

Methoden moderner Röntgenphysik II

Streuung und Abbildung

Stephan V. Roth
DESY
03.06.2014

- > Methoden moderner Röntgenphysik II:
Streuung und Abbildung
- > Vorlesung zum Haupt/Masterstudiengang Physik
SS 2014 (Nr. 66-362)
- > G. Grübel, M. Martins, E. Weckert
- > Location:
Hörs AP, Physik, Jungiusstrasse
Tuesdays 12.45 – 14.15
Thursdays 8:30 – 10.00
- > Übungen: 2 SWS: Dienstag 14:30 – 16:00 (wenn vereinbart)
(Nr. 66-363)
SemRm 4
- > **EXKURSION: 1.7.2014, ab 14:15? Bestätigung!!!**

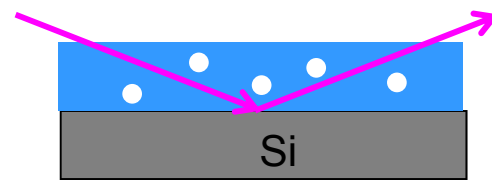
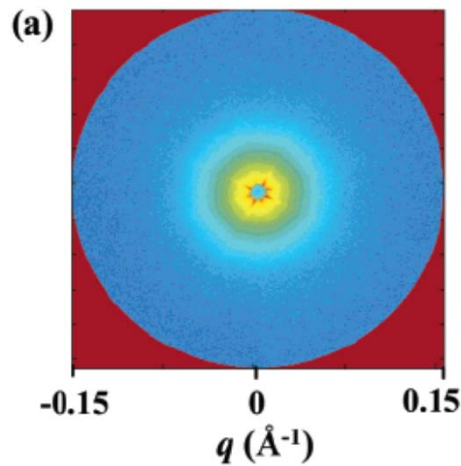
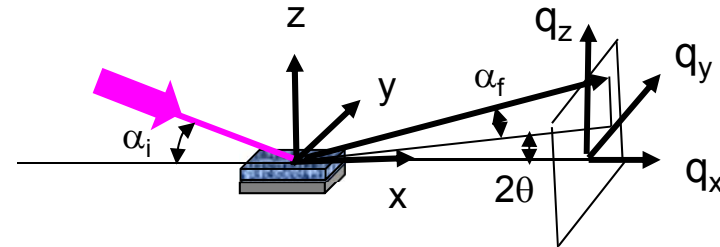
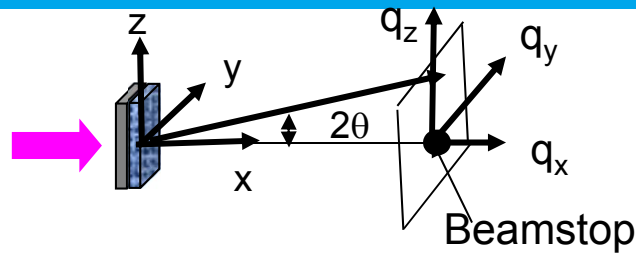


Outline I

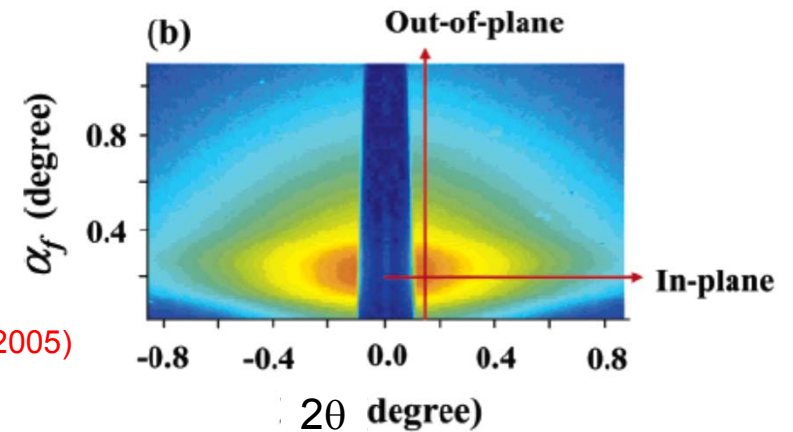
- > 03.06 : Small-Angle X-ray Scattering (SAXS)
- > 05.06 : Applications &
A short excursion into Polymeric materials
- > 17.06 : Grazing incidence SAXS (GISAXS)
- > 19.06 : In-situ studies of metal layer growth (M. Schartzkopf)
- > 24.06 : The polymer-metal interface – application of GISAXS
On the route to organic electronics



T-SAXS vs. GISAXS



Lee et al., *Macromolecules*, 38, 8991 (2005)



- Easy measurement
- Easy analysis
- In-plane information (q_y, q_z)
- Any possible scattering from substrate
- Transparency of substrate
- High energy

- Strong intensity
- Easy preparation of samples
- Full information (q_x, q_y, q_z)
- Scattering from surface / internal structure
- Scattering from reflected AND transmitted beam
- Refraction effects (DWBA)
- Special setup

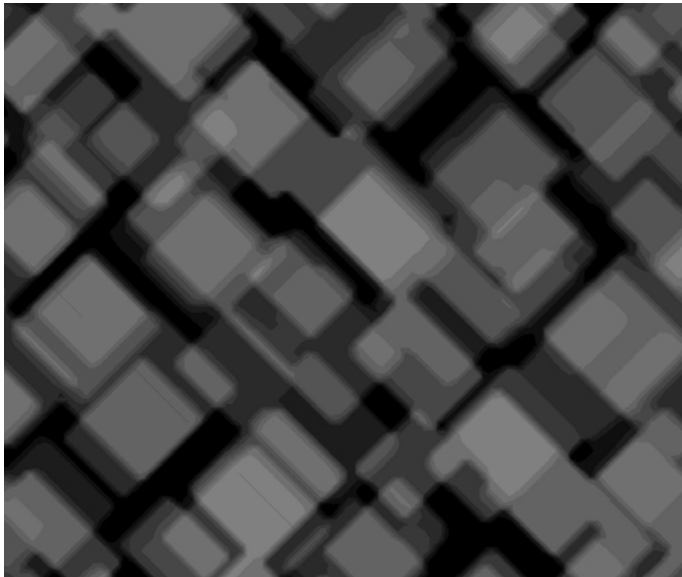


Aim

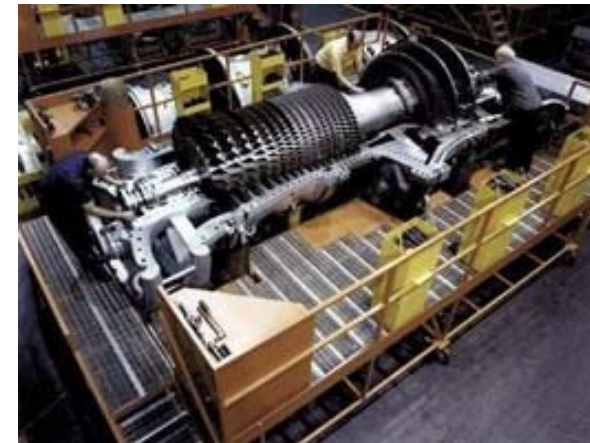
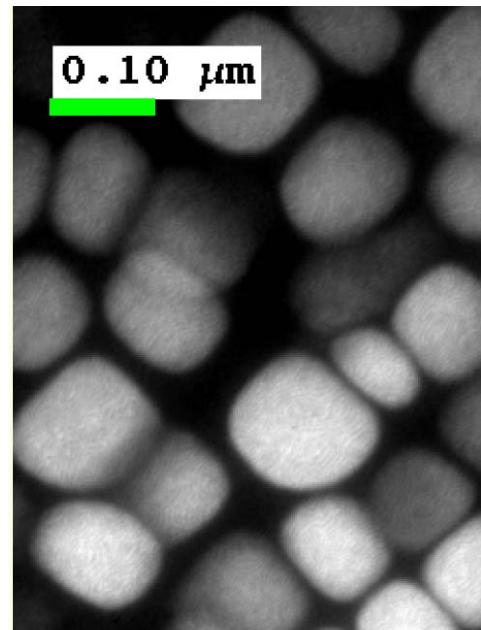
> To understand the **structure – property relation** of materials on **multiple length scales**

- **q-resolution**
- Maximum q-value
- **Beam size**

- Real pieces & materials
- Model systems
- Nanotechnology



Courtesy: R. Gilles (TUM)



<http://news.thomasnet.com/companystory/GE-Gas-Turbine-Technology-Selected-for-Pearl-GTL-Project-in-Qatar-495497>

Outline II - today

SAXS – Introduction

Neutrons, X-rays and Light: Scattering Methods Applied to Soft Condensed Matter.
Eds: P. Lindner, Th. Zemb. North Holland Delta Series, Elsevier, Amsterdam (2002)
ISBN: 0-444-51122-9

> Instrumentation

- P03/MiNaXS @ PETRA III

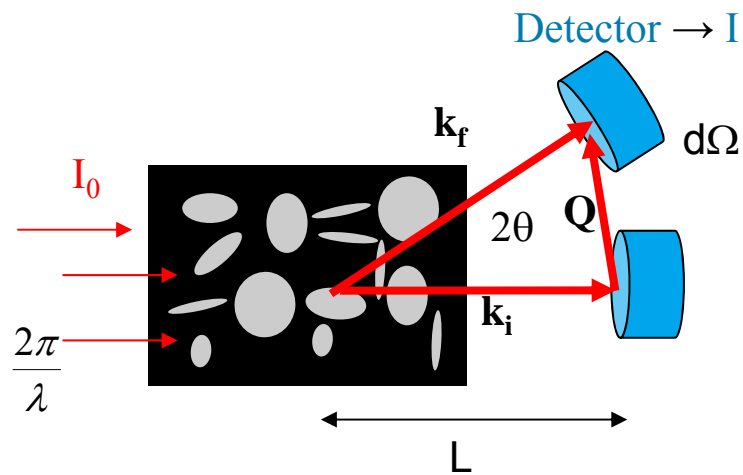
> Bulk materials → Transmission U/SAXS:

- Porous materials
- Ni-base superalloys
- Droplet drying



Cross-section

> Differential cross section



$$d\sigma = \frac{I}{I_0} (L^2 d\Omega)$$

$$\frac{d\sigma}{d\Omega} = \frac{I}{I_0} (L^2) \Rightarrow \frac{d\Sigma}{d\Omega} = \frac{1}{V} \frac{d\sigma}{d\Omega}$$

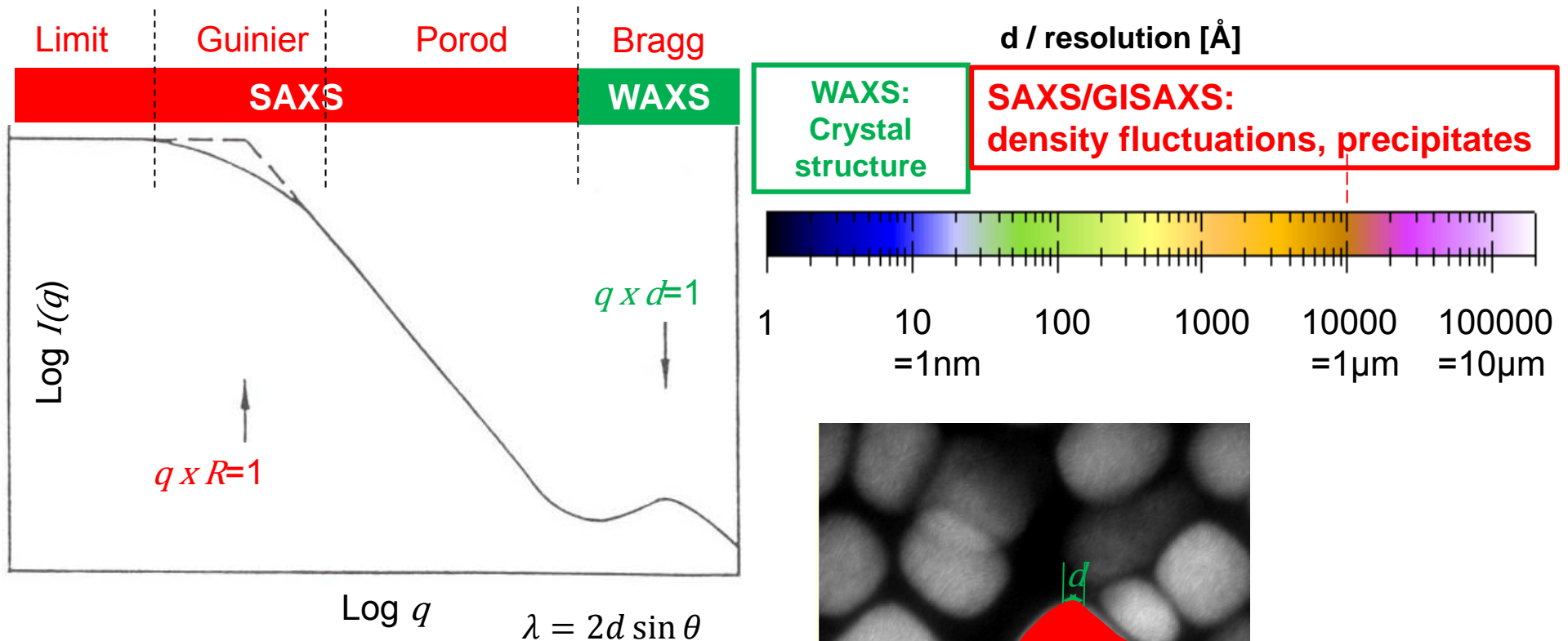
V = Sample volume

$$\vec{q} = \vec{k}_f - \vec{k}_i \quad |\vec{k}_f| = |\vec{k}_i| = \frac{2\pi}{\lambda} \quad |\vec{q}| = 2 \frac{2\pi}{\lambda} \sin(\theta)$$

> Scattering occurs due to density differences

WAXS, SAXS, GISAXS...

Source: Streumethoden zur Untersuchung kondensierter Materie
1996; ISBN 978-3-89336-180-9

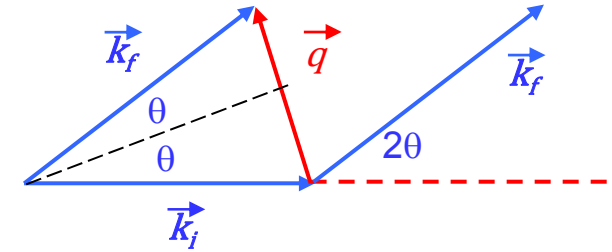
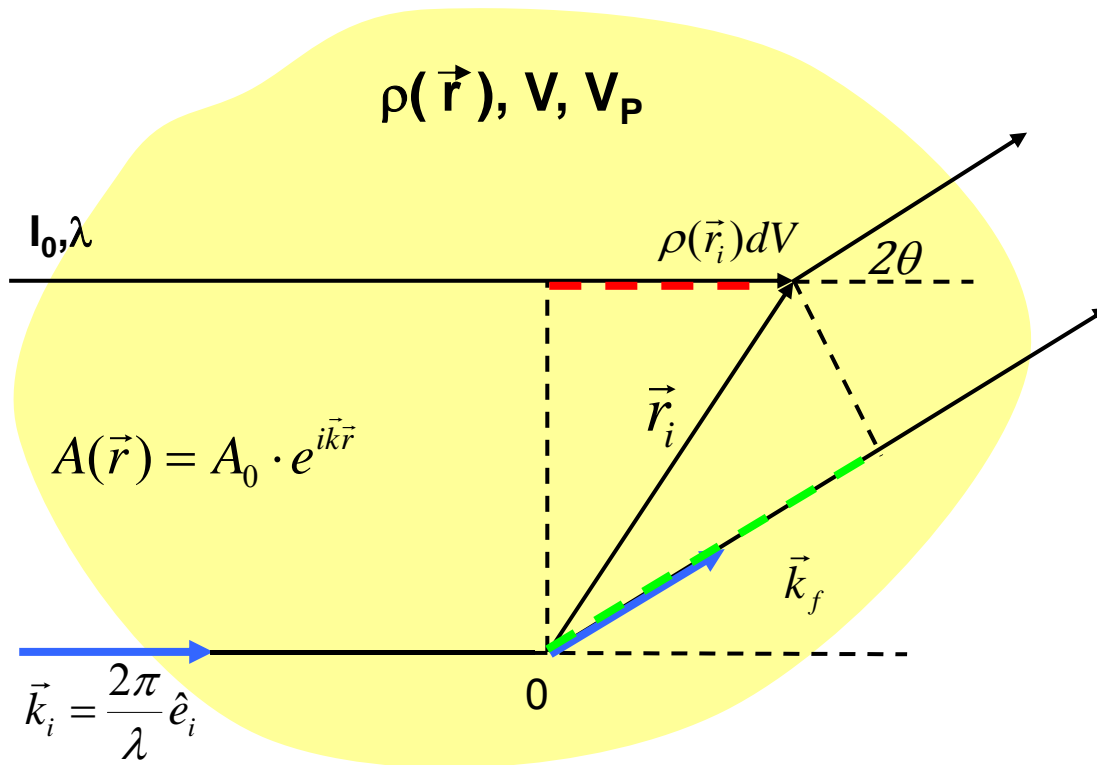


- > R ~particles "radius"
- > d ~interatomic distance
- > SAXS: $\theta < 5^\circ$



Scattering amplitude

- > Interference in far field



$$|\vec{q}| = \frac{4\pi}{\lambda} \sin \theta$$

- > Phase difference: $\Delta\varphi_i = (\vec{k}_f - \vec{k}_i) \cdot \vec{r}_i = \vec{q} \cdot \vec{r}_i$
- > Scattering amplitude: $A(\vec{q}) = \int \rho(\vec{r}) e^{-i\vec{q}\vec{r}} dV = \int \rho(\vec{r}) e^{-i\vec{q}\vec{r}} d^3\vec{r}$
- > Intensity: $I(\vec{q}) = \frac{1}{V} |A(\vec{q})|^2$

Form factor and structure factor: Fourier transform

Single particle: Fourier transformation

$$A(\vec{q}) = \int \rho_P(\vec{r}) e^{-i\vec{q}\vec{r}} dV = \int \rho_P(\vec{r}) e^{-i\vec{q}\vec{r}} d^3\vec{r}$$

Particle distribution function $G(\mathbf{r}) \rightarrow$ Electron density distribution

$$\rho(\vec{r}) = \sum_i \rho_P(\vec{r}_i) = \int \rho_P(\vec{r}') G(\vec{r} - \vec{r}') d^3r' = \rho_P(\vec{r}) * G(\vec{r})$$

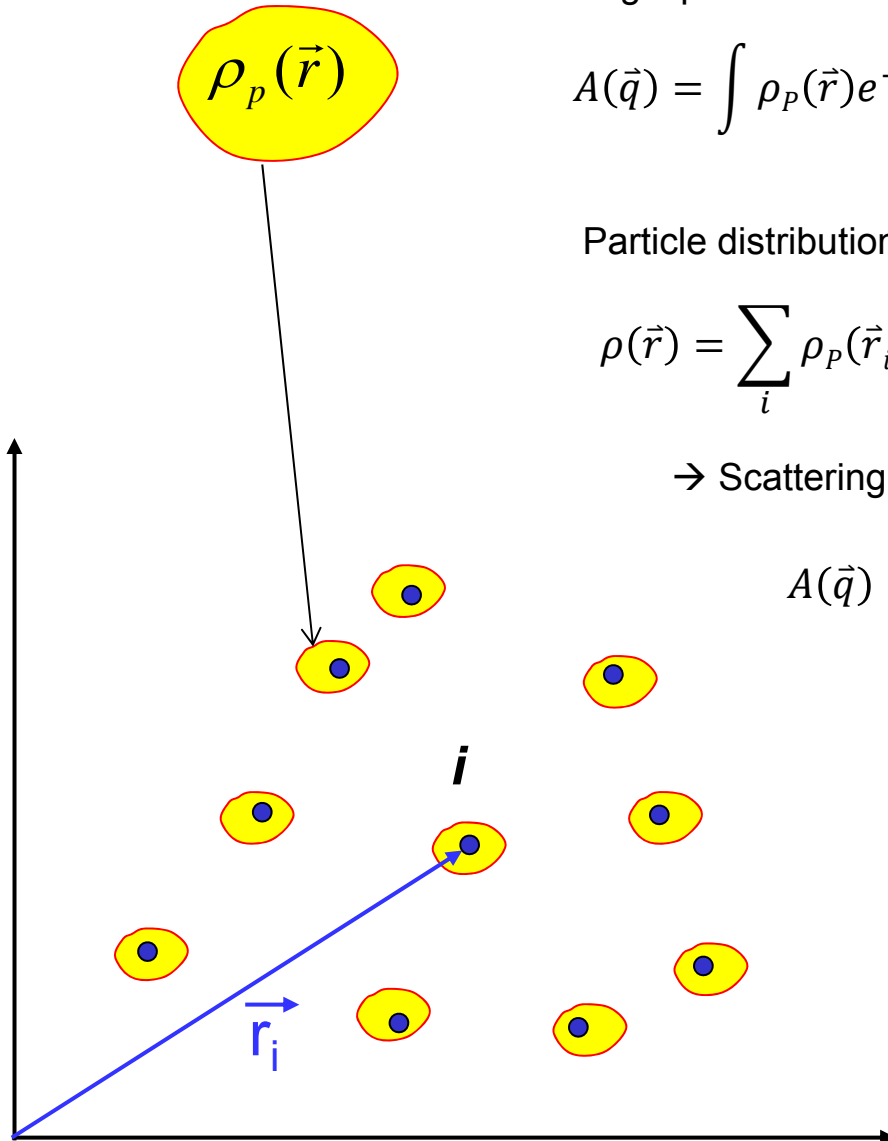
\rightarrow Scattering amplitudes of the whole arrangement

$$\begin{aligned} A(\vec{q}) &= \int \rho(\vec{r}) e^{-i\vec{q}\vec{r}} dV = \int [\rho_P(\vec{r}) * G(\vec{r})] e^{-i\vec{q}\vec{r}} d^3\vec{r} \\ &= \int \rho_P(\vec{r}) e^{-i\vec{q}\vec{r}} dV \cdot \int G(\vec{r}) e^{-i\vec{q}\vec{r}} dV \end{aligned}$$

\rightarrow Scattered Intensity

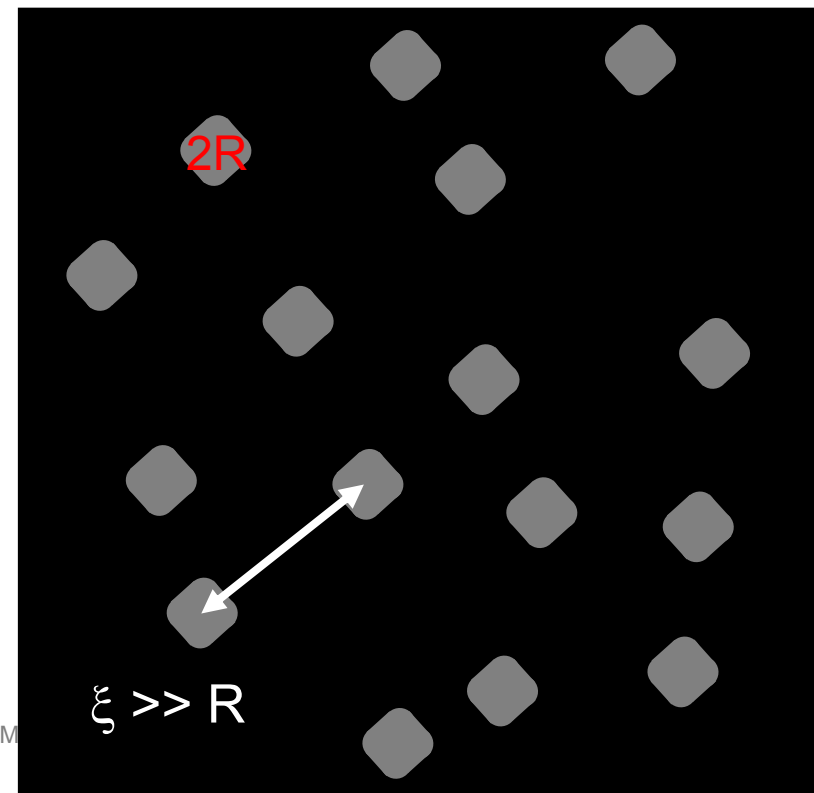
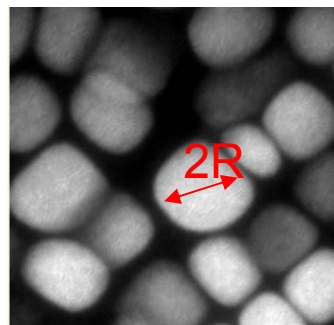
$$I(\vec{q}) = \frac{1}{V} |A(\vec{q})|^2 = P(\vec{q}) S(\vec{q})$$

Form factor Structure factor



Two-phase model: Dilute systems

- > Only form of particle relevant
- > Matrix M , volume fraction Φ
Particles P , volume fraction $(1-\Phi)$
Electron density: $\rho_{M,P} = n_{M,P} * f_{M,P}$
- $f_{M,P}$: atomic form factor („extension of the electron cloud“, resonances)
- $n_{M,P}$: number density of atoms
- > Consider $\rho_{M,P}$ as constant resp.



Two phase Model

- > Scattering amplitude:

$$A(\vec{q}) = \int \rho(\vec{r}) e^{-i\vec{q}\vec{r}} d^3\vec{r} = \int_{\Phi V} \rho_M(\vec{r}) e^{-i\vec{q}\vec{r}} d^3\vec{r} + \int_{(1-\Phi)V} \rho_P(\vec{r}) e^{-i\vec{q}\vec{r}} d^3\vec{r}$$

$$A(\vec{q}) = (\rho_M - \rho_P) \int_{\Phi V} e^{-i\vec{q}\vec{r}} d^3\vec{r}$$

$$A(\vec{q}) = \Delta\rho \int_{\Phi V} e^{-i\vec{q}\vec{r}} d^3\vec{r}$$

- > $I(\vec{q}) = \frac{1}{V} |A(\vec{q})|^2 \sim \Delta\rho^2$

- > Porod Invariant Q (Porod, 1982):

$$Q = \int I(\vec{q}) d^3\vec{q} = 4\pi\Phi(1-\Phi)\Delta\rho^2$$

Ableiten!

Mittlung <..> erklären S.25, S.51

- > Only dependent on density contrast $\Delta\rho$



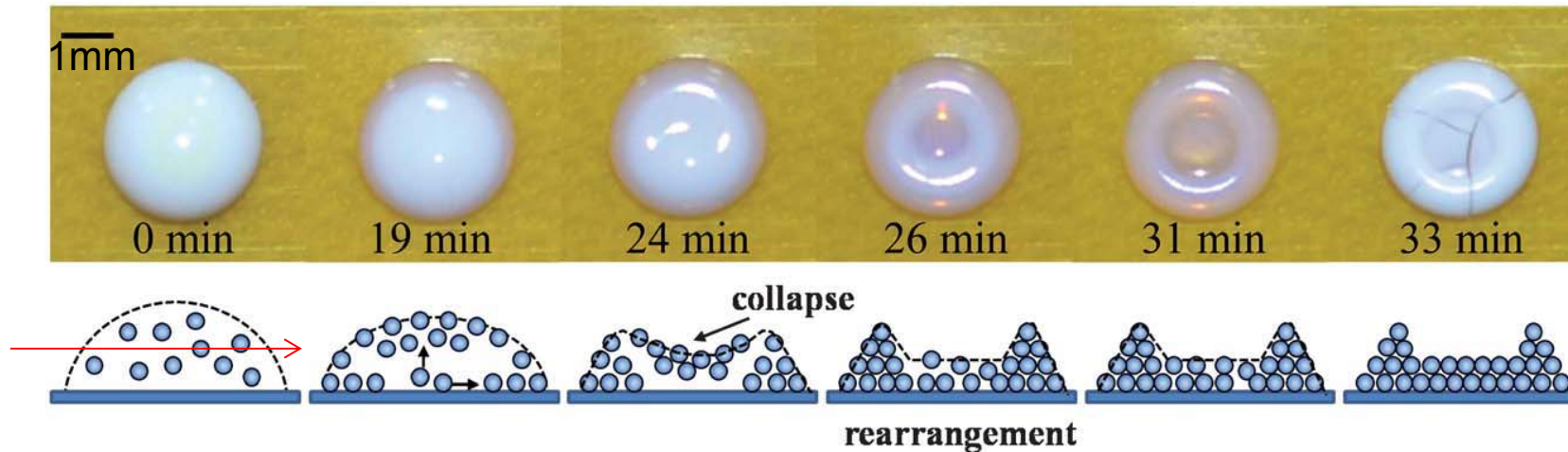
Herleitung Parod-Invariante

> Siehe Handzettel und Übung



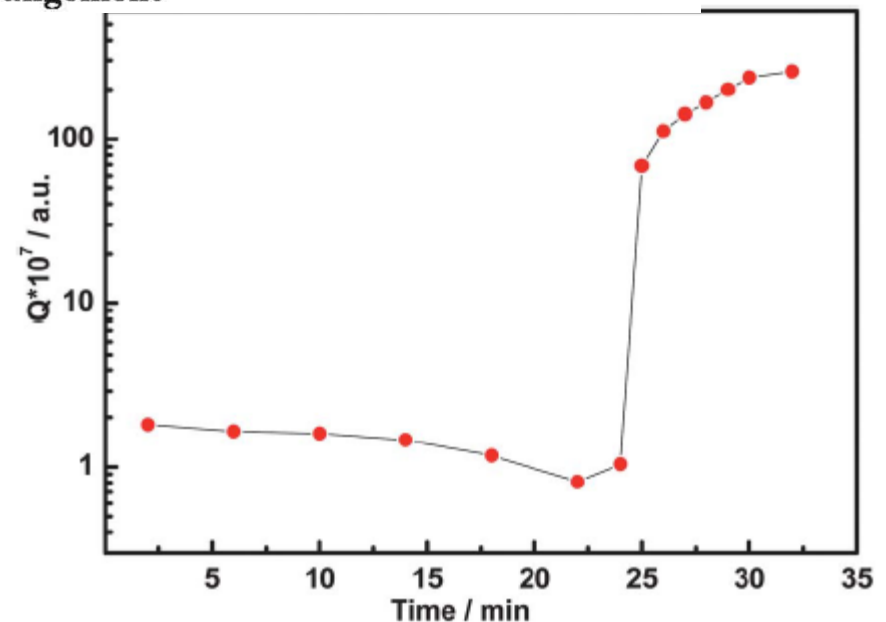
Porod Invariant - practical application

> Colloidal solution: drying thick droplet



> Evaporation of water:

- Irradiated volume becomes smaller: shrinking
- Distance of colloidal particles decreases, $\Phi \rightarrow 1$
- $\Delta\rho$ increases (air!), as water removed from interstitial sites



Stephan V. Roth | Mod

