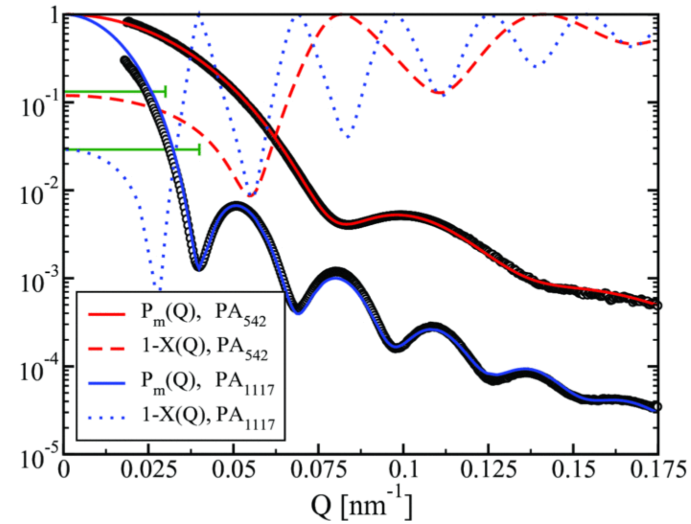






Example 1: Structure and Formfactors from charge stabilized colloids



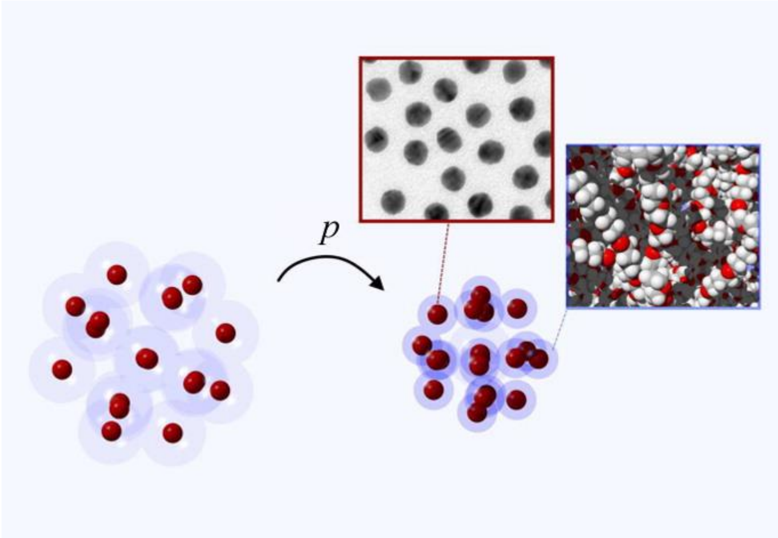


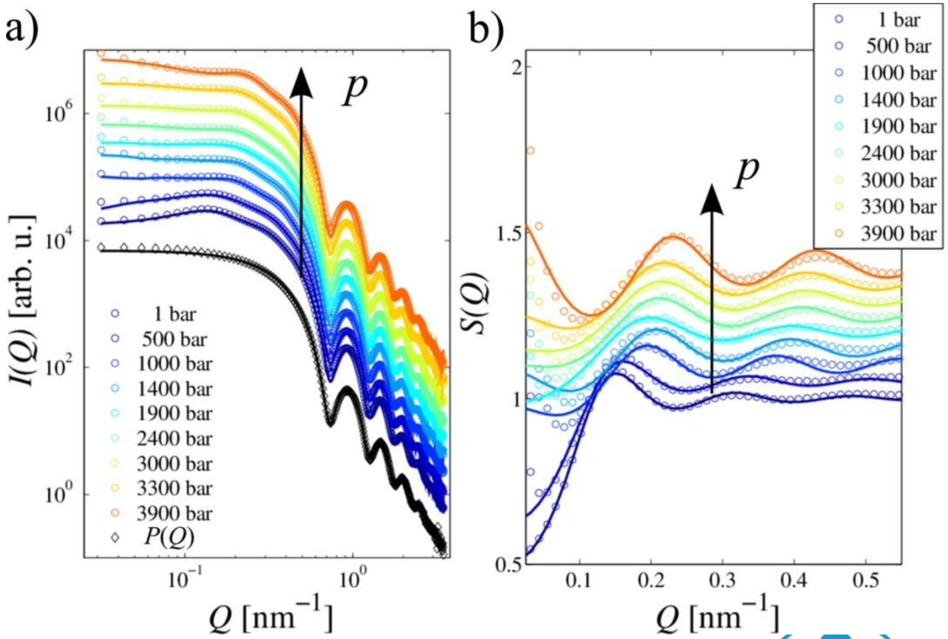
PMMA spheres in water

Westermeier et al. JCP 137, 114504 (2012)

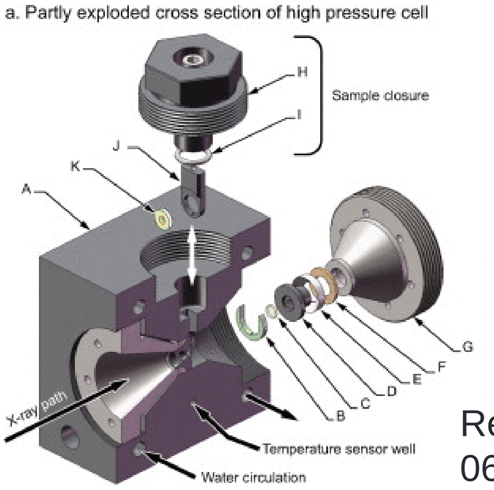
Example 2: High pressure studies

- Structure at high pressures → solid sample chambers (diamond windows)
- X-rays to penetrate diamond windows
- Functionalized core-shell particles at pressures up to 4 kbar: transition from repulsion to attraction (sticky hard spheres!)



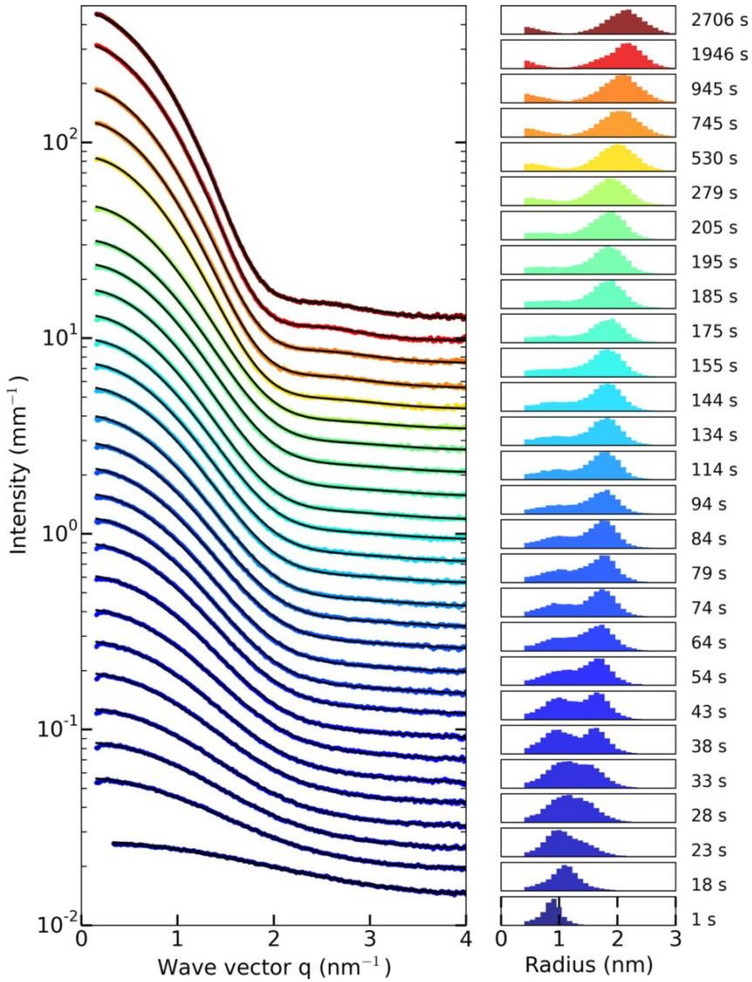


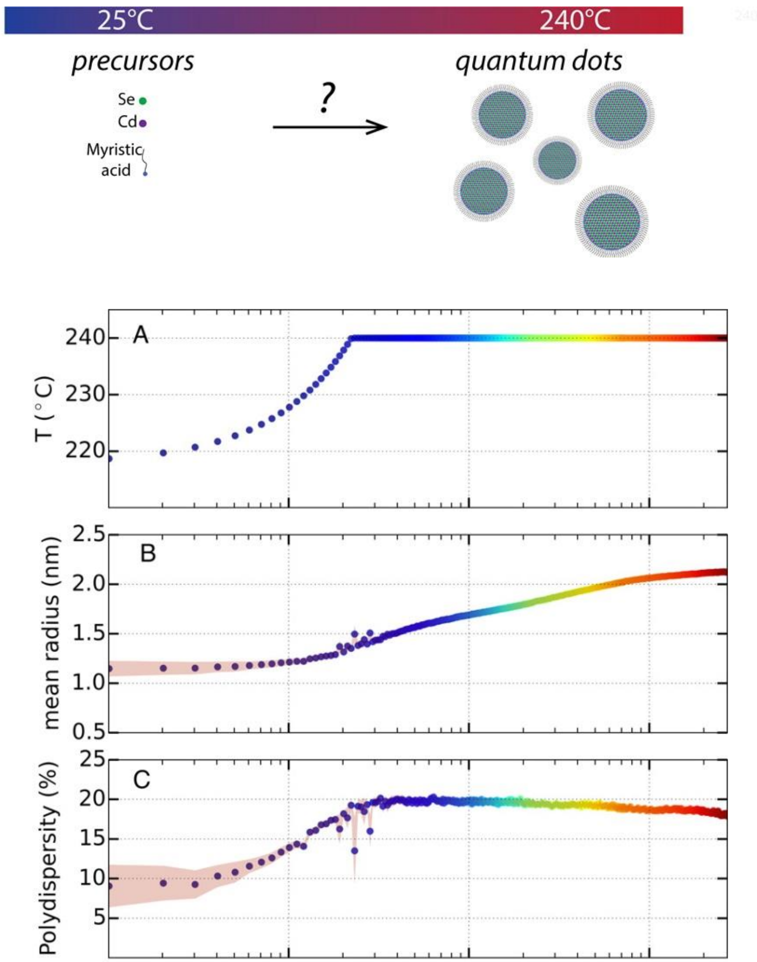
J. Phys. Chem. C 2016, 120, 19856-19861



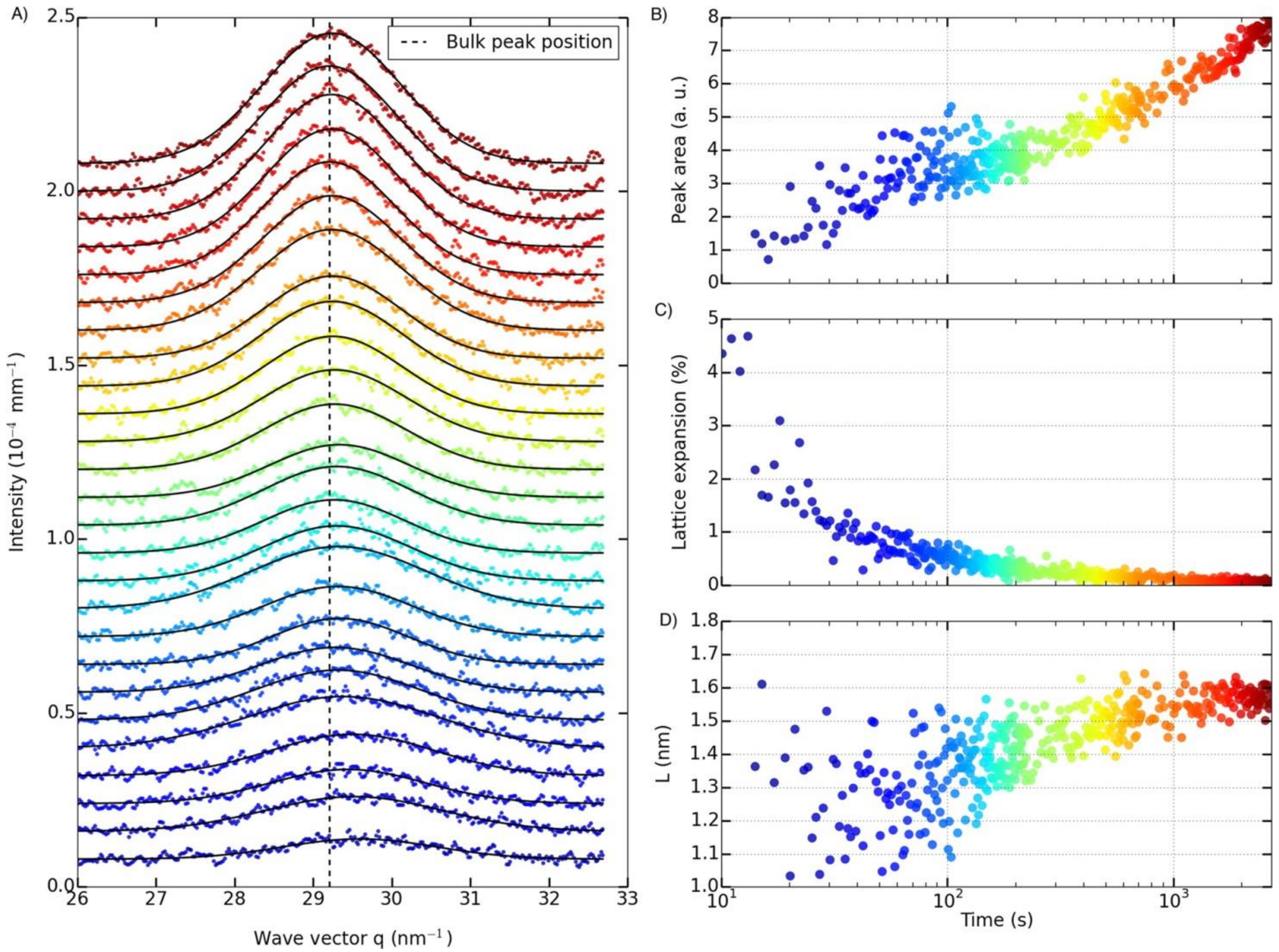
Rev. Sci. Instrum. 81, 064103 (2010).

Example 3: nucleation and growth of quantum dots





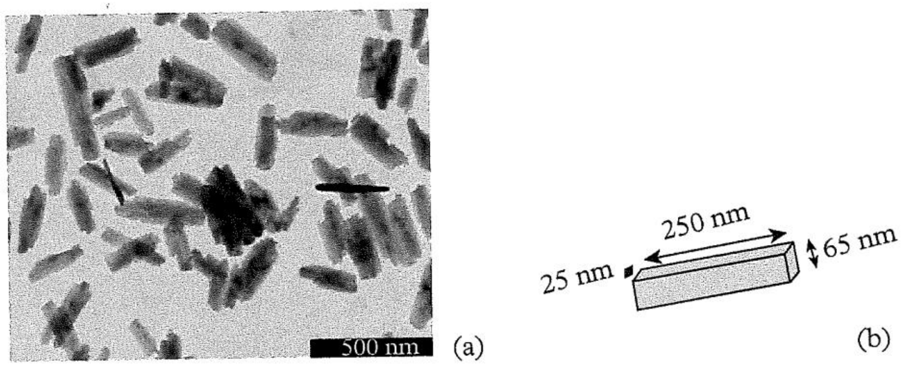
B. Abecassis et al. Nano Lett. 15, 2620 (2015)

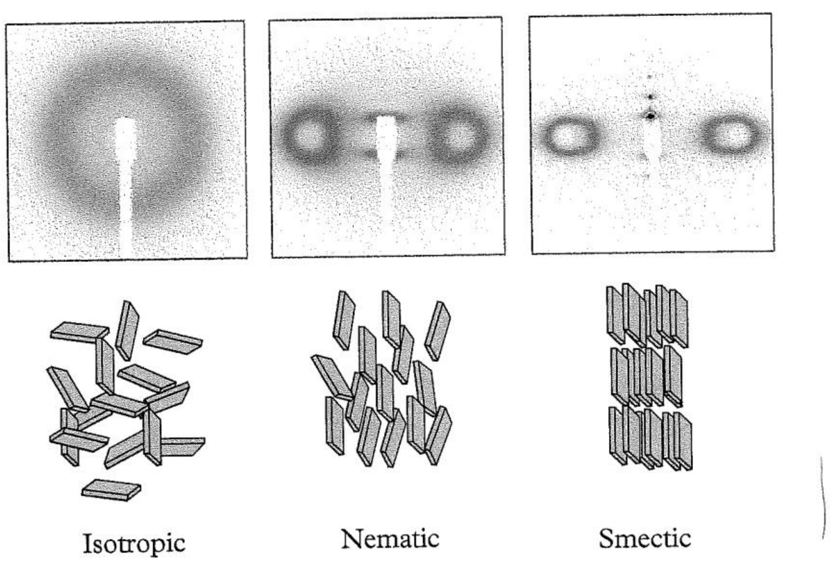


Combination of SAXS & XRD

→ cristallinity of nanoparticle

Example 4: Phase transitions in liquid crystals





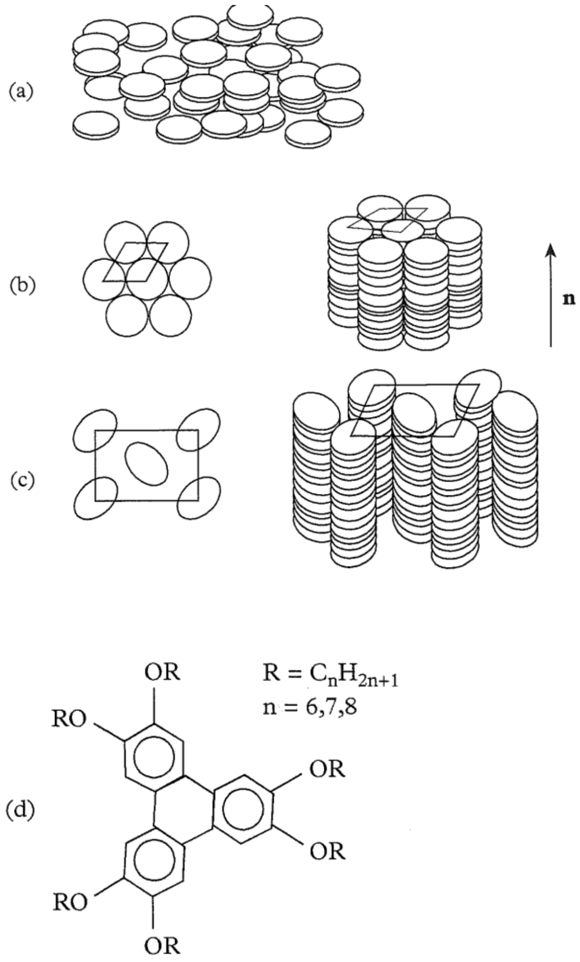
Goethite [α-FeO(OH)] particles in water may form

- Isotropic
- Nematic
- Smectic

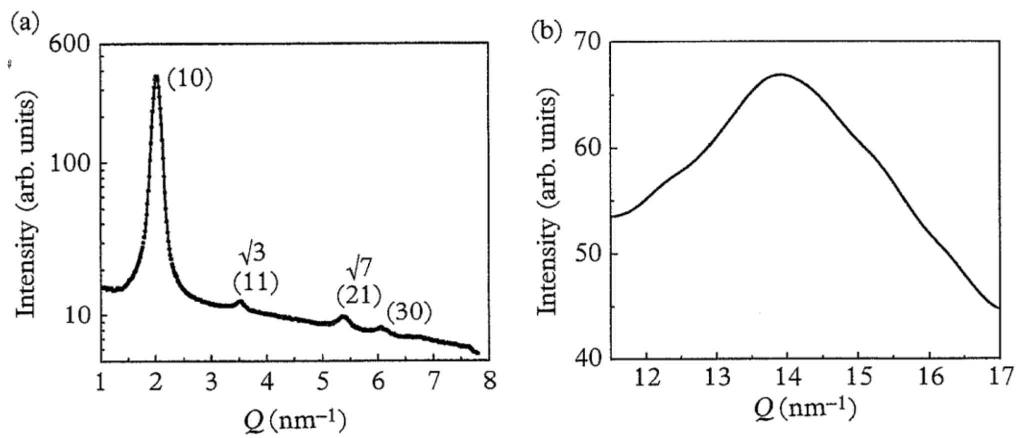
Phases → SAXS

de Jeu: "Basic X-ray scattering for Soft Matter", 2016

Example 4: Phase transitions in liquid crystals



Disc-systems
(a) Discotic nematic phase
(b) Hexagonal columnar phase
(c) Rectangular columnar phase

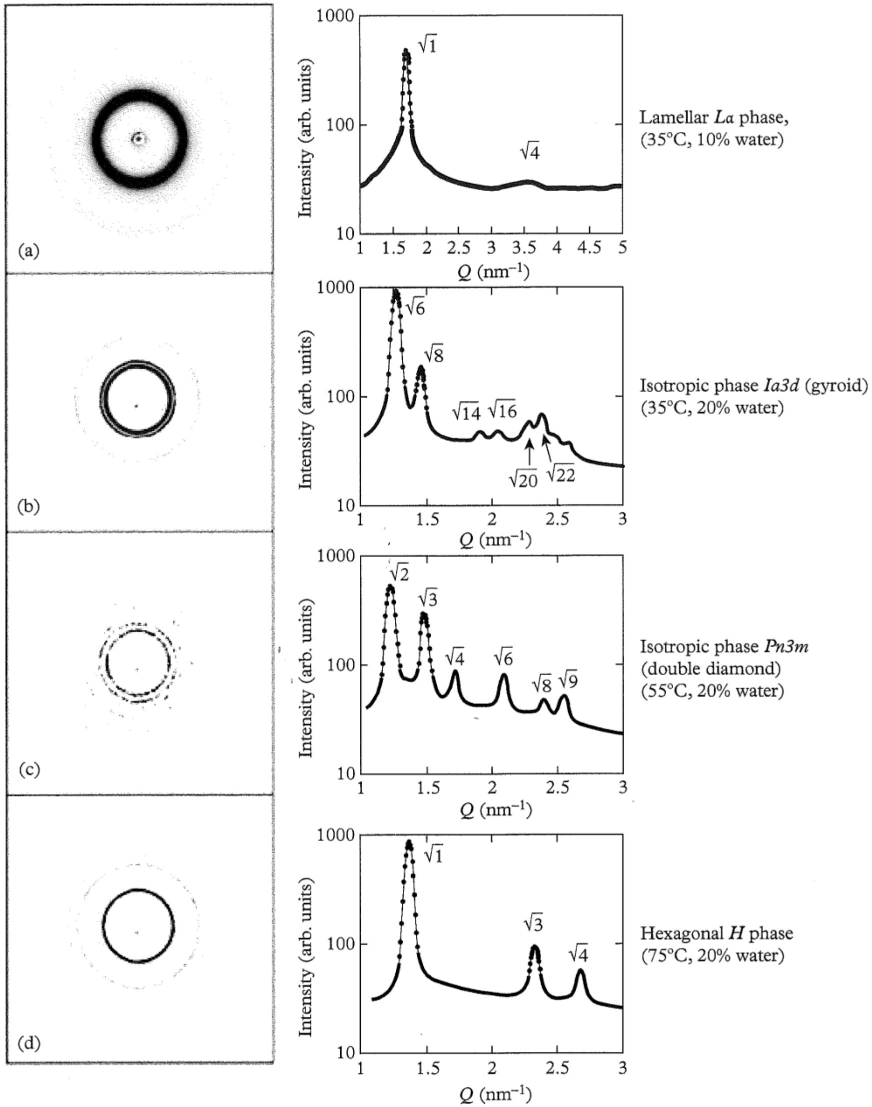


Combined SAXS/WAXS from columnar phase

- SAXS: hexagonal intercolumnar order
- WAXS: disorder inside column

de Jeu: "Basic X-ray scattering for Soft Matter", 2016

Example 4: Phase transitions in liquid crystals



Liquid crystal phase of the system monoglyceride-water

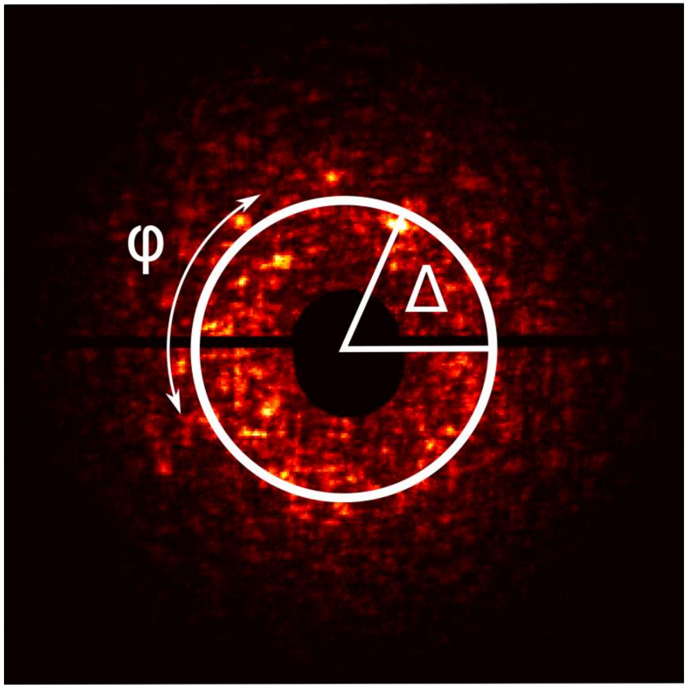
de Jeu: "Basic X-ray scattering for Soft Matter", 2016

Further methods and applications

- Anomalous SAXS → ASAXS
- Scanning SAXS
- Phase transitions and self-assembly
- Time resolved techniques
- SAXS tomography
- BioSAXS
- …

X-ray cross correlation analysis

SAXS: 1D information (typically)
→ How to make use of the 2D information obtained from a 2D scattering pattern?
→ Angular correlations



1D information (standard SAXS)

- $I(\mathbf{q}) = \langle I(q,\varphi)\rangle_{\varphi} = I(q)$

2D information: Angular correlations

- $C(q,\Delta) = \frac{\langle I(q,\phi) I(q,\phi+\Delta)\rangle_{\phi} - \langle I(q,\phi)\rangle_{\phi}^{2}}{\langle I(q,\phi)\rangle_{\phi}^{2}}$, i.e. correlations of fluctuations
- Coherent X-rays
- Two possibilities:
  - Solve structures in solution
  - Hidden symmetries

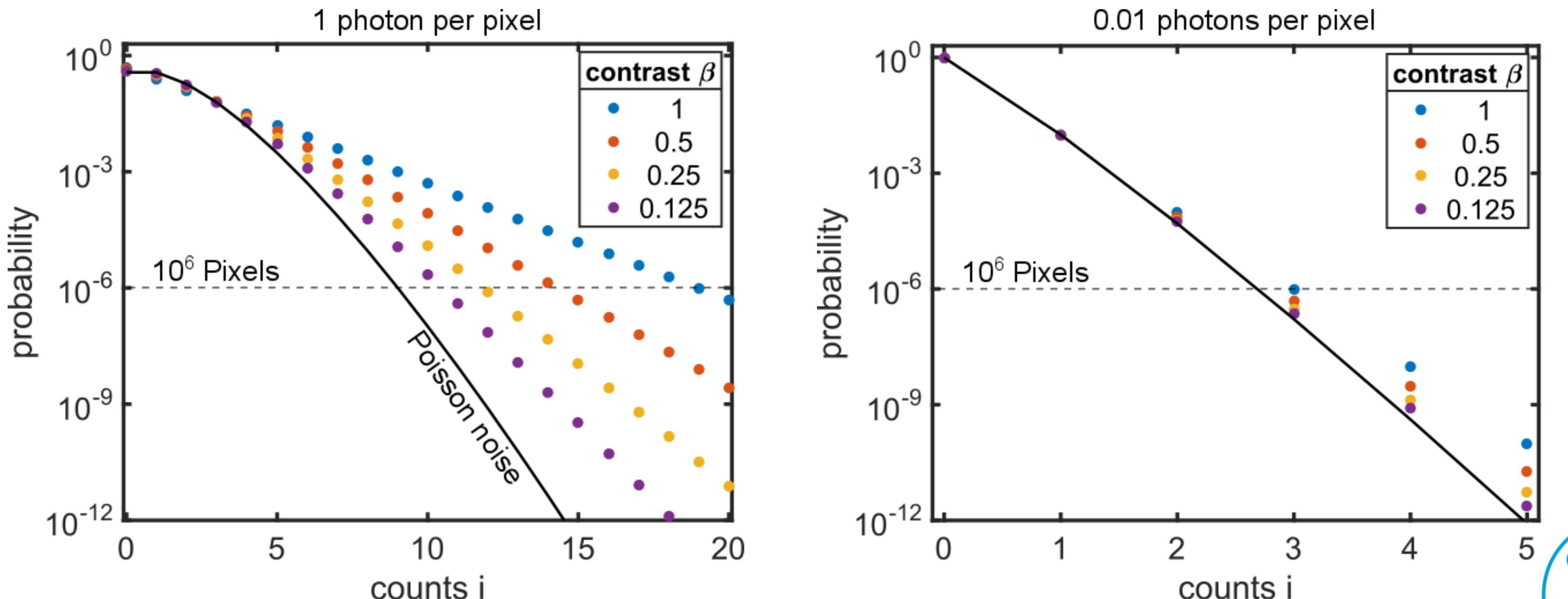
Reminder: coherent X-rays

- Correlations of speckles → coherent X-rays
- Reminder: degree of coherence of partial coherent source → speckle contrast

$$\beta = \frac{\sigma^2}{\langle I \rangle^2} = \frac{\mathrm{var}(I)}{\langle I \rangle^2} \leq 1$$

- Intensity follows Gamma distribution (Lecture 10)
- Low intensities → Poisson noise → Negative binomial probability function

$$P_{nb}(i) = \frac{\Gamma(i+M)}{\Gamma(M)\Gamma(i+1)} \left(1 + \frac{M}{\langle i \rangle}\right)^{-i} \left(1 + \frac{\langle i \rangle}{M}\right)^{-M}$$, with number of modes $M = \frac{1}{\beta}$



Reminder: coherent X-rays

Excercise: Contrast calculation at low count rates

X-ray cross correlation analysis

Consider coherent X-ray scattering experiment in transmission geometry (e.g. SAXS) with 2D detector on disordered sample of N identical particles

$$A_j(\mathbf{q}) = \int \rho_j(\mathbf{r}) e^{i\mathbf{qr}} \mathrm{d}\mathbf{r} \;\rightarrow \mathrm{I}(\mathbf{q}) = \sum_{j_1,j_2=1}^{N} e^{i\mathbf{qR}(j_1,j_2)} A_{j_1}^*(\mathbf{q}) A_{j_2}(\mathbf{q})$$

$$= \sum_{j_1,j_2=1}^{N} \int\int \rho_{j_1}^*(\mathbf{r_1}) \rho_{j_2}(\mathbf{r_2}) e^{i\mathbf{q}(\mathbf{R}(j_1,j_2)+\mathbf{r}_{21})} d\mathbf{r_1} d\mathbf{r_2}$$

Partially coherent illumination and dilute system (particles distance > coherence length) → interparticle correlations can be neglected:

$$I(\mathbf{q}) = \sum_{j=1}^{N} I_j(\mathbf{q}) = \sum_{j=1}^{N} \left|A_j(\mathbf{q})\right|^2$$

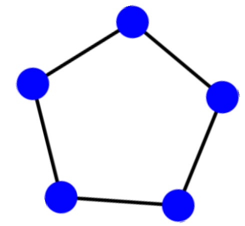
Angular information: Fourier decomposition

$$I(\mathbf{q}) = I(q,\phi) = \sum_{l=-\infty}^{\infty} \hat{\mathrm{I}}_\ell(q) e^{il\phi} \; ; \qquad \hat{\mathrm{I}}_\ell(q) = \frac{1}{2\pi} \int_0^{2\pi} I(q,\phi) e^{-i\ell\phi} \, \mathrm{d}\phi$$

X-ray cross correlation analysis

Now consider 2D disordered system in the dilute limit, e.g. pentagonal arrangement of particles (polar coordinates, R_0 radius of pentagon, $\theta_j = \frac{2\pi j}{5}$)

$$\rho(r,\theta) = \frac{\delta(r - R_0)}{R_0} \sum_{j=1}^{5} \delta(\theta - \theta_j)$$



Expansion of scattering amplitude in Fourier series yields

$$A(q,\phi) = \sum_{\ell=-\infty}^{\infty} \hat{a}_\ell(q) e^{il\phi} \quad (1)$$

with Fourier coefficients

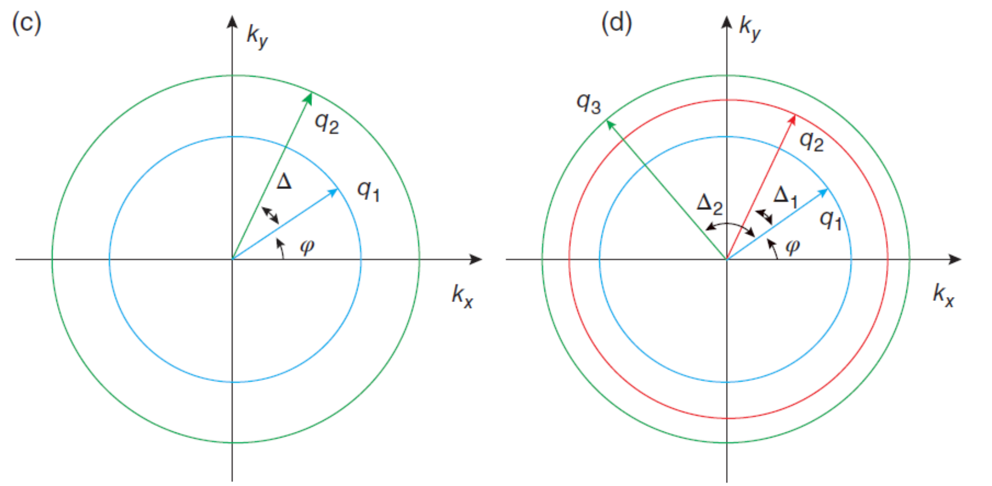
$$\hat{a}_\ell(q) = i^{-\ell} J_\ell(qR_0) \sum_{j=1}^{5} e_j^{il\theta_j} \quad (2)$$

- Pentagonal symmetry: only contribution if $\ell = 0 \bmod 5$ in (2).
- Odd terms cancel out pairwise (e.g. $\ell = 5$ and $\ell = -5$) in (1) → Friedel's law!
- Only contributions with $\ell = 0 \bmod 10$
- $F_l(q) \propto J_\ell(qR_0)$ → higher-order terms at large q

X-ray cross correlation analysis

- Corresponding correlation function $C(q,\Delta) = \frac{\langle I(q,\phi)I(q,\phi+\Delta)\rangle_\phi - \langle I(q,\phi)\rangle_\phi^2}{\langle I(q,\phi)\rangle_\phi^2}$ with Fourier coefficients $\hat{c}_\ell(q) = \left|\hat{\mathrm{I}}_\ell(q)\right|^2$ (Wiener–Khinchin theorem)

- Correlations between different q possible

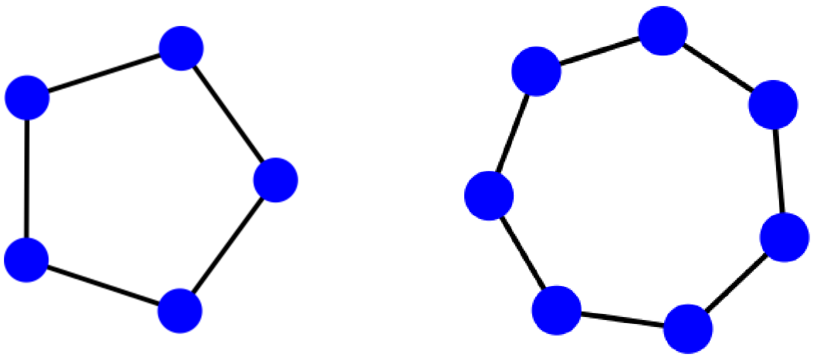


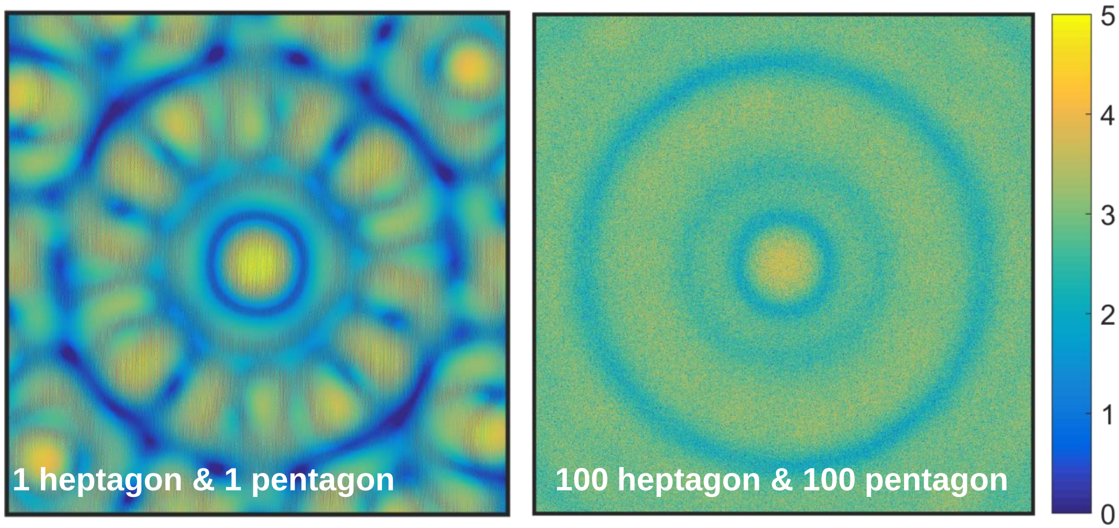
Adv. Chem. Phys. 161, 1 (2016)

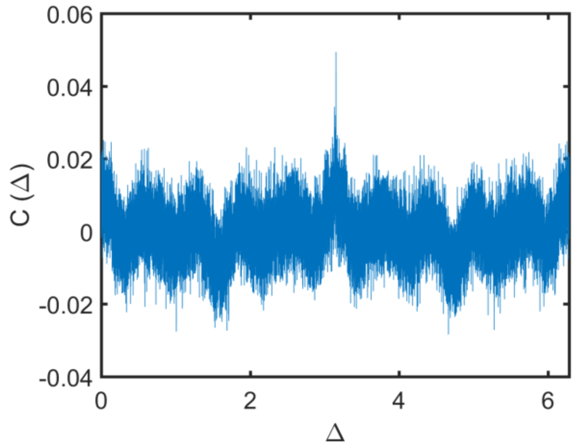
- 3D systems: curvature of Ewald sphere → odd symmetries

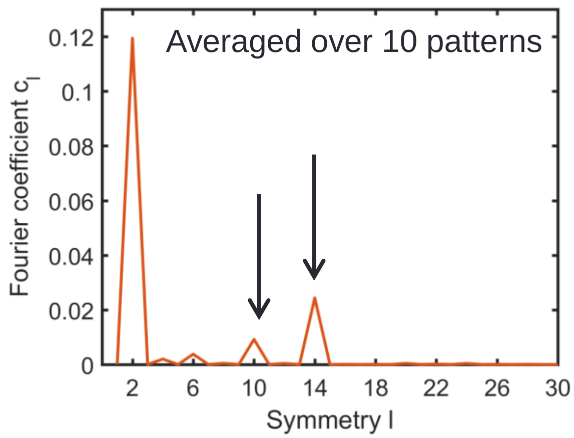
X-ray cross correlation analysis

2D model system: Heptagons and Pentagons



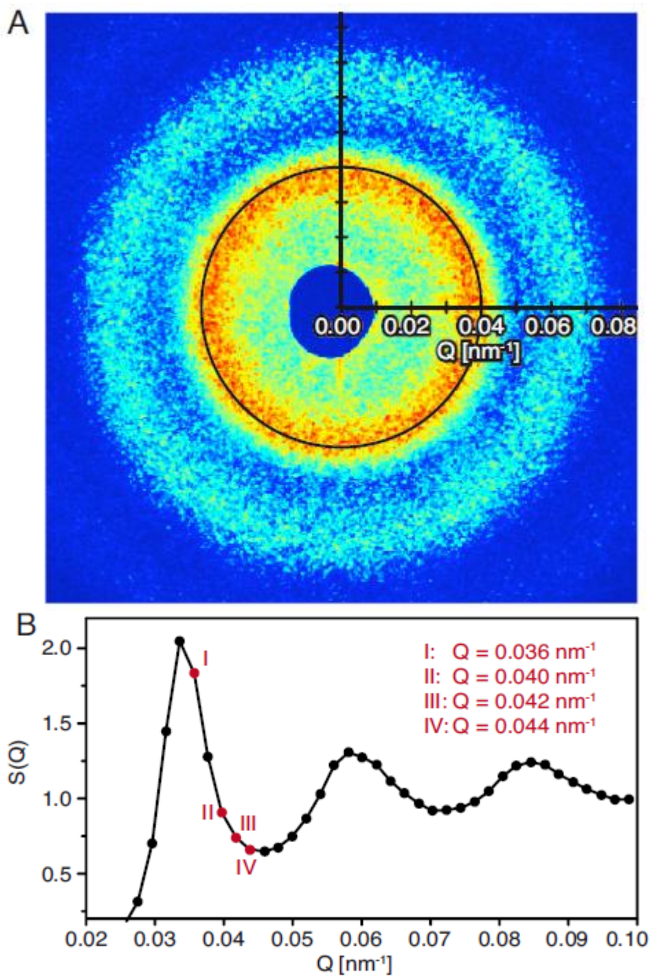


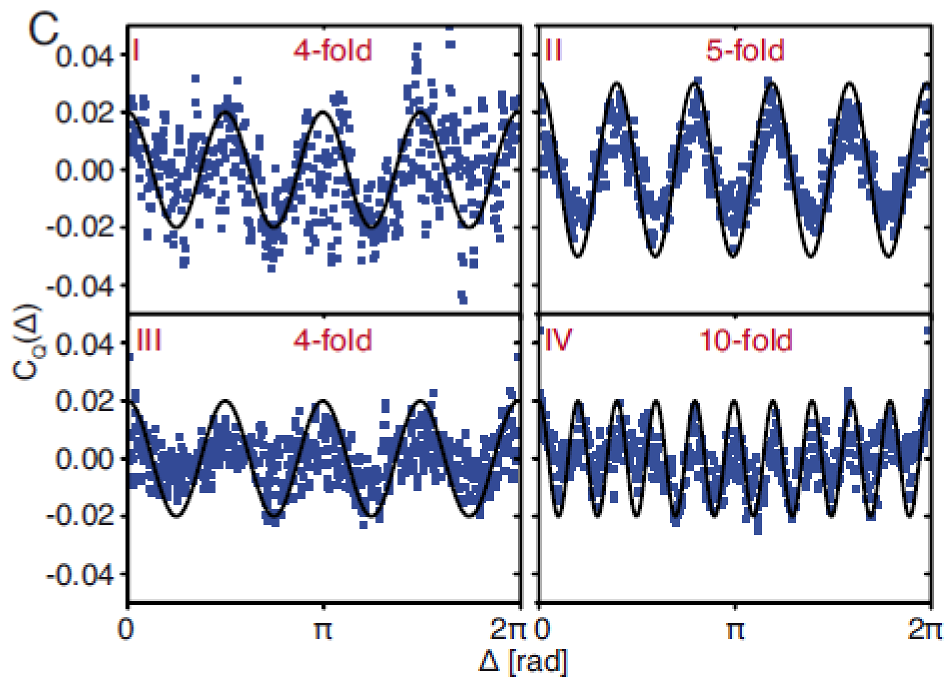




X-ray cross correlation analysis

Hard-sphere glass

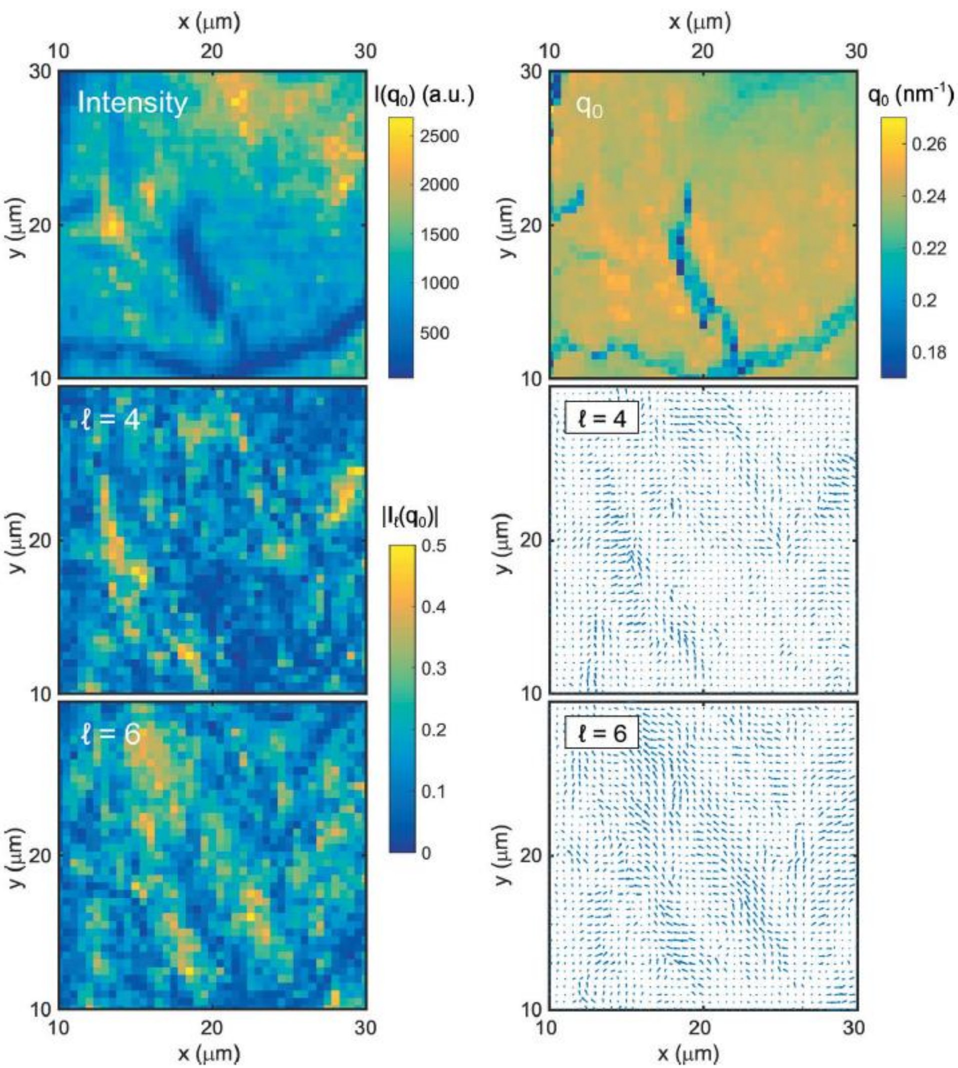




→ Hidden symmetries
→ Structural information beyond SAXS

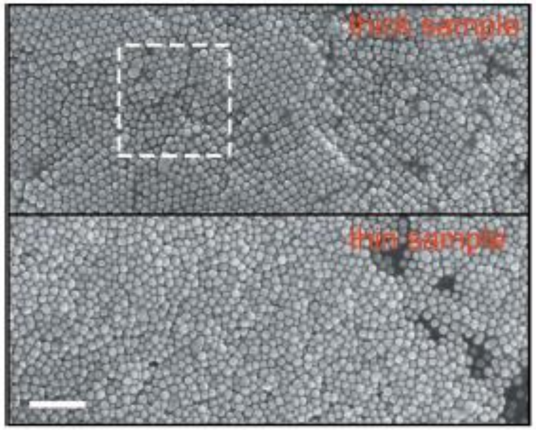
PNAS 109, 11511 (2009)

XCCA & microscopy



Thin colloidal films

Orientational order with 500 nm resolution



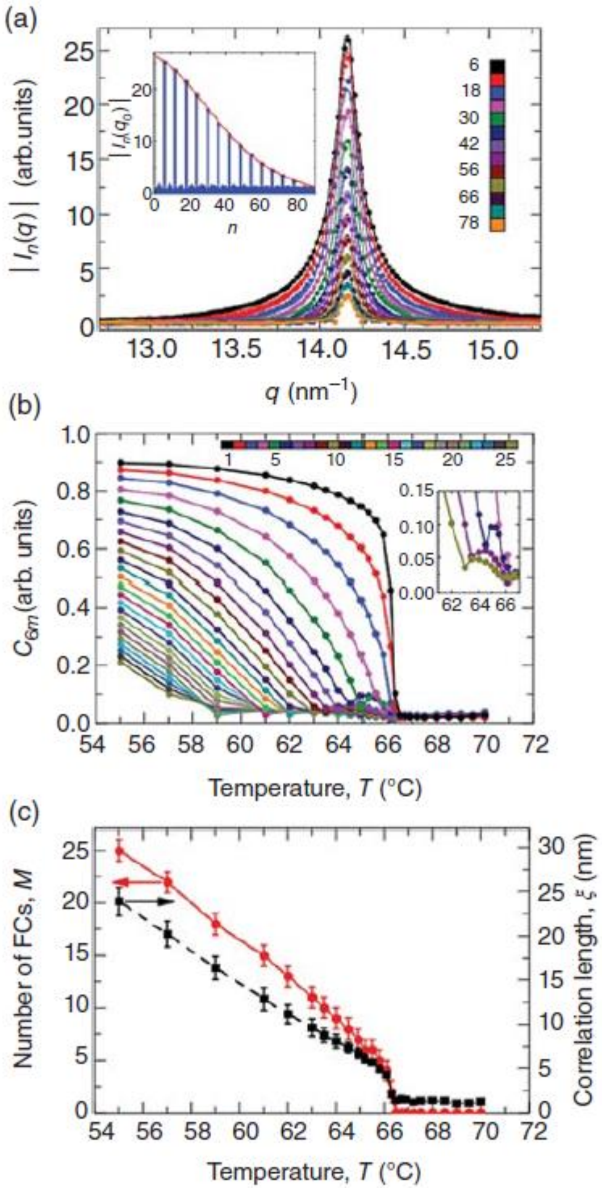
IUCrJ 5, 354 (2018)

Liquid crystals

High number of symmetries → strongly developed hexatic order

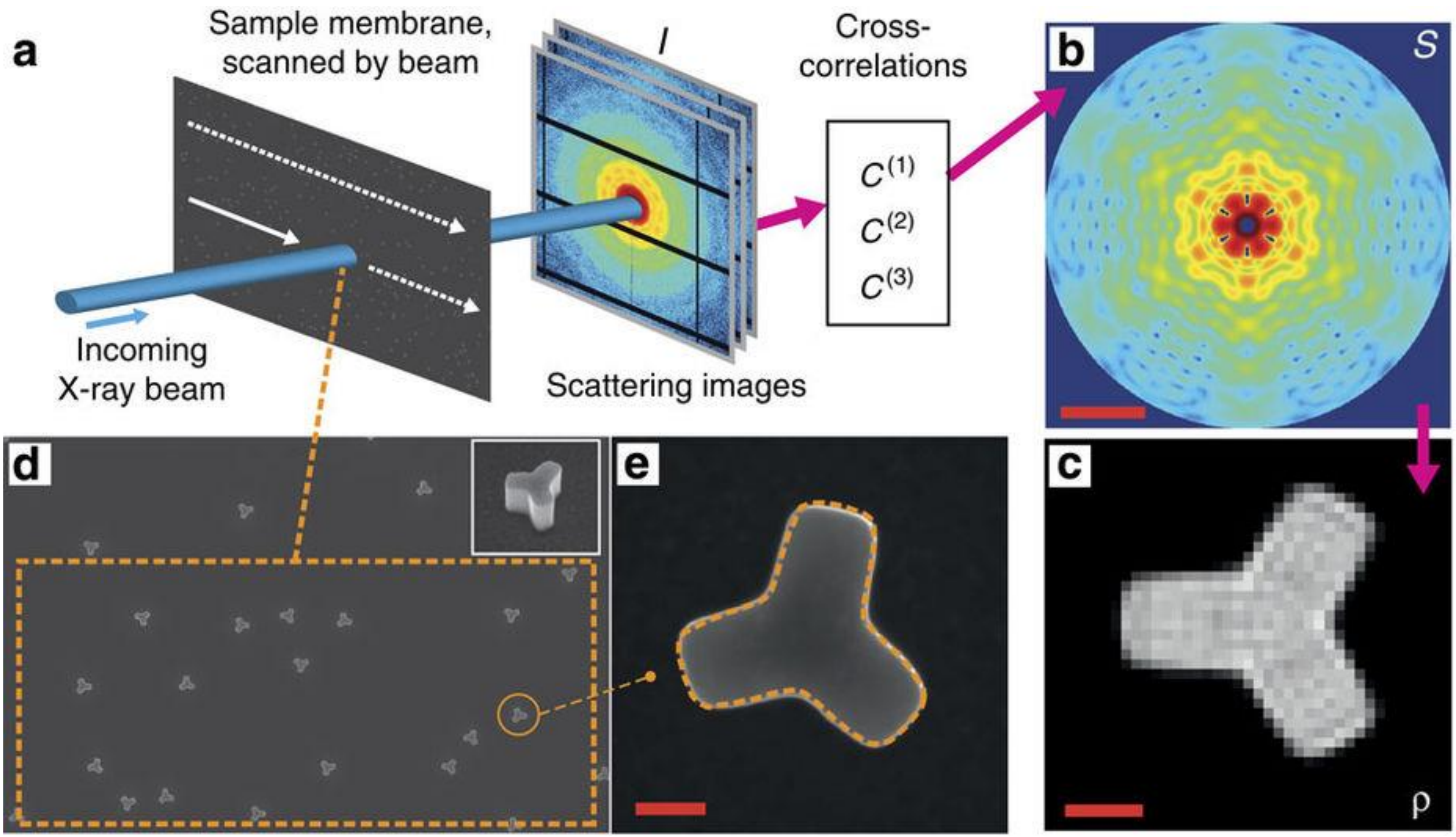
Measure of correlation length

XCCA to provide measure of degree of order and as order parameter for phase transitions



Adv. Chem. Phys. 161, 1 (2016)

XCCA – sample reconstruction



Nat. Comm. 4, 1647 (2013)