

# Methoden moderner Röntgenphysik: Streuung und Abbildung

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Lecture 3	Vorlesung zum Haupt- oder Masterstudiengang Physik, SoSe 2018 G. Grübel, A. Philippi-Kobs, O. Seeck, L. Frenzel, F. Lehmkuhler, M. Martins, W. Wurth		
Location	Lecture hall AP, Physics, Jungiusstraße		
Date	Tuesdays	13:00 - 14:30	(starting 3.4.)
	Thursdays	8:30 - 10:00	(until 12.7.)



# Methoden moderner Röntgenphysik: Streuung und Abbildung

## Part I:

### Basics of X-ray Physics

by Gerhard Grübel (GG)

- [ 3.4.] Organisation and Introduction
- [ 5.4.] X-ray Scattering Primer
- [10.4.] Sources of X-rays, Synchrotron Radiation
- [12.4.] Refraction and Reflection
- [17.4.] Kinematical Scattering Theory (I)
- [19.4.] Kinematical Scattering Theory (II), Applications
- [24.4.] Small Angle Scattering and Soft Matter
- [26.4.] Anomalous Scattering
- [ 3.5.] Introduction: Coherence I
- [ 8.5.] Coherence II; Applications of Coherent X-ray Beams



# Methoden moderner Röntgenphysik II: Streuung und Abbildung

## Part I:

### Basics of X-ray Physics

by Gerhard Grübel (GG)

#### Introduction

Overview, Introduction to X-ray Scattering

#### X-ray Scattering Primer

Elements of X-ray Scattering

#### Sources of X-rays, Synchrotron Radiation

Laboratory Sources, Accelerator Bases Sources



#### Reflection and Refraction from Interfaces

Snell's Law, Fresnel Equations

#### Kinematical Diffraction (I)

Diffraction from an Atom, a Molecule, from Liquids, Glasses, ...

#### Kinematical Diffraction (II)

Diffraction from a Crystal, Reciprocal Lattice, Structure Factor, ...



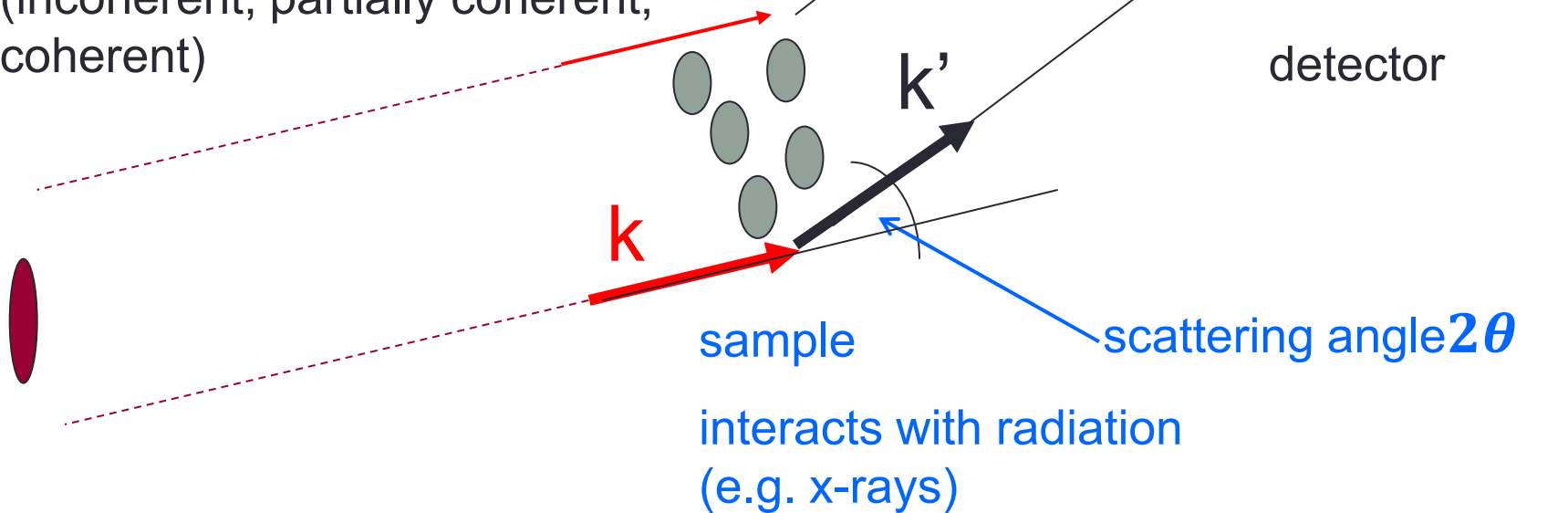
# Set-up for Scattering Experiments

source (visible light, x-rays,...)

source parameters: source

size,  $\lambda$ ,  $\frac{\Delta\lambda}{\lambda}$ ...

coherence properties:  
(incoherent, partially coherent,  
coherent)



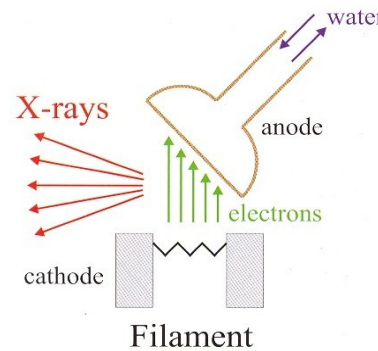
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# Source of X-Rays

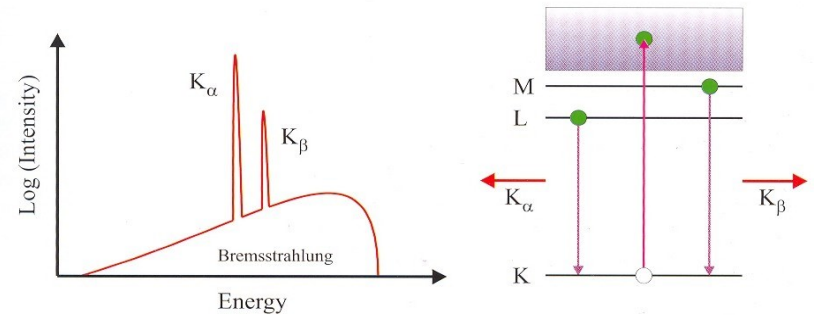
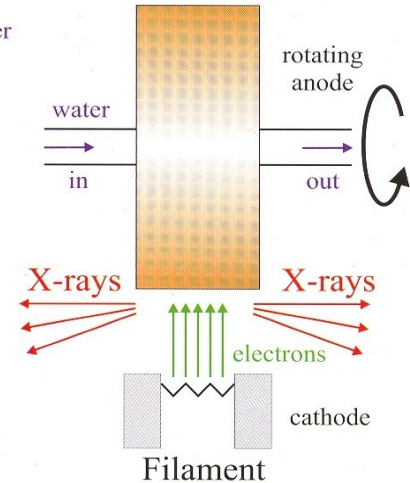
- 1895 Discovered by W.C. Röntgen
- 1912 First diffraction experiment (v. Laue)
- 1912 Coolidge tube (W.D. Coolidge, GE)
- 1946 Radiation from electrons in a synchrotron, GE, Physical Review, 71,829 (1947)



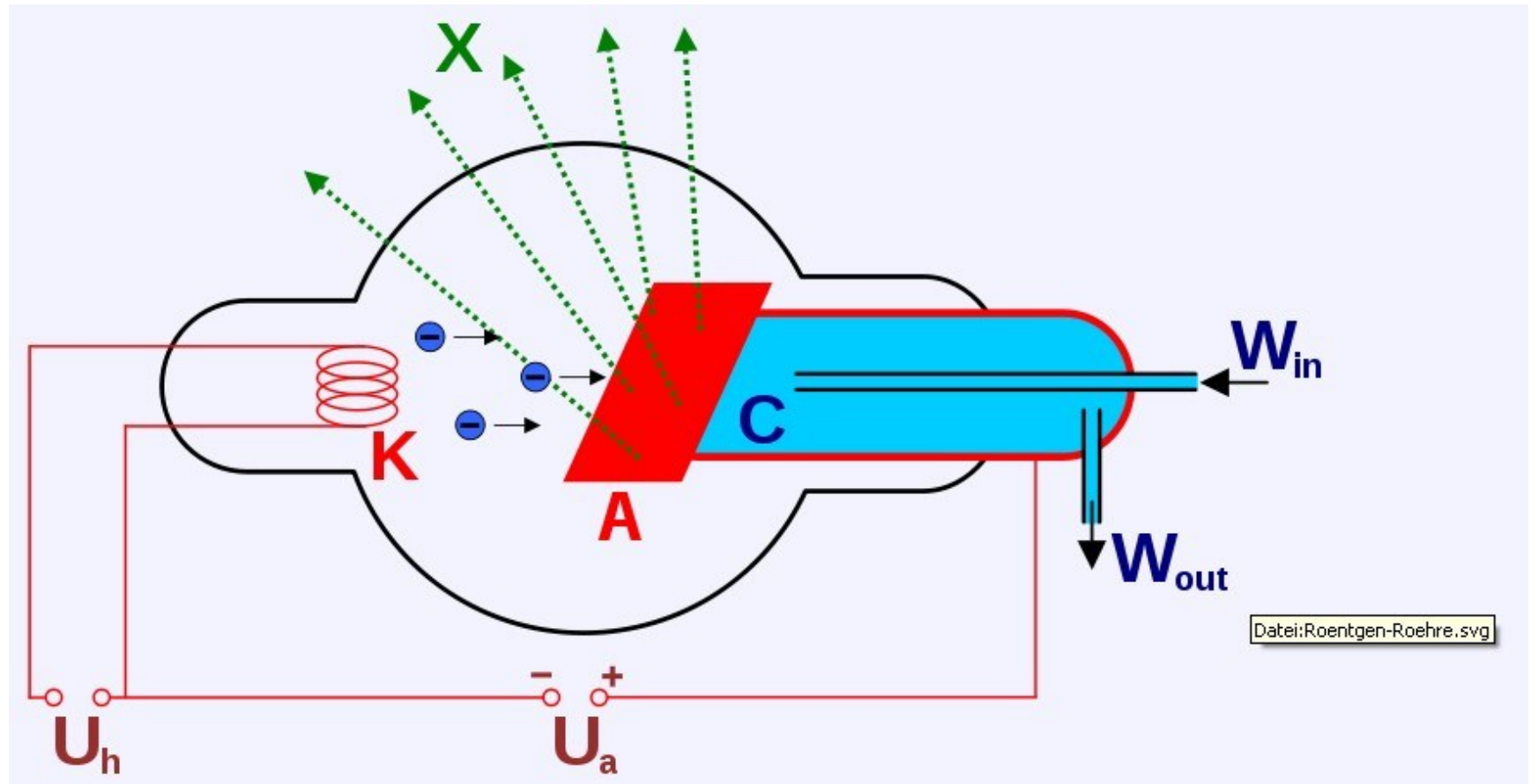
Coolidge Tube



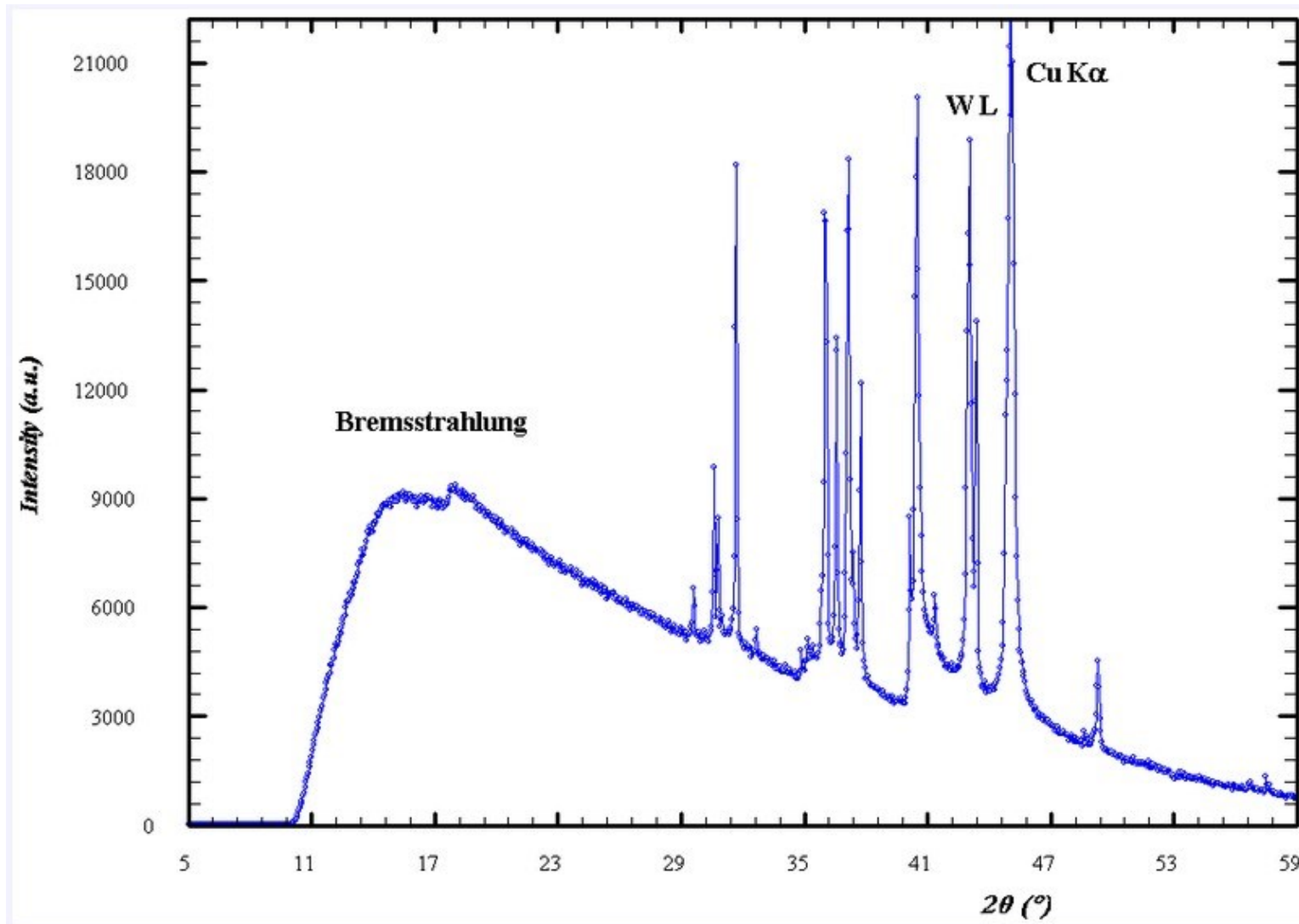
Rotating Anode



# X-Ray Tube

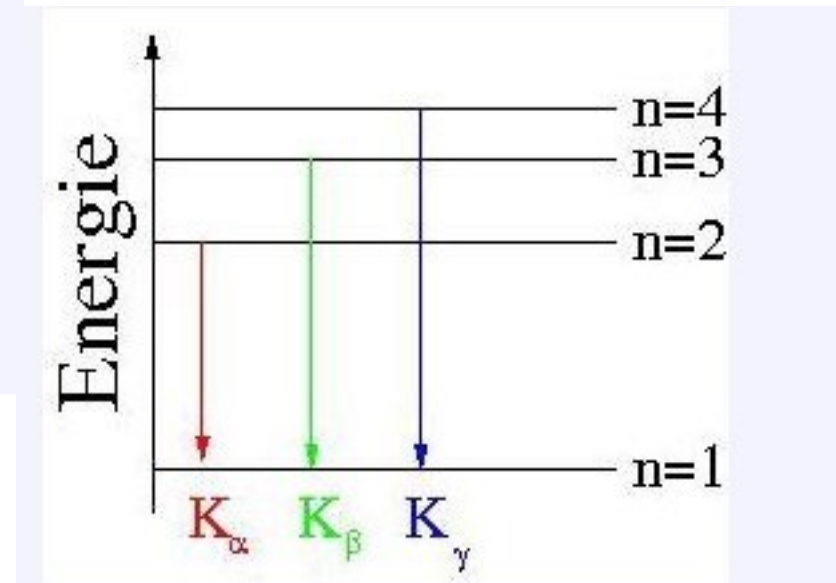
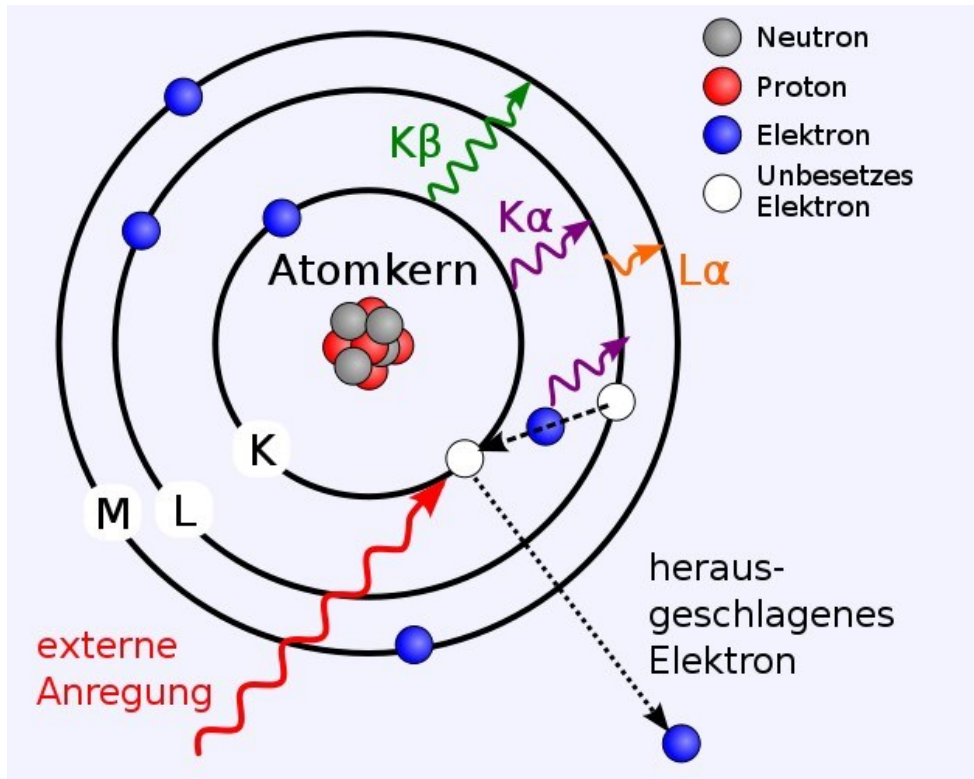


# X-Ray Tube



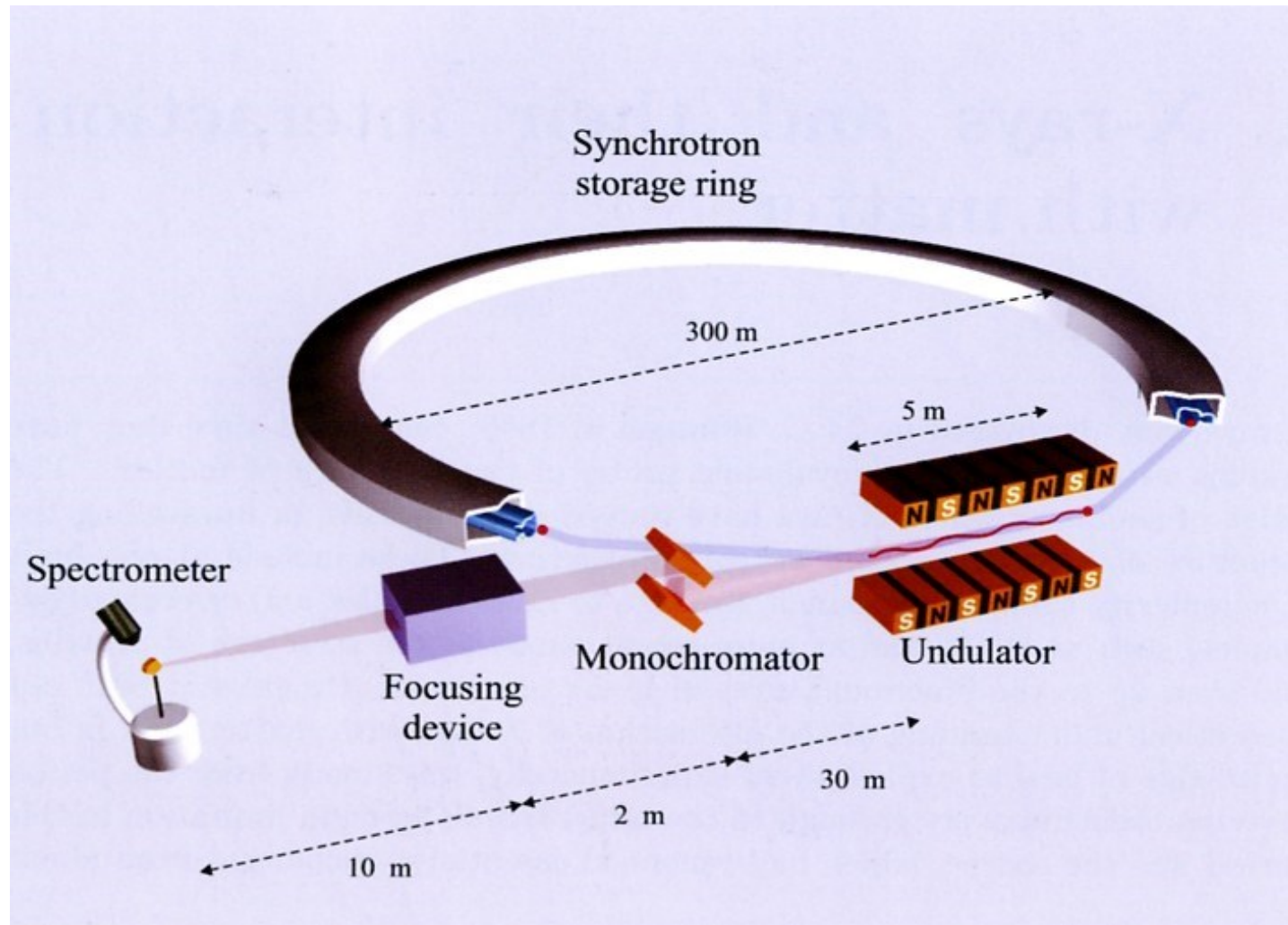


# X-Ray Tube





# Synchrotron Radiation Storage Ring



# Circular Accelerators

Cyclotron

Microtron

Synchrotron

Storage Ring

# Cyclotron

- Proposed in 1930 by E.O. Lawrence
- Electrons circulate in a homogeneous magnetic field  $B$
- Frequency for one cycle is given by

$$\omega_c = \left(\frac{e}{m}\right) B_Z$$

- For non-relativistic electrons  $\omega_c$  is independent of the velocity  $v$

$$\left(\frac{v}{c} < 0.15\right)$$

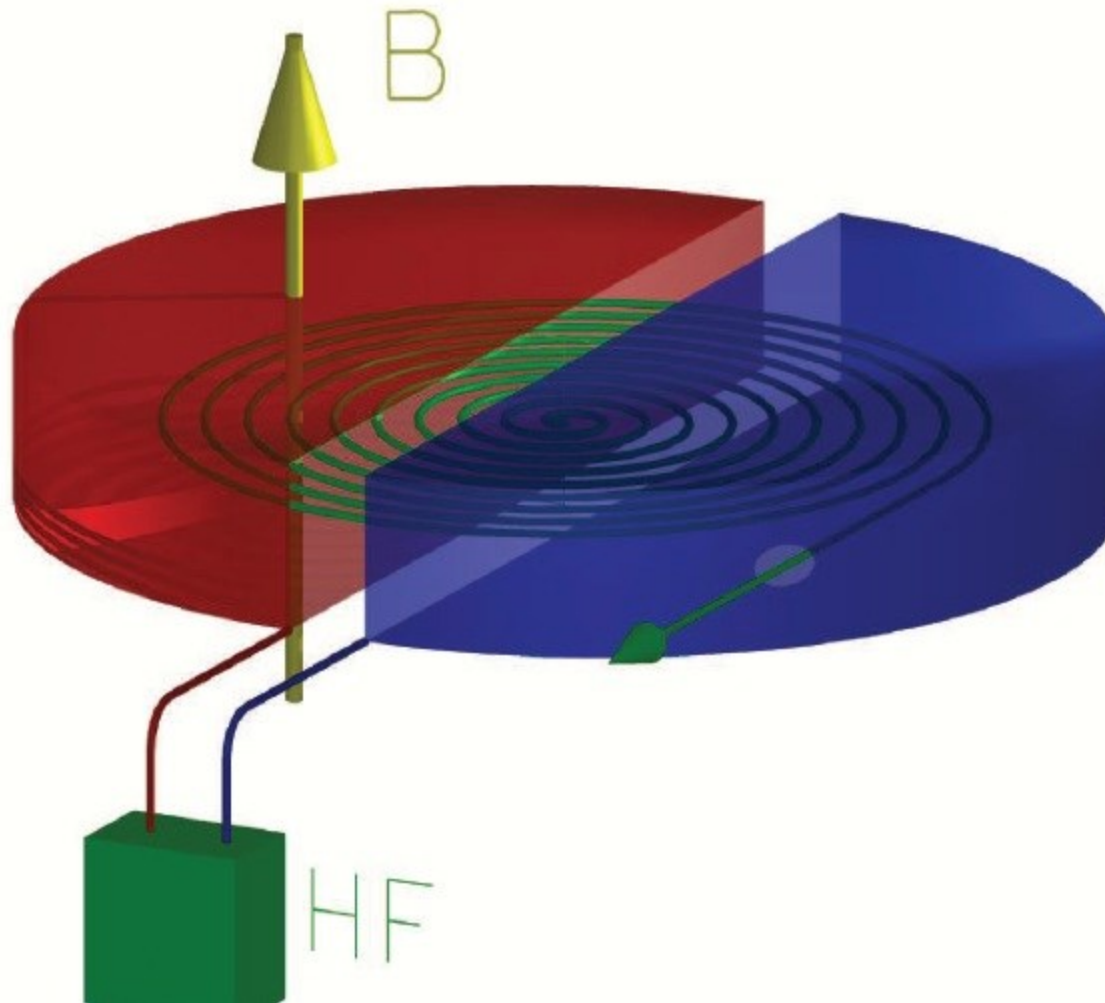
- At high energies the mass changes and the frequency of the field needs to be adapted.

Example:  $E_{\text{kin}} = 10\text{keV} = eU = m_e \frac{v_e^2}{2} \Rightarrow \frac{v_e}{c} = 0.2!$

- Electrons at 10 keV are already relativistic!



# Cyclotron



# Cyclotron



Zyklotron der  
Uni Bonn

# Microtron

- Acceleration with a linear accelerator
- Circular bend similar to a cyclotron
- Bending radius  $R$  in magnetic field  $B$  for relativistic particles

Lorentz Force = Radial Force

$$evB = m \frac{v^2}{R}$$

$$\Rightarrow R = \frac{mv}{eB} = \frac{vmc^2}{ec^2B} = \left( \frac{v}{ec^2B} \right) E$$

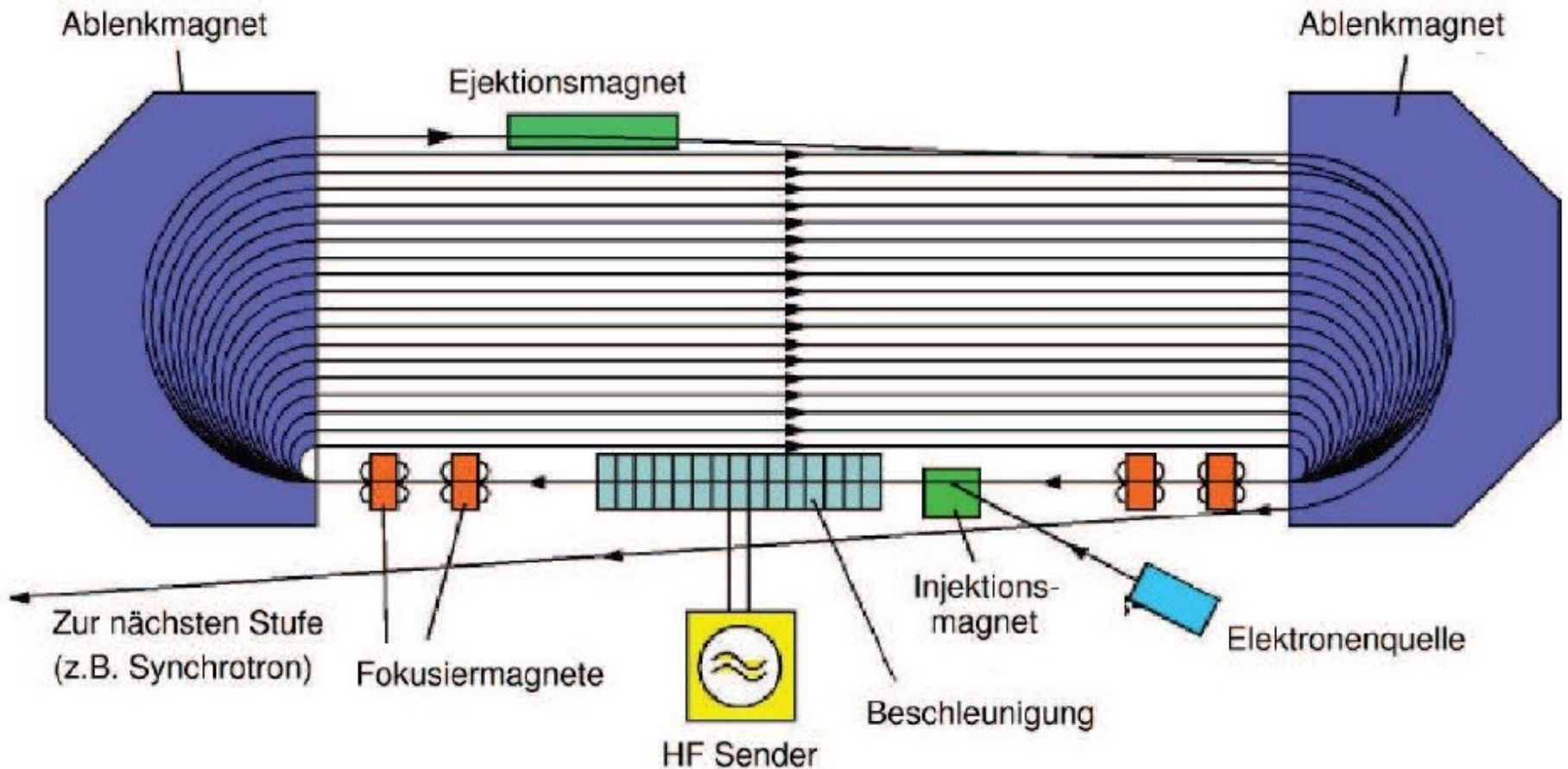
- Such Acceleration that electrons are in phase with RF field
- Energies up to 100 MeV can be reached

Example: BESSY II



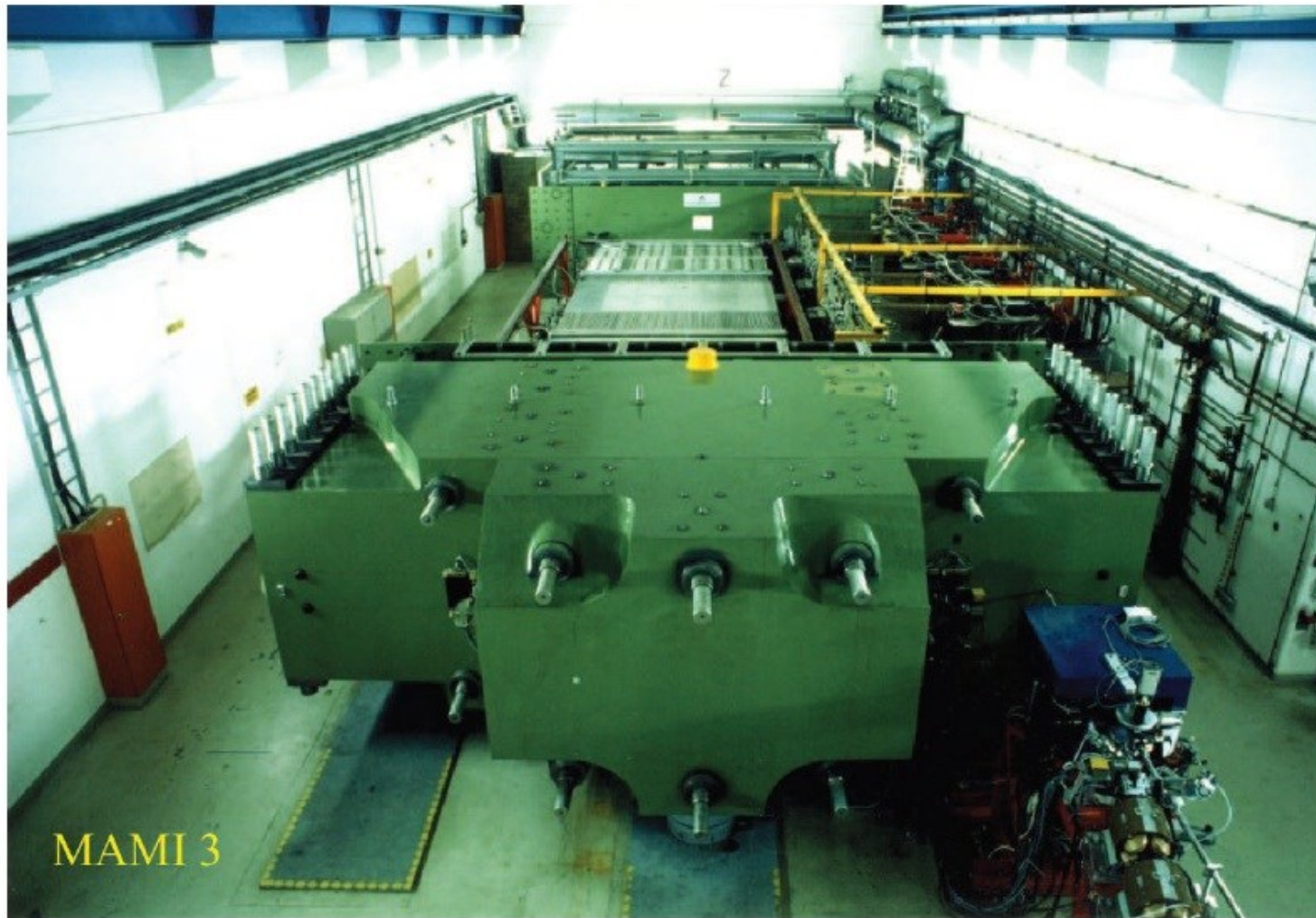


# Microtron





# Microtron



# Synchrotron

- For relativistic particles  $v \cong c$  in a B field, the radius is given by

$$R = \frac{E}{ecB}$$

- For  $E > 1 \text{ GeV}$  and  $B = 5\text{T}$  :  $R > \text{several meter}$
- Technically difficult
- Enforce trajectory with constant radius

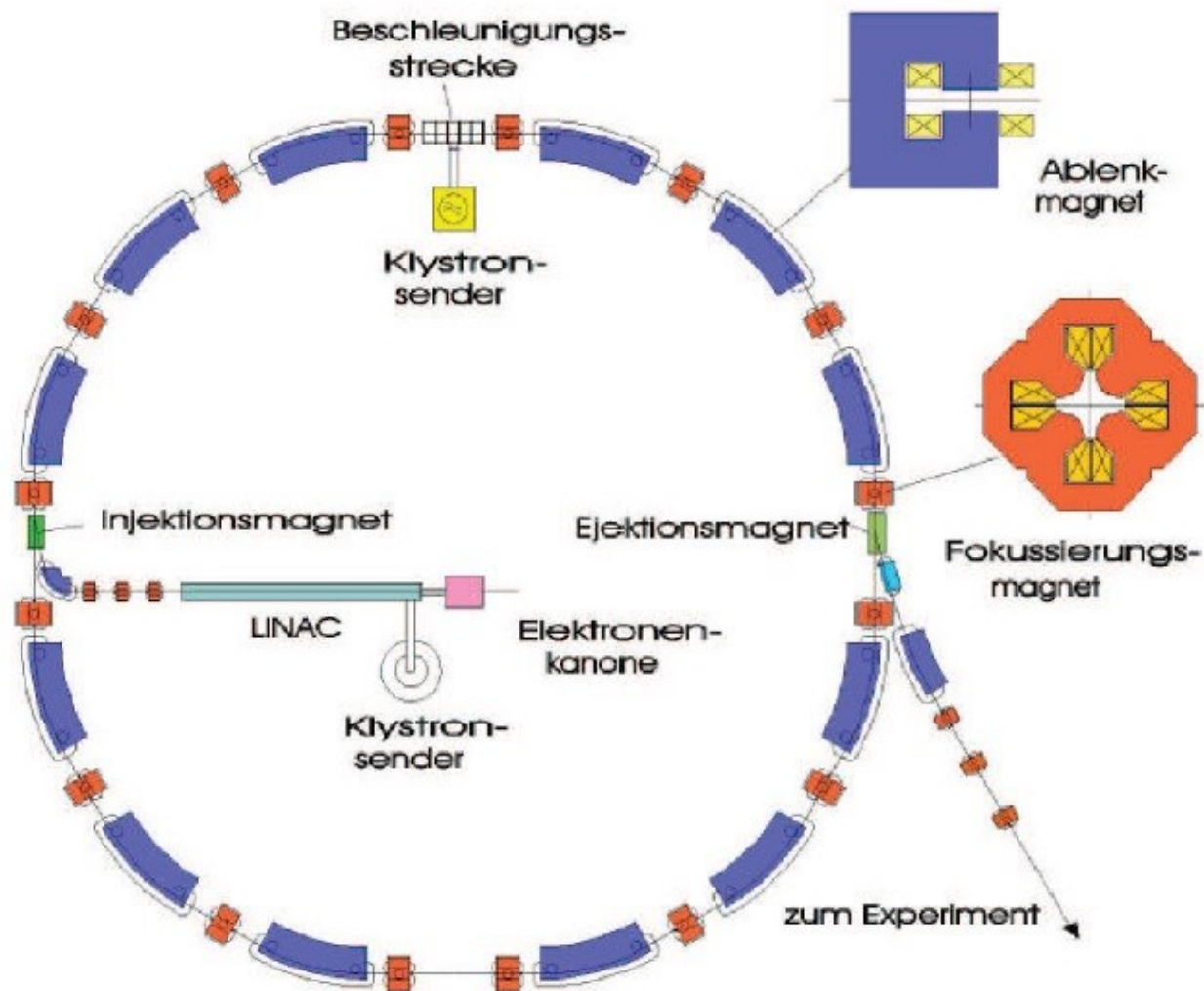
Bends in small , local magnets

$\frac{E}{B} = \text{const.} \Rightarrow \text{synchronous ramping of E and B}$

$\Rightarrow$  Synchrotron



# Synchrotron

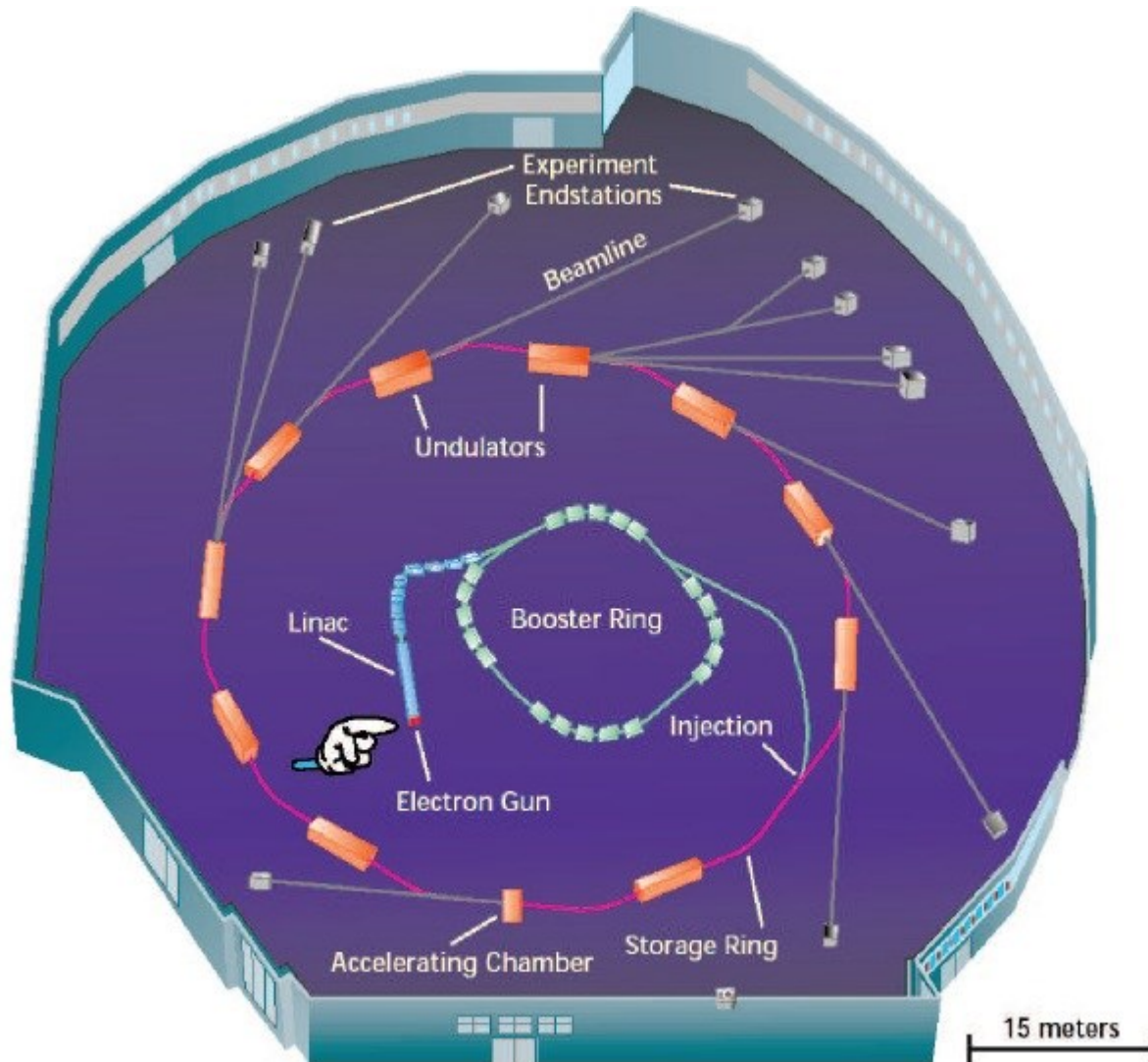


# Synchrotron

- Modern synchrotron radiation sources are built as storage rings
- Synchrotron cannot operate at  $E=0$  since it requires  $B \neq 0$ .
  - ⇒ Use LINAC or Microtron as pre-accelerator
  - Use synchrotron to reach the final energy  $E$
  - Use storage ring to keep electrons at energy
- The storage ring supplies the energy lost by radiation in each turn.
- Typical parameters: Lifetime: up to 30 h  
Current: 100 – 500 mA
- Current losses through interaction with residual gas ⇒ UHV
- Current supplied in bunches.



# Storage Rings





# Storage Rings



# Photon Machines

The three largest and most powerful synchrotrons in the world



APS, USA



ESRF, Europe-France



Spring-8, Japan





# Synchrotron Radiation Primer

Radiation of a non-relativistic, accelerated particle:

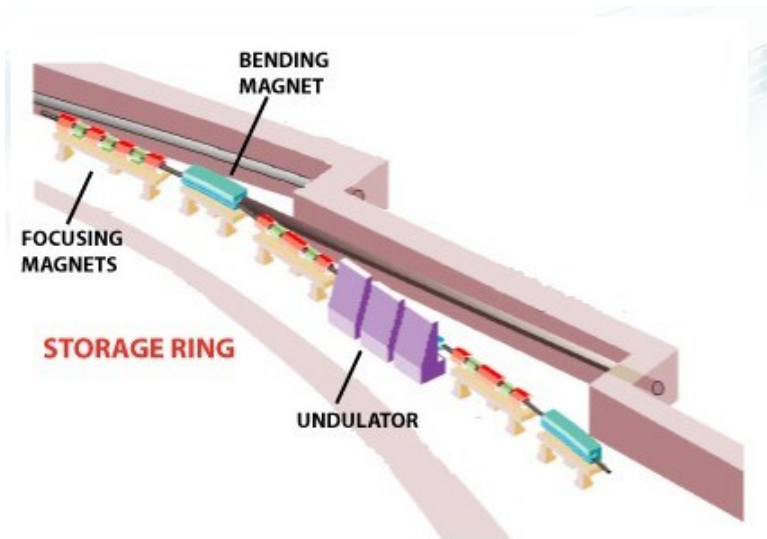
$$P = \left( \frac{e^2}{6\pi\epsilon_0 m_0^2 c^3} \right) \left( \frac{dp}{dt} \right)^2$$

Angular distribution resembles the one of a Hertz dipole:

$$\left( \frac{dP}{d\Omega} \right) = \left( \frac{e^2}{16\pi^2 \epsilon_0 m_0^2 c^3} \right) \left( \frac{dp}{dt} \right)^2 \sin^2(\Psi)$$

Radiation is emitted (similar to the dipole) in the direction perpendicular to the acceleration

# Synchrotron Radiation Primer



**Energy  $E_e$  of an electron at speed  $v$ :**

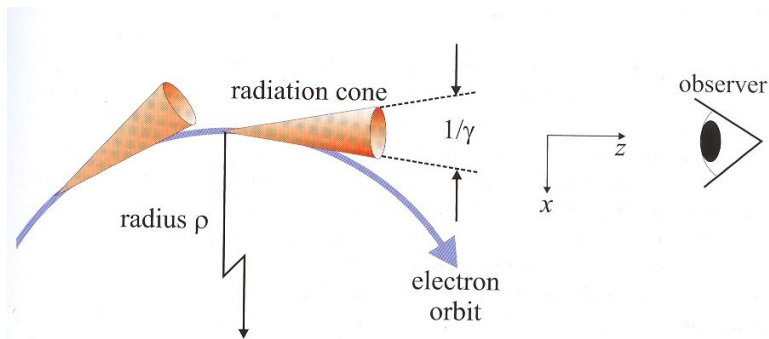
$$E_e = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma mc^2$$

For 5GeV and  $mc^2=0.511$  MeV get  $\gamma \approx 10^4$

**Centrifugal=Lorentz force yields for radius:**

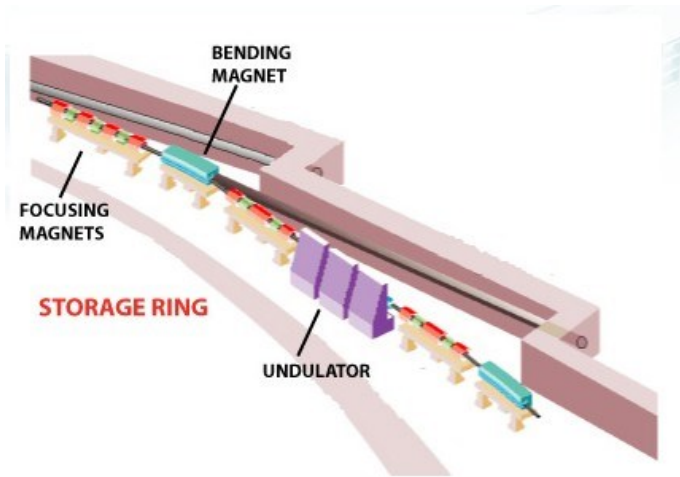
$$\rho = \frac{\gamma mc}{eB} = \frac{3.3 E [\text{GeV}]}{B[\text{T}]} \approx 25 \text{ m}$$

$$E_e = 6 \text{ GeV}, \quad B = 0.8 \text{ T}$$



Opening angle is of order  $\frac{1}{\gamma} \approx 0.1$  mrad

# Bending Magnets



**Characteristic energy  $\hbar\omega_c$  for bend or wiggler:**

$$\hbar\omega_c[\text{keV}] = 0.665 E_e^2 [\text{GeV}] B(\text{T}) \approx 20 \text{ keV}$$

$$\text{Flux} \sim E^2$$

**Energy loss by synchrotron radiation per turn:**

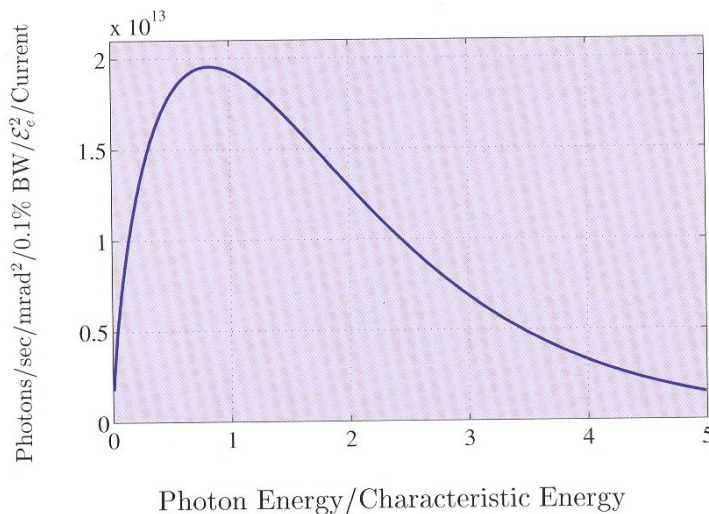
$$\Delta E[\text{keV}] = \frac{88.5 E^4[\text{GeV}]}{\rho[\text{m}]}$$

For 1 GeV and  $\rho = 3.33 \text{ m}$ :  $\Delta E = 26.6 \text{ keV/turn}$

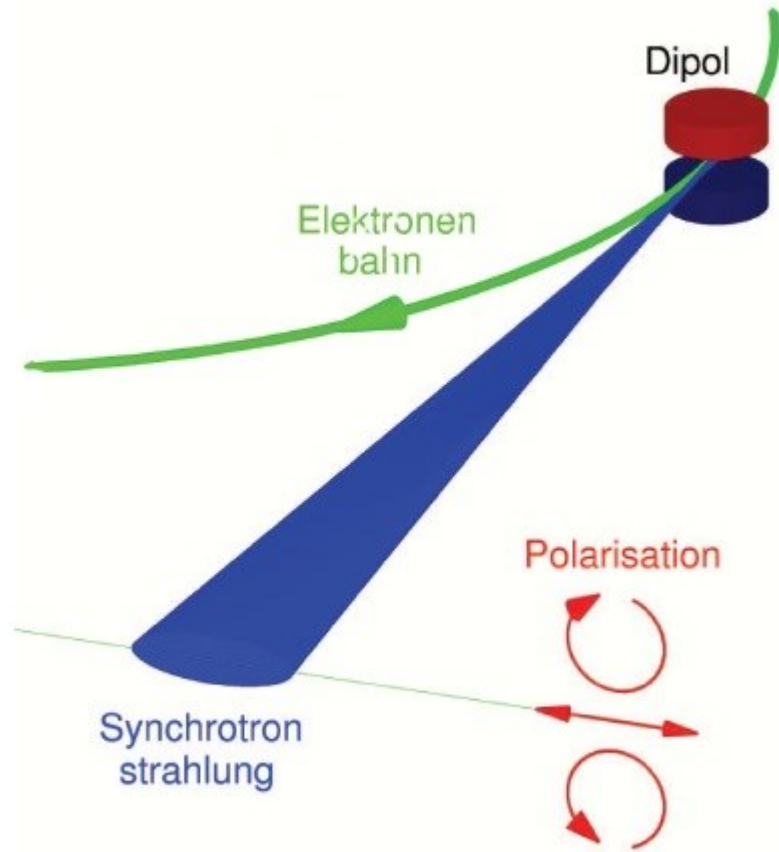
$$\text{For } I = 500 \text{ mA} \equiv 0.5 \frac{\text{Cb}}{\text{s}} = 0.5 \times 6.25 \times 10^{18} \frac{e^-}{\text{s}}$$

$$\rightarrow P = 0.5 \times 6.25 \times 10^{18} \frac{e^-}{\text{s} \times 26.6 \text{ keV}}$$

$$= 8.3125 \times 10^{22} \times 1.6 \times 10^{-19} = 13.3 \frac{\text{kJ}}{\text{s}} = 13.3 \text{ KW}$$



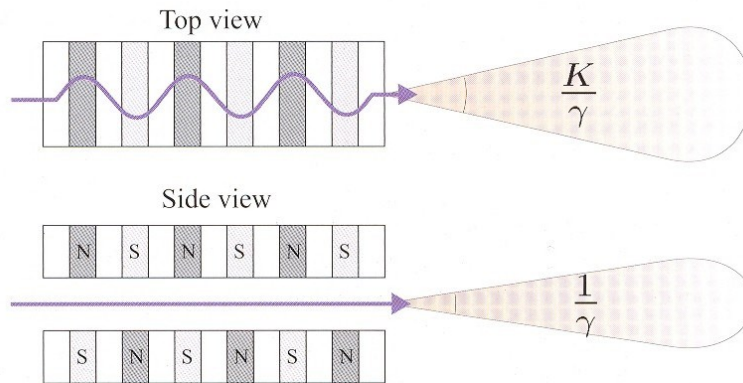
# Polarization



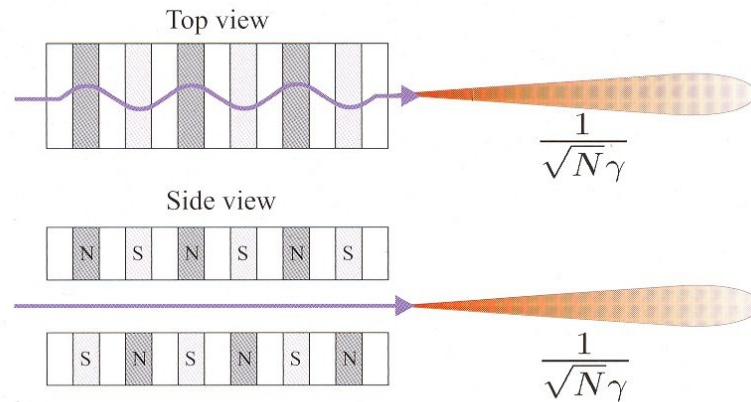
- Synchrotron radiation is polarized linearly in the plane of the orbit
- Above and below the orbital plane of the polarization is circular
- Important applications for magnetic x-ray scattering

# Insertion Devices (Wigglers and Undulators)

(a) Wiggler



(b) Undulator



## Wiggler:

$$P[\text{kW}] = 0.633 E_e^2 [\text{GeV}] B^2 [\text{T}] L [\text{m}] I [\text{A}]$$

$$\text{Flux} \sim E^2 \times N$$

N: number poles

## Undulator:

$$k = \frac{eB}{mc} \quad k_u = 0.934 \lambda_u [\text{cm}] B_0 [\text{T}]$$

with  $\lambda_u$  undulator period

~~undulator fundamental:~~

$$\lambda_0 = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{k^2}{2} + \gamma\theta \right)$$

on axis

$$\text{Flux} \sim E^2 \times N^2$$

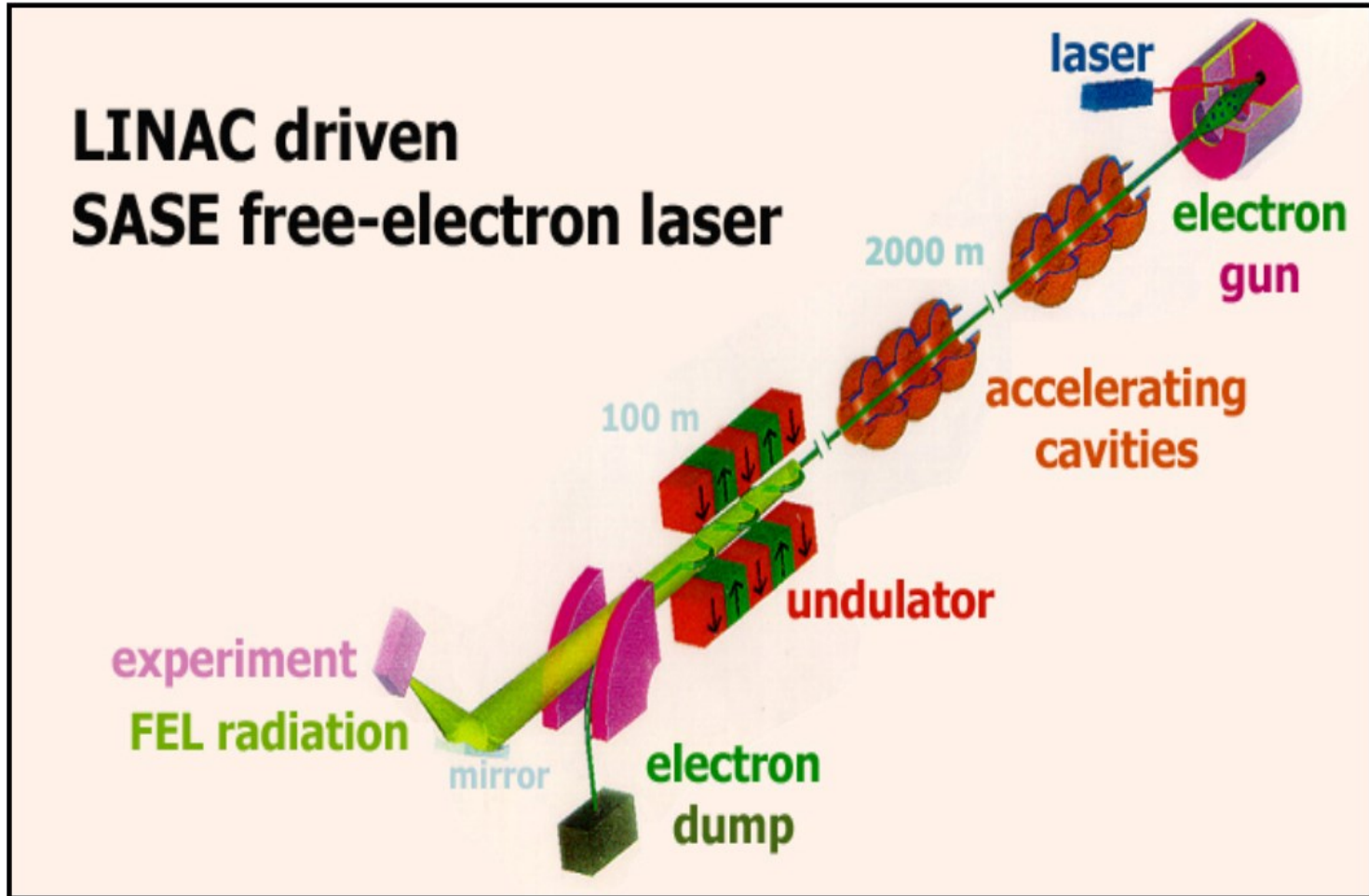
$$\text{bandwidth: } \frac{\Delta\lambda}{\lambda} \sim \frac{1}{nN}$$



# Towards Diffraction Limited Light Sources: MAX IV (Lund)

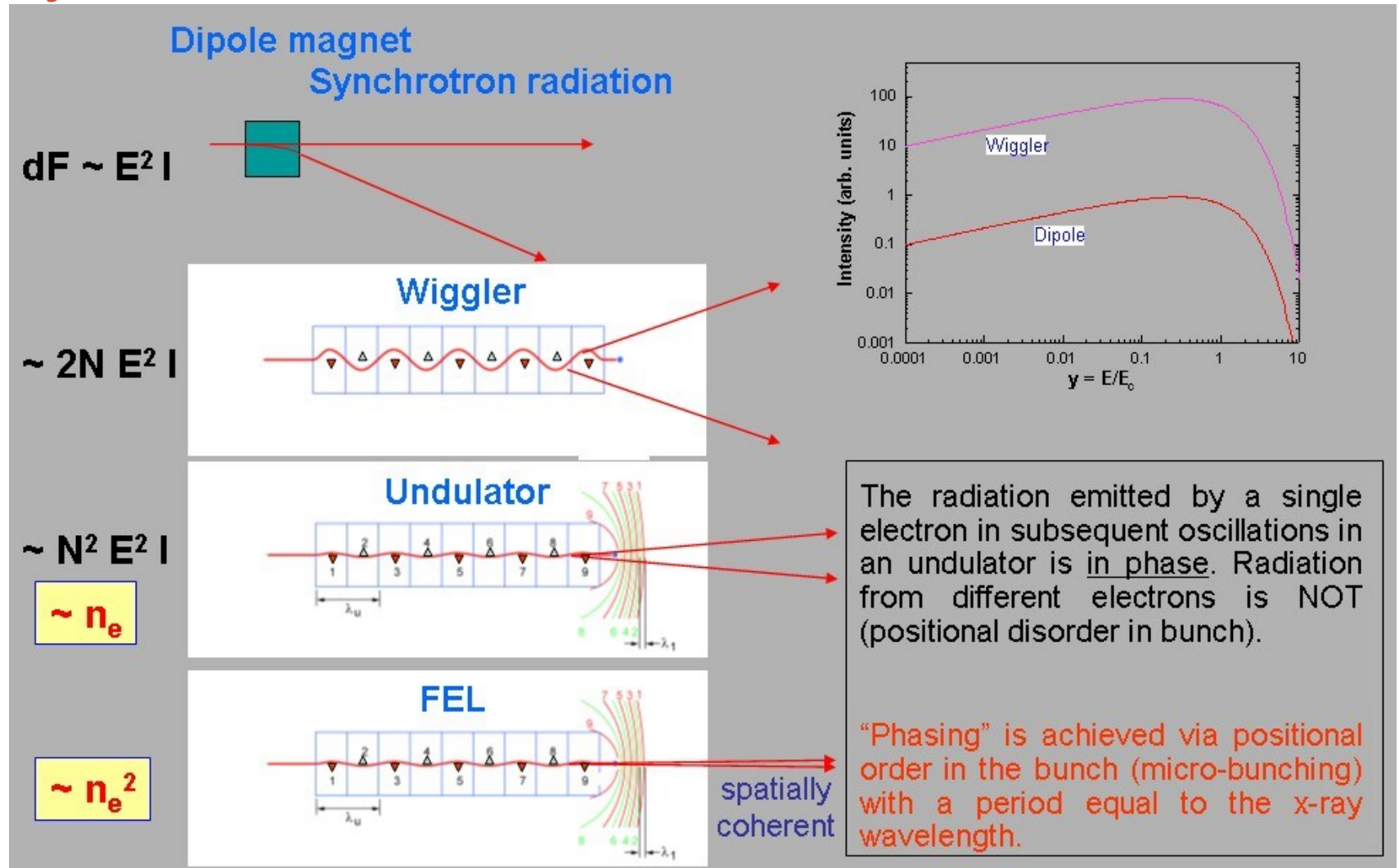


# Free Electron Lasers (FELs)

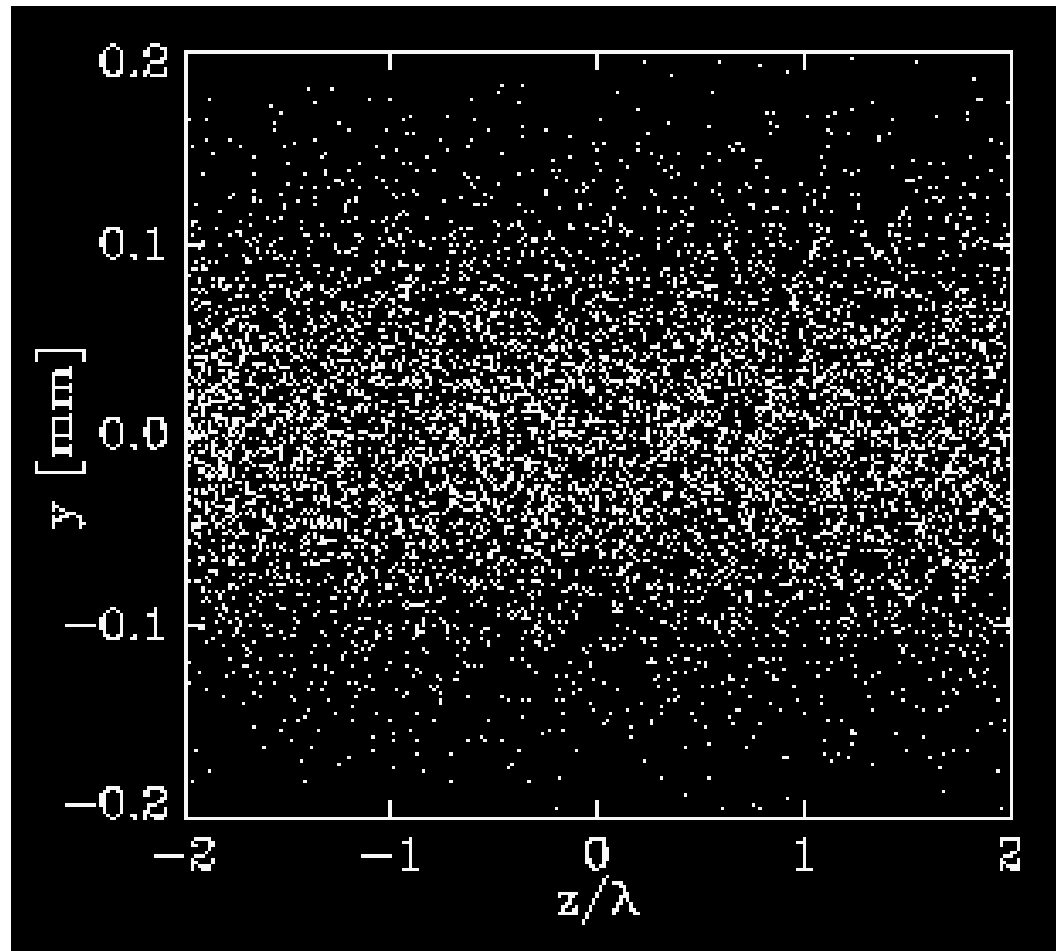




# Synchrotron and FEL Sources



# Electron Bunching

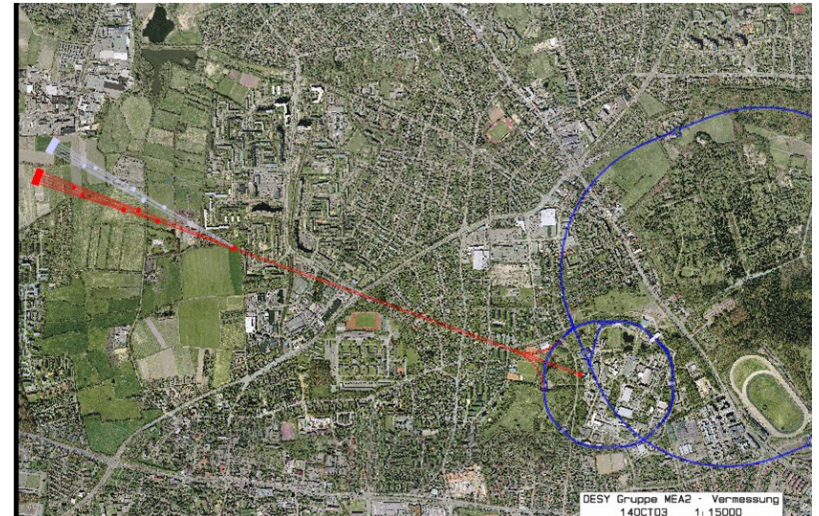


GENESIS – simulation for TTF parameters

Courtesy Sven Reiche  
(UCLA)



# VUV and X-Ray FELs



# Brilliance

$$B = \frac{\text{photons}}{\text{s mm}^2 \text{ mrad}^2 \text{ 0.1\% BW}}$$

