Methoden moderner Röntgenphysik II: Streuung und Abbildung

Lecture 9	Vorlesung zum Haupt- oder Masterstudiengang Physik, SoSe 2015 G. Grübel, M. Martins, E. Weckert	
Location	Lecture hall AP, Physics, Jungiusstraße	
Date	Tuesday	12:45 - 14:15
	Thursday	8:30 - 10:00

Methoden moderner Röntgenphysik II: Streuung und Abbildung

Part I:

Basics of X-ray Physics

by Gerhard Grübel (GG)

Introduction
Overview, Introduction to X-ray Scattering

X-ray Scattering Primer
Elements of X-ray Scattering

Sources of X-rays, Synchrotron Radiation
Laboratory Sources, Accelerator Bases Sources

Reflection and Refraction from Interfaces
Snell's Law, Fresnel Equations

Kinematical Diffraction (I)
Diffraction from an Atom, a Molecule, from Liquids, Glasses, …

Kinematical Diffraction (II)
Diffraction from a Crystal, Reciprocal Lattice, Structure Factor, …

Methoden moderner Röntgenphysik II: Streuung und Abbildung

Small Angle Scattering, and Soft Matter
Introduction, Form Factor, Structure Factor, Applications, ...

Anomalous Diffraction
Introduction into Anomalous Scattering, ...

Introduction into Coherence
Concept, First Order Coherence, ...

Coherent Scattering
Spatial Coherence, Second Order Coherence, ...

Applications of Coherent Scattering
Imaging and Correlation Spectroscopy, ...

The Concept of Coherence: Classical Light

First Order Coherence

Coherence and Emission Spectrum

Spatial Coherence

Second Order Coherence

Chaotic Light

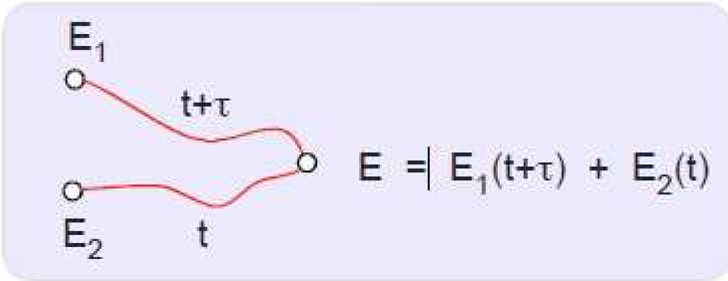
Basic concepts:

The quantum theory of light
Rodney Loudon, Oxford University Press (1990)

Quantum optics
Marlan O. Scully, M. Suhal Zubairy,
Cambridge University Press (1997)

The Concept of Coherence

Consider harmonic fields E_1, E_2 at positions r_1, r_2 at time:



$$< I_n > \equiv < E_n(t+\tau)E_n^*(t) >, \qquad n \in \{1,2\}$$

$$< I > = < EE^* > = < I_1 > + < I_2 > + 2\,\mathrm{Re}\,[< E_1(t+\tau)E_2^*(t) >] \qquad \text{with } < f > \equiv \lim_{T\to\infty} < f >_T$$

$$< f >_T \equiv \left(\frac{1}{T}\right) A \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)dt$$

Here the limes T→∞ means that T is finite but sufficiently large such that $<f>_T$ does not depend on T

Normalized pair correlation function: $\gamma_{12}(\tau) \equiv \dfrac{<E_1(t+\tau)E_2^*(t)>}{(<I_1><I_2>)^{\frac{1}{2}}}$

$$\Rightarrow < I > = < I_1 > + < I_2 > + 2\,(< I_1 >< I_2 >)^{\frac{1}{2}}\,\mathrm{Re}[\gamma_{12}(\tau)]$$

DESY

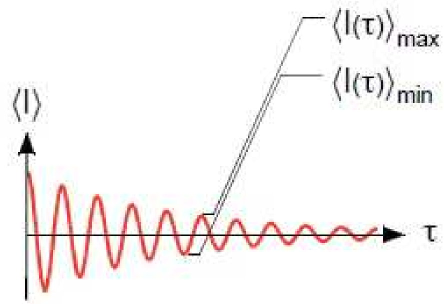
$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)| e^{i\phi_{12}(\tau)} \Rightarrow I = <I_1> + <I_2> + 2(<I_1><I_2>)^{\frac{1}{2}} |\gamma_{12}(\tau)| \cos(\phi_{12}(\tau))$$

Assume: $\phi_{12}(\tau)$ changes much faster than $|\gamma_{12}(\tau)|$ (quasi-coherent light)

$$\Rightarrow <I>_{max/min} = <I_1> + <I_2> \pm 2(<I_1><I_2>)^{\frac{1}{2}} |\gamma_{12}(\tau)|$$

Interference visibility:

$$\kappa \equiv \left| \frac{<I>_{max} - <I>_{min}}{<I>_{max} + <I>_{min}} \right| = 2\frac{(<I_1><I_2>)^{\frac{1}{2}}}{<I_1> + <I_2>} |\gamma_{12}(\tau)|$$



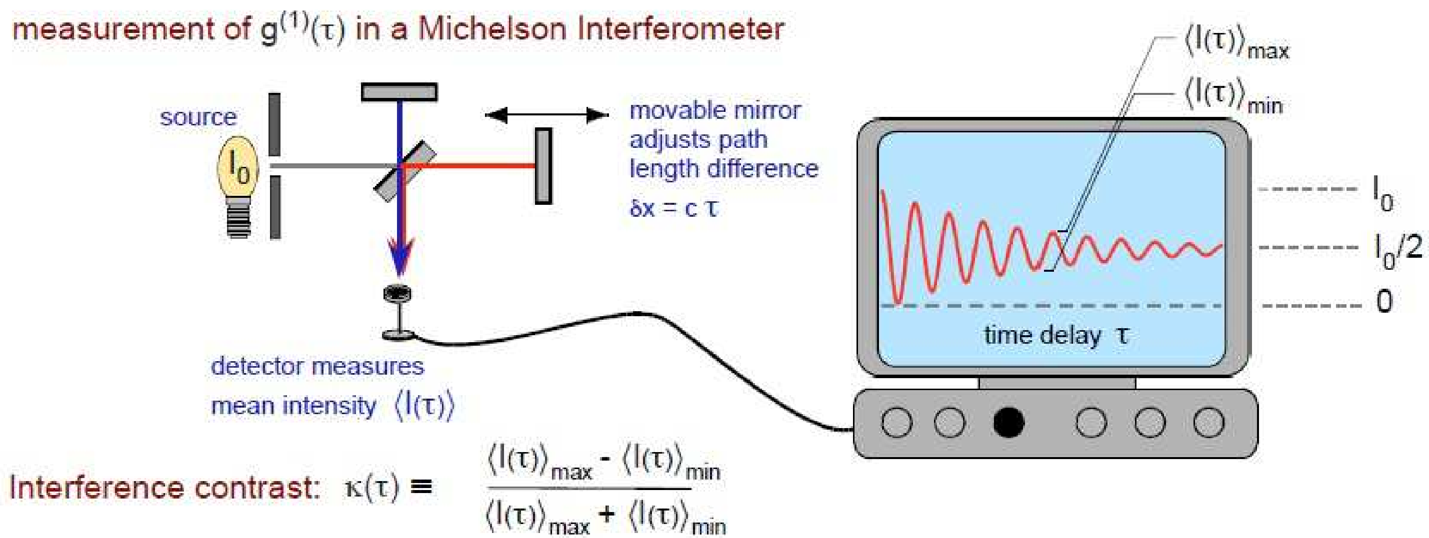
$$<I_1> = <I_2> \Rightarrow \kappa = |\gamma_{12}(\tau)|$$

Definition:	$\lvert\gamma_{12}(\tau)\rvert = 1$	for all τ	$\Rightarrow$	complete coherence
	$0 < \lvert\gamma_{12}(\tau)\rvert < 1$	for some τ	$\Rightarrow$	partial coherence
	$\lvert\gamma_{12}(\tau)\rvert = 0$	for all τ	$\Rightarrow$	no coherence

Normalized autocorrelation function: $$g^{(1)}(\tau) \equiv \frac{<E(t+\tau)E^*(t)>}{<I>}$$

$$\text{with } g^{(1)}(0) = 1 \text{ and } g^{(1)}(-\tau) = g^{(1)*}(\tau)$$

Measurement of $g^{(1)}(\tau)$ in a Michelson Interferometer

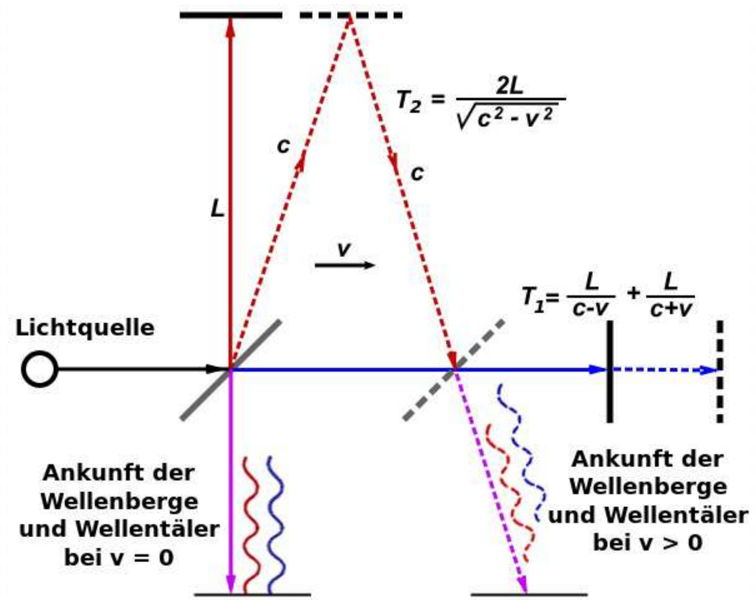


Maximal coherence: Interference contrast maximal for all τ 

Partial coherence: Interference contrast decreases for large τ 

The Michelson-Morley Experiment

by Albert Abraham Michelson: 1881 in Potsdam



Normalized autocorrelation function:

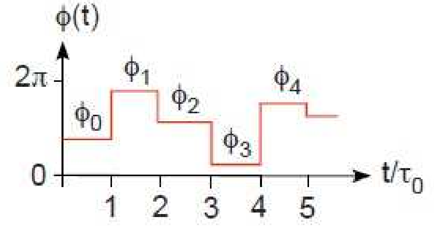
$$g^{(1)}(\tau) \equiv \frac{<E(t+\tau)E^*(t)>}{<I>}$$

with $g^{(1)}(0) = 1$ and $g^{(1)}(-\tau) = g^{(1)*}(\tau)$

Example: successive wave trains of duration τ_0 and length $c\tau_0$

$$E(t) = E_0 e^{i\omega t + i\phi(t)} \quad \text{with } \phi(t):$$

$$\Rightarrow g^{(1)}(\tau) = e^{i\omega t} < e^{i(\phi(t+\tau)-\phi(t))} >$$

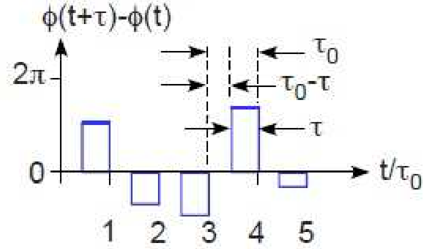


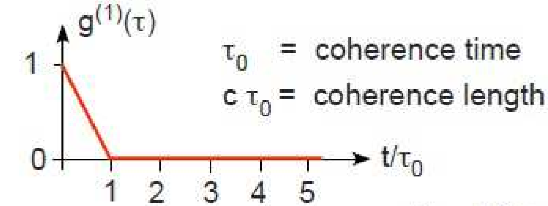
$0 \leq \tau \leq \tau_0$:

$$< e^{i(\phi(t+\tau)-\phi(t))} > = \frac{1}{N\tau_0} \sum_{n=0}^{N+1} \int_{n\tau_0}^{(n+1)\tau_0} dt\, e^{i(\phi(t+\tau)-\phi(t))}$$

$$= \frac{1}{N\tau_0} \sum_{n=0}^{N+1} \{(\tau_0 - \tau) + \tau\, e^{i(\phi_{n+1} - \phi_n)}\}$$

$$\Rightarrow g^{(1)}(\tau) = e^{i\omega t} \frac{(\tau_0 - \tau)}{\tau_0} \quad \begin{matrix} \text{x } 0 & \text{if } \tau_0 < \tau \\ \text{x } 1 & \text{if } 0 \leq \tau \leq \tau_0 \end{matrix}$$

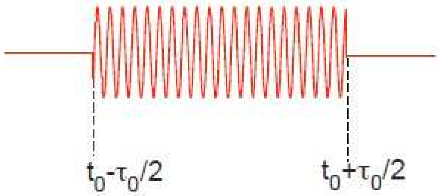




Note: τ_0: **coherence time**; $\xi_l = \frac{\lambda}{2}\frac{\lambda}{\Delta\lambda} = c\tau_0$: **longitudinal coherence length**

Coherence and Emission Spectrum

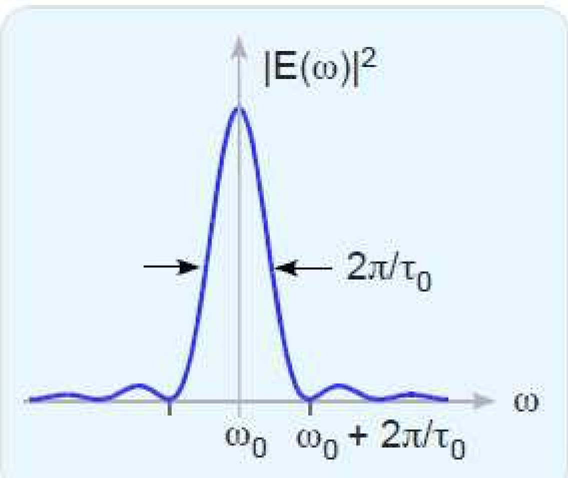
Consider single wave train of duration τ_0, phase ϕ_0, frequency ω_0:



$$E(t) = e^{[-i\omega_0 t - i\phi_0]} \times 1 \ \left(\text{if} \frac{t_0-\tau_0}{2} \le t \le \frac{t_0+\tau_0}{2}\right)$$
$$\times 0 \quad \text{otherwise}$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\ E(t)\ e^{i\omega t} = \sqrt{\frac{2}{\pi}} \frac{\sin\left(\frac{(\omega-\omega_0)\tau_0}{2}\right)}{(\omega-\omega_0)} \bullet e^{-i\phi_0}$$

N wave trains with the same frequency ω_0 but arbitrary phases ϕ_n, durations τ_n, starting times t_n:



$$E(\omega) = \sum_{n=1}^{N} \sqrt{\frac{2}{\pi}} \frac{\sin\left(\frac{(\omega-\omega_0)\tau_0}{2}\right)}{(\omega-\omega_0)} \bullet e^{i(\omega-\omega_0)t_n - i\phi_n}$$

$$|E(\omega)|^2 = \sum_{n=1}^{N} |E_n(\omega)|^2 = \frac{2}{\pi} \sum_{n=1}^{N} \frac{\sin^2\left(\frac{(\omega-\omega_0)\tau_0}{2}\right)}{(\omega-\omega_0)^2}$$

Emission bandwidth $\Delta\nu \cong \frac{1}{\tau}$ with $\tau \equiv \frac{1}{N}\sum_{n=1}^{N} \tau_n$

Example: Collision Broadened Light Source

Molecules of a gas radiate light $E(t) = E_0 e^{-i(\omega_0 t - \phi(t))}$ at frequency ω_0. Collisions yield random phase jumps, i.e., phase $\phi(t) \in [0, 2\pi]$ fluctuates.

Probability for a free flight of duration $t \in [\tau, \tau + d\tau]$: $P(t) = \frac{1}{\tau_0} e^{\frac{-\tau}{\tau_0}}$

kinetic gas theory (τ_0 means duration of free flight)

Coherence function: $g^{(1)}(\tau) = e^{i\omega_0\tau} < e^{i(\phi(t+\tau) - \phi(t))} >$

$e^{i(\phi(t+\tau) - \phi(t))}$ $= 1$ for free flights with duration $> \tau$

$= e^{i\chi}$ with $\chi \in [0, 2\pi]$ random for free flights with duration $< \tau$

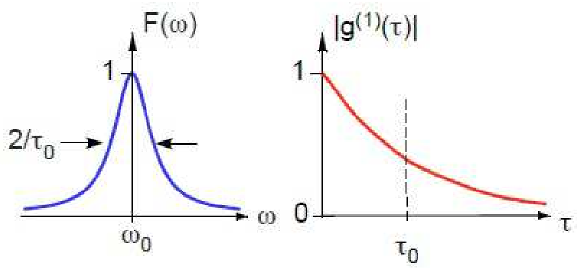
i.e., only flights of duration $t>\tau$ yield contribution to $< e^{i(\phi(t+\tau) - \phi(t))} >$:

$$\Rightarrow \quad g^{(1)}(\tau) = e^{i\omega_0\tau} \int_\tau^\infty P(s)\, ds = e^{i\omega_0\tau} e^{\frac{-\tau}{\tau_0}}$$

$$\Rightarrow \quad |g^{(1)}(\tau)| = e^{\frac{-\tau}{\tau_0}}$$

$$\Rightarrow \quad F(\omega) = \frac{1}{1+(\omega-\omega_0)^2\tau_0^2}$$

(Wiener-Khinchin Theorem)



Wiener Khinchin Theorem

$$E(\omega) \equiv \mathcal{F}[E(t)] \equiv \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\ E(t)\ e^{i\omega t}$$

$$F(\omega) \equiv \frac{|E(\omega)|^2}{\int_{-\infty}^{\infty} dt |E(\omega)|^2}$$

normalized spectral density

$$\Rightarrow F(\omega) = \frac{1}{\sqrt{2\pi}} \mathcal{F}\left[g^{(1)}\right],$$

$\mathcal{F} \equiv$ Fourier-Transform

Wiener Khinchin Theorem

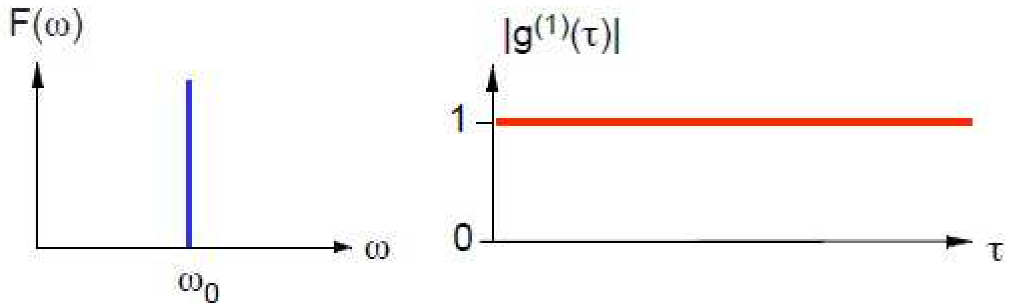
The spectral power density $F(\omega)$ is the Fourier transform of the normalized autocorrelation function $g^{(1)}(\tau)$

Example: Monochromatic Light

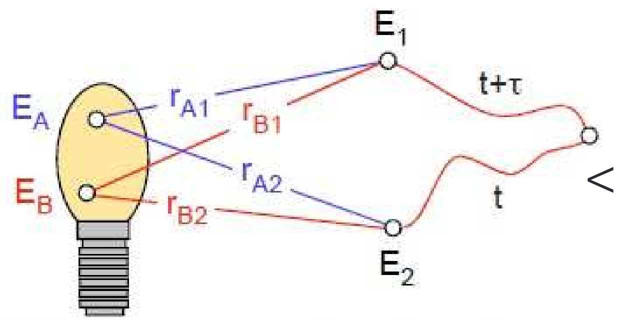
$$E(t) = e^{-i(\omega_0 t - \phi)}$$

$$g^{(1)}(\tau) = e^{i\omega_0 \tau}$$

$$\left|g^{(1)}(\tau)\right| = 1$$



Spatial Coherence



Light Source: mutually incoherent point sources $g^{(1)}(\tau) = \exp(i\omega\tau - \tau/\tau_0)$

$$E_1 = E_{A1} + E_{B1} \qquad E_{An} = E_A e^{\frac{i r_{An}\omega}{c}}$$

$$E_2 = E_{A2} + E_{B2} \qquad E_{Bn} = E_B e^{\frac{i r_{Bn}\omega}{c}}$$

$$<E_1(t+\tau)E_2^*(t)> = <E_{A1}(t+\tau)E_{A2}^*(t)> + <E_{B1}(t+\tau)E_{B2}^*(t)> + <E_{A1}(t+\tau)E_{B2}^*(t)> + <E_{B1}(t+\tau)E_{A2}^*(t)>$$

$$<I_n> = <E_n(t)E_n^*(t)> = <E_{An}(t)E_{An}^*(t)> + <E_{Bn}(t)E_{Bn}^*(t)> + <E_{An}(t)E_{Bn}^*(t)> + <E_{Bn}(t)E_{An}^*(t)>$$

$$\Rightarrow <I_1> = <I_2>$$

$$<E_{A1}(t+\tau)E_{A2}^*(t)> = <E_A(t+\tau)E_A^*(t)> e^{\left[\frac{i(r_{A1}-r_{A2})\omega}{c}\right]} = <E_A(t+\tau_A)E_A^*(t)> \text{ with} \tau_A \equiv \tau + \frac{(r_{A1}-r_{A2})}{c}$$

$$<E_{B1}(t+\tau)E_{B2}^*(t)> = <E_B(t+\tau)E_B^*(t)> e^{\left[\frac{i(r_{B1}-r_{B2})\omega}{c}\right]} = <E_B(t+\tau_B)E_B^*(t)> \text{ with} \tau_B \equiv \tau + \frac{(r_{B1}-r_{B2})}{c}$$

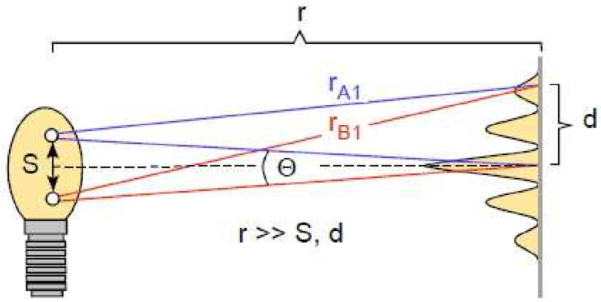
$$\Rightarrow <E_1(t+\tau)E_2^*(t)> = <E_A(t+\tau_A)E_A^*(t)> + <E_B(t+\tau_B)E_B^*(t)>$$

$$\gamma_{12}(\tau) \equiv \frac{<E_1(t+\tau)E_2^*(t)>}{(<I_1><I_2>)^{\frac{1}{2}}} = \frac{1}{2}\left[g^{(1)}(\tau_A) + g^{(1)}(\tau_B)\right] = \frac{1}{2}\left[e^{i\omega\tau_A - \frac{\tau_A}{\tau_0}} + e^{i\omega\tau_B - \frac{\tau_B}{\tau_0}}\right]$$

$$4|\gamma_{12}(\tau)|^2 \equiv \left|g^{(1)}(\tau_A)\right|^2 + \left|g^{(1)}(\tau_B)\right|^2 + 2\left|g^{(1)}(\tau_A)\right|\left|g^{(1)}(\tau_B)\right|\cos(\omega(\tau_A - \tau_B))$$ **interference term**

$$4|\gamma_{12(\tau)}|^2 \equiv |g^{(1)}(\tau_A)|^2 + |g^{(1)}(\tau_B)|^2 + 2|g^{(1)}(\tau_A)|\,|g^{(1)}(\tau_B)|\,\cos(\omega(\tau_A - \tau_B))$$

Interference term depends on $\tau_A - \tau_B = \frac{r_{A1} - r_{A2}}{c} - \frac{r_{B1} - r_{B2}}{c}$



Light Source: mutually incoherent point sources $g^{(1)}(\tau) = \exp(i\omega\tau - \tau/\tau_0)$

symmetric: $r_{A2} - r_{B2} = 0 \;\Rightarrow \tau_A - \tau_B = \frac{r_{A1} - r_{B1}}{c}$

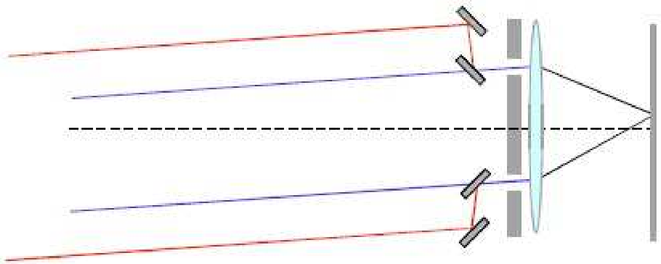
$$r_{A1} \cong r + \frac{\left(\frac{d-S}{2}\right)^2}{2r}, \quad r_{B1} \cong r + \frac{\left(\frac{d+S}{2}\right)^2}{2r}$$

$$\Rightarrow \tau_A - \tau_B \cong -\frac{Sd}{2rc}$$

First minimum of $|\gamma_{12}(\tau)|^2$:

$\omega(\tau_A - \tau_B) = \pi; \quad S \cong r\theta \;\Rightarrow\; d \cong \frac{\lambda}{\theta}$

transverse coherence length



Michelson stellar interferometer: adjustable slits, extension of slit separation by mirrors

Measurement of angular diameter of stars, angular separation of double stars, etc.