

- Coherence of light and matter:
from basic concepts to modern applications

Part II

Script 4

Vorlesung im GrK 1355

WS 2013

A. Hemmerich & G. Grübel

Location: SemRm 052, Gebäude 69, Bahrenfeld

Thursdays 12.15 – 13.45

G.Grübel (GR), A.Hemmerich (HE)

Literature

Basic concepts: [The quantum theory of light](#)

Rodney Loudon, Oxford University Press (1990)

[Quantum Optics](#)

Marlan O. Scully, M. Suhail Zubairy, Cambridge University Press (1997)

[Dynamic Light Scattering with Applications](#)

B.J. Berne and R. Pecora, John Wiley&Sons (1976)

[Elements of Modern X-Ray Physics](#)

J. A. Nielsen and D. McMorrow, J. Wiley&Sons (2001)

Matter Waves: [Bose-Einstein Condensation in Dilute Gases](#)

C. J. Pethick and H. Smith, Cambridge University Press (2002)

Lecture Notes

Part I: http://photon.physnet.uni-hamburg.de/fileadmin/user_upload/ILP/Hemmerich/teaching.html/Coherence.pdf

Part II: http://photon-science.desy.de/research/studentsteaching/lectures__seminars/ws_13_14/coherence_of_light_grk1355/.....

- **Coherence of light and matter:
from basic concepts to modern applications**

Part II: G. Grübel

Coherence based X-ray techniques

Overview, Introduction to X-ray Scattering, Sources of Coherent X-rays, Speckle pattern and their analysis

Imaging techniques

Phase Retrieval, Sampling Theory, Reconstruction of Oversampled Data, Fourier Transform Holography, Applications

X-ray Photon Correlation Spectroscopy (XPCS)

Introduction, Equilibrium Dynamics (Brownian Motion), Surface Dynamics, Non-Equilibrium Dynamics

Imaging and XPCS at FEL Sources

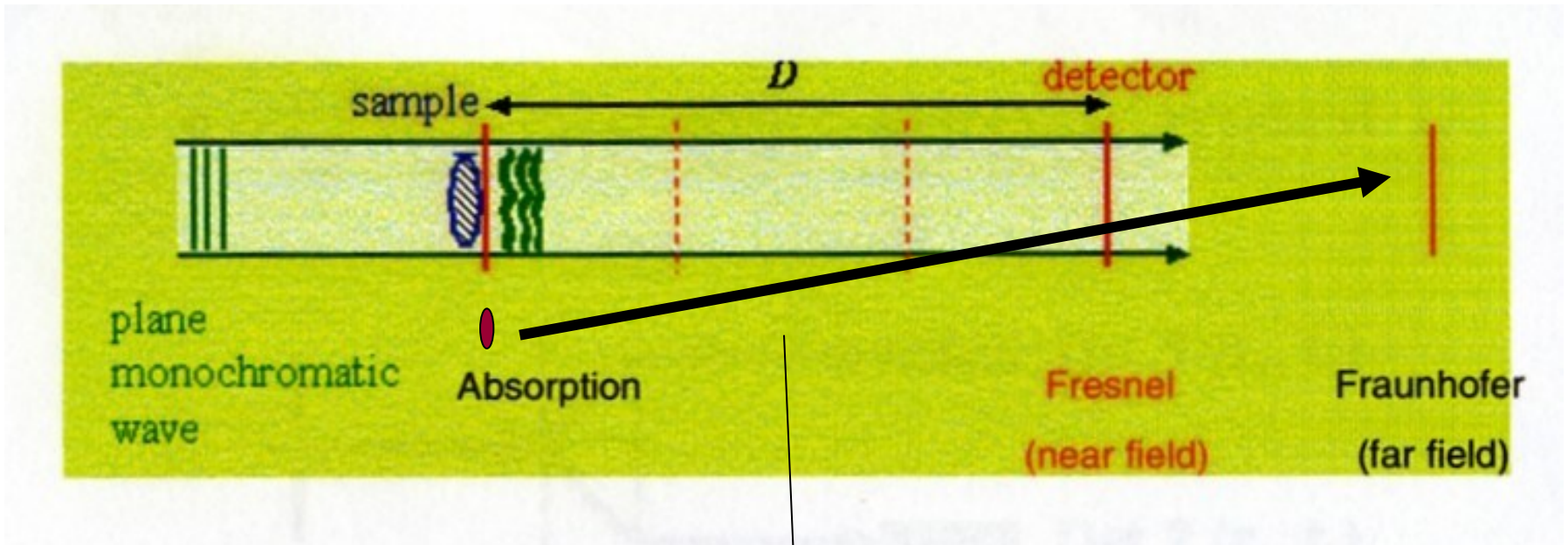
■

Imaging techniques:

Fourier Transform Holography

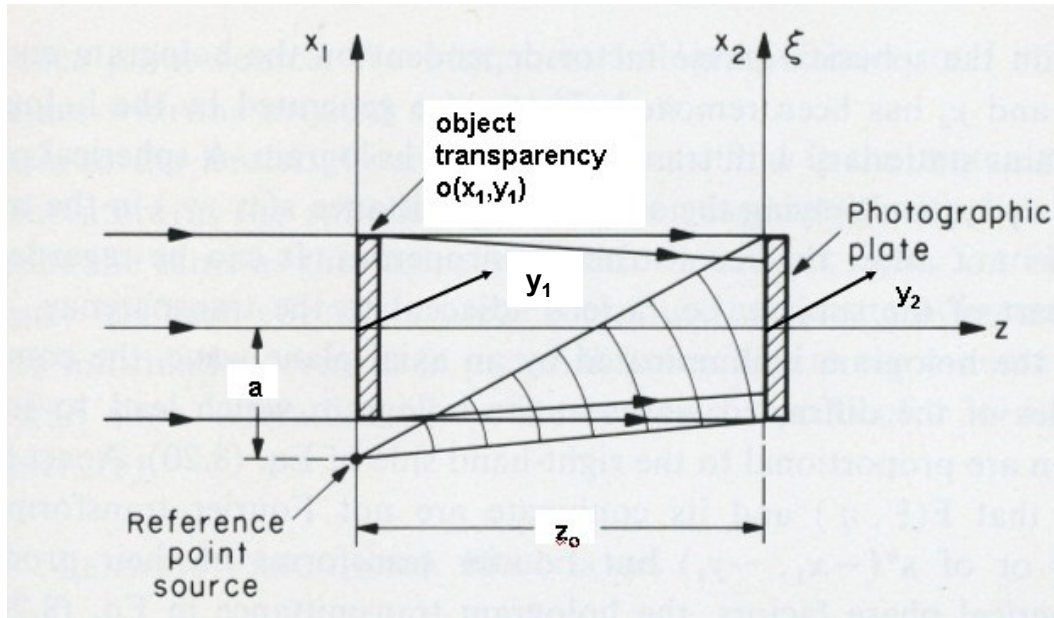
- Imaging techniques:

Parallel beam, reference beam



reference beam: holography

Fourier Transform Holography – FTH (1)



$o(x_1, y_1)$: amplitude of the wave transmitted through object o

$r(x_1, y_1)$: reference wave

R.J. Collier, C.B. Burckhardt, L.H. Lin "Optical Holography", Academic Press (1971)

Fresnel Kirchhoff Theory

$$o(x_2, y_2) = (i/\lambda z_0) \exp\{i\pi/\lambda z_0 (x_2^2 + y_2^2)\} \bar{O}(\xi, \eta)$$

$$r(x_2, y_2) = (i/\lambda z_0) \exp\{i\pi/\lambda z_0 (x_2^2 + y_2^2)\} \check{R}(\xi, \eta) \exp\{-2i\pi\xi a\}$$

with $\bar{O}(\xi, \eta) = \text{FT}\{o(x_1, y_1)\}$, $\check{R}(\xi, \eta) = \text{FT}\{r(x_1, y_1)\}$, $\xi = x_2/\lambda z_0$, $\eta = y_2/\lambda z_0$

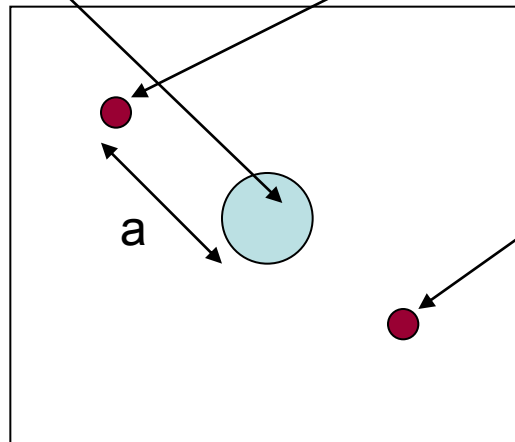
Fourier Transform Holography (2)

$$I(x_2, y_2) = |r(x_2, y_2) + o(x_2, y_2)|^2$$

$$I(x_2, y_2) = |r(x_2, y_2)|^2 + |o(x_2, y_2)|^2 + r^*(x_2, y_2) o(x_2, y_2) + r(x_2, y_2) o^*(x_2, y_2)$$

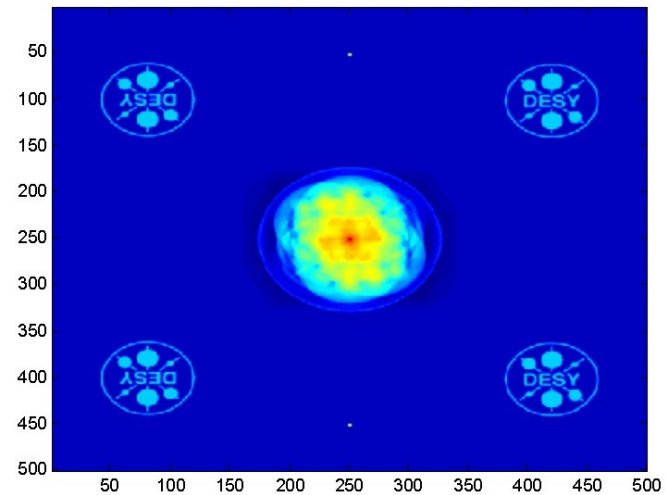
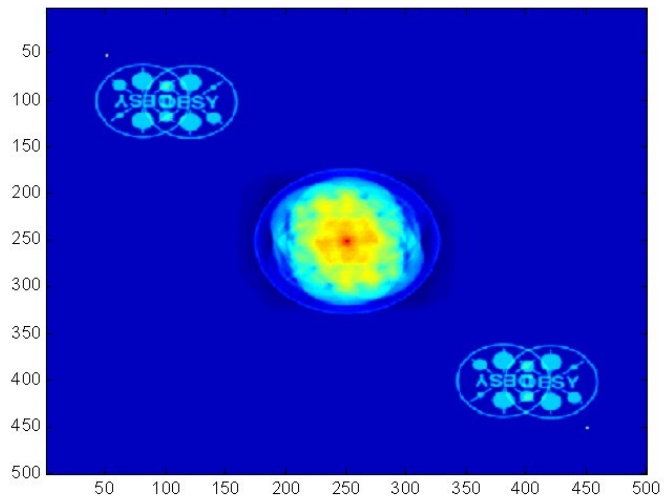
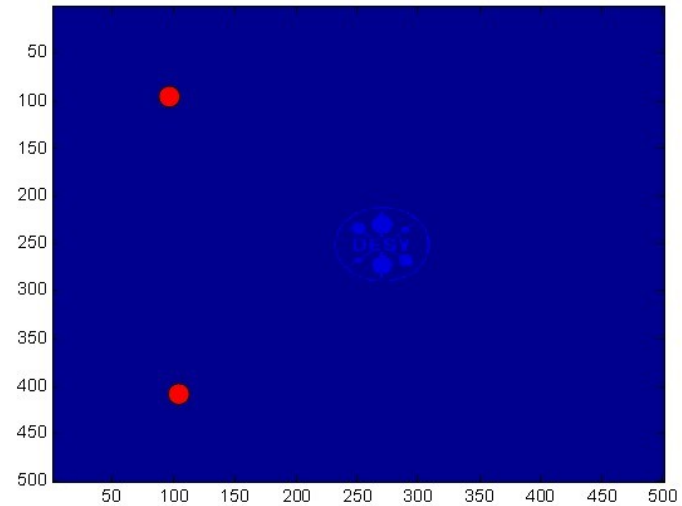
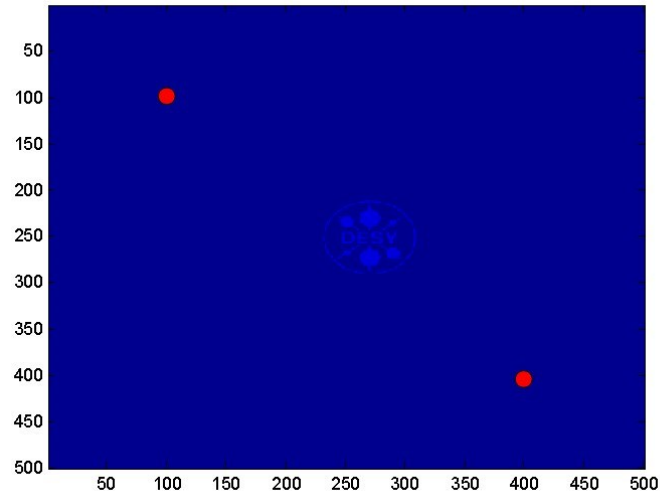
$$I(x_2, y_2) \approx \underbrace{|\check{R}(\xi, \eta)|^2}_{\text{reference}} + \underbrace{|\check{O}(\xi, \eta)|^2 + \check{R}^*(\xi, \eta) \check{O}(\xi, \eta) e^{i\pi a \xi} + \check{R}(\xi, \eta) \check{O}^*(\xi, \eta) e^{-i\pi a \xi}}_{\text{object}}$$

$$FT\{r^*(-x, -y) \otimes o(x+a, y)\} + FT\{r(x, y) \otimes o(-x-a, -y)\}$$



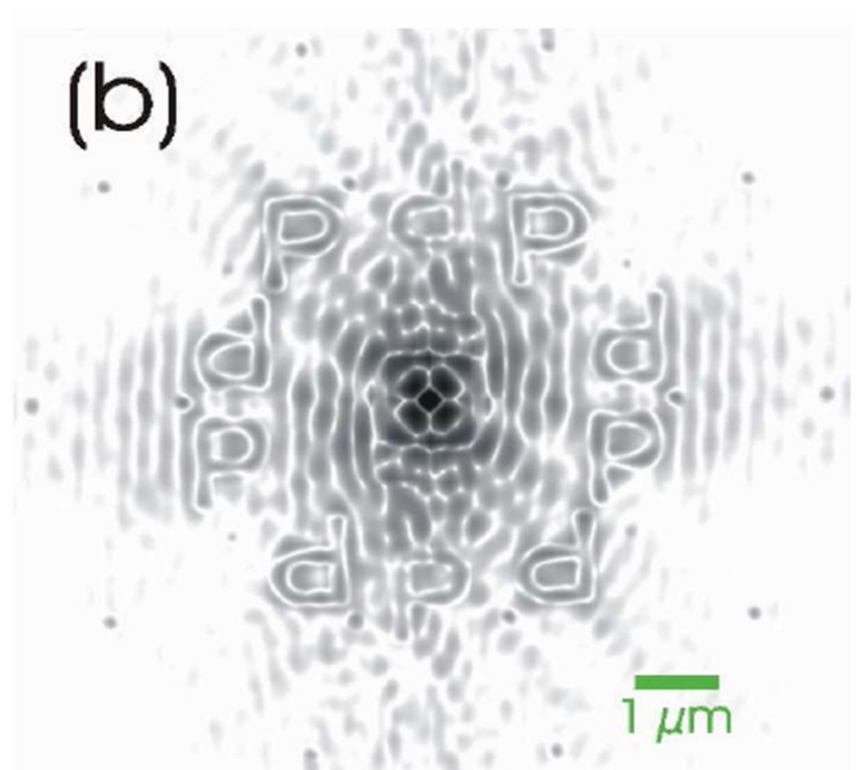
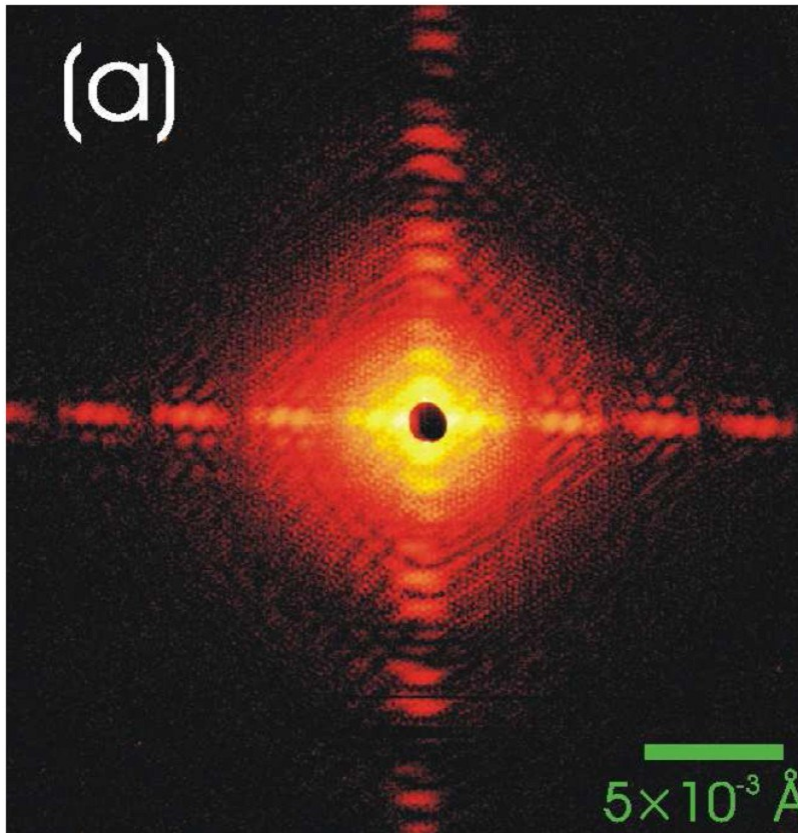
note: resolution determined by size of reference aperture

- FTH (9): more than one reference hole

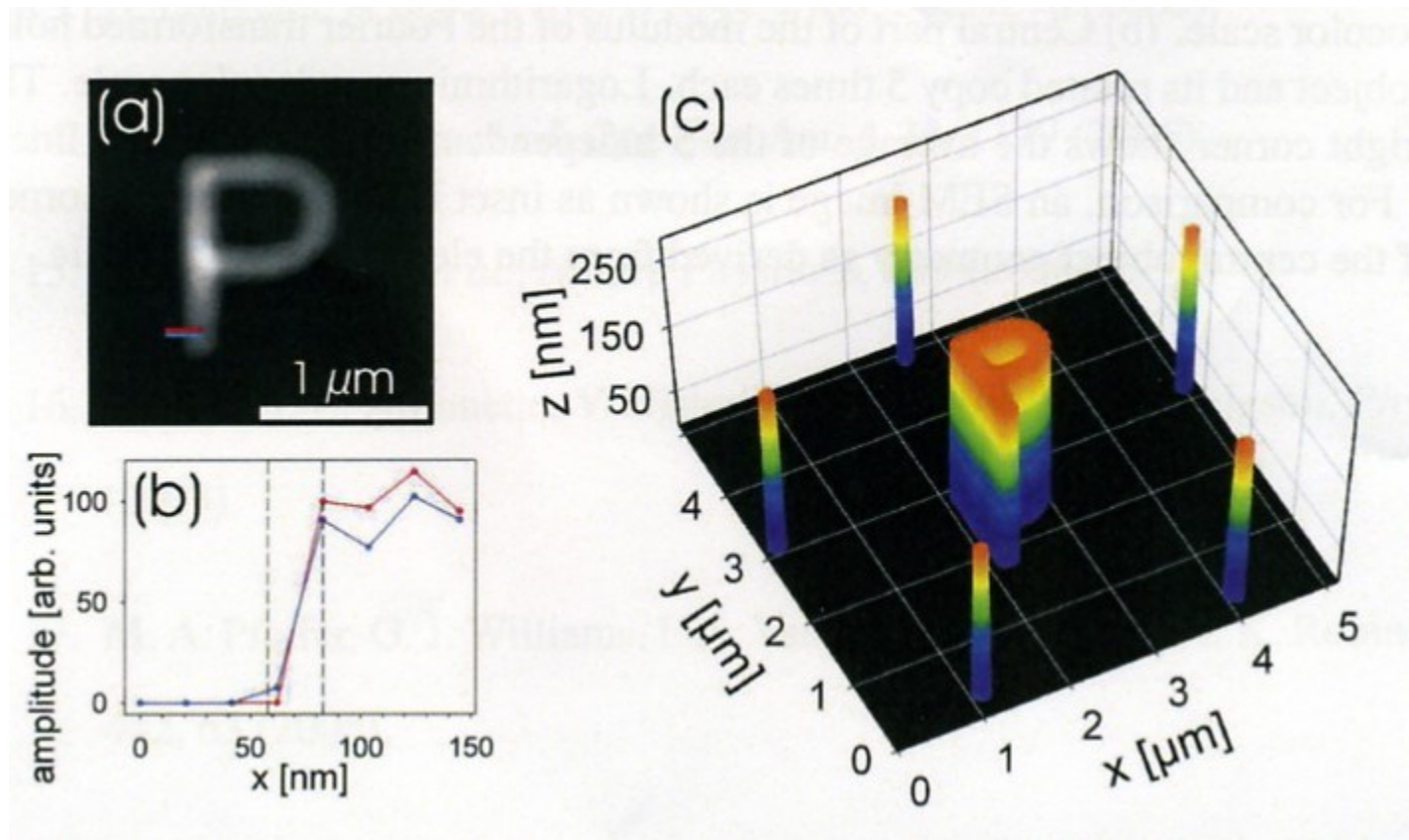


▪ FTH (10)

Au reference structure (letter P): letter elements 200 nm in width and 220 nm height on 50 nm Si_3N_4 -membrane. 5 Au reference dots of 175 nm diameter and 220 nm height on a circle of 2.5 μm around the sample. 200x 3s exposures ($\approx 1.4 \times 10^8$ ph/s through $10 \times 10 \mu\text{m}^2$ at 8 keV).



FTH (10):



(a) average of 100 phase retrieval runs

(b) slices yielding a resolution of ≈ 25 nm

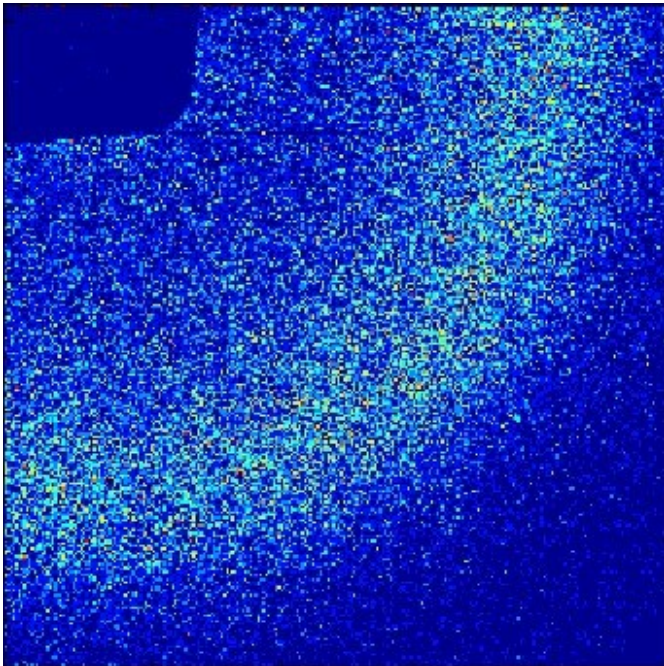
(c) visualization of the object as determined from the electron density profile

▪ (Magnetic) Scattering with coherent X-rays

If coherent light is scattered from a disordered system it gives rise to a random (grainy) diffraction pattern, known as “speckle”. A speckle pattern is an interference pattern and related to the exact spatial arrangement of the scatterers in the disordered system.

$$I(Q,t) \sim S_c(Q,t) \sim \left| \sum (f_j^{\text{charge}} + f_j^{\text{magnetic}}) e^{iQR_j(t)} \right|^2$$

j in coherence volume $c = \xi_t^2 \xi_l$

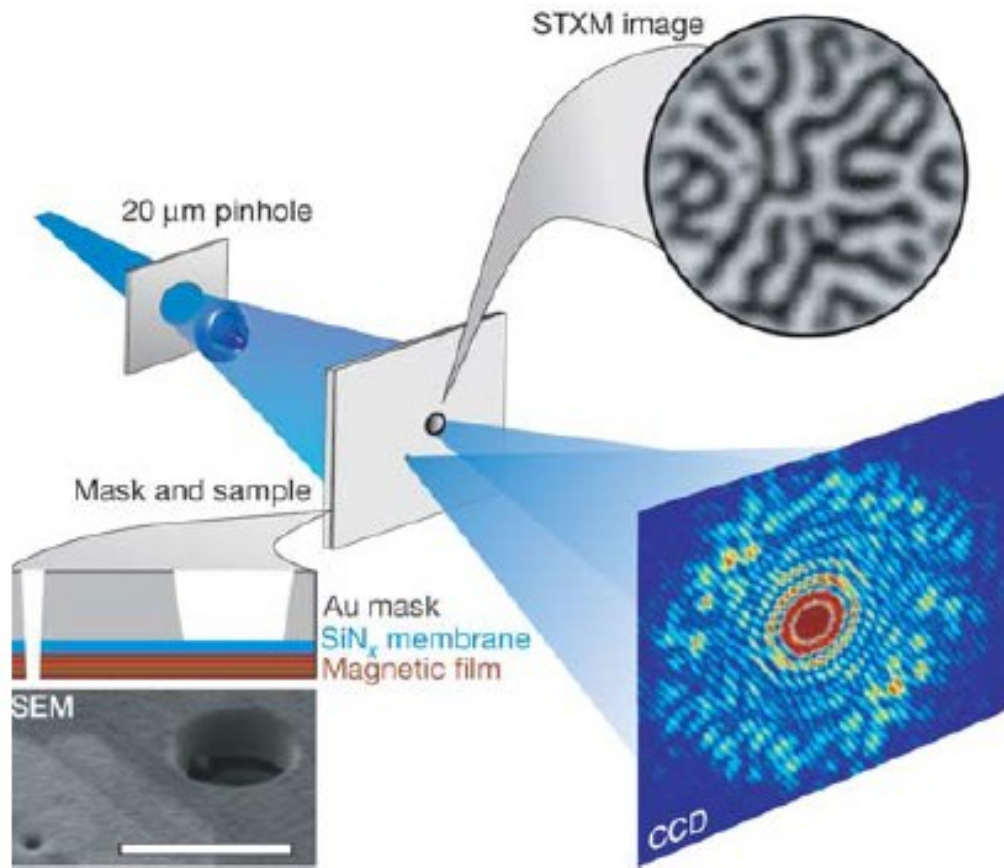


Incoherent Light:

$$S(Q,t) = \langle S_c(Q,t) \rangle_{V \gg c}$$

ensemble average

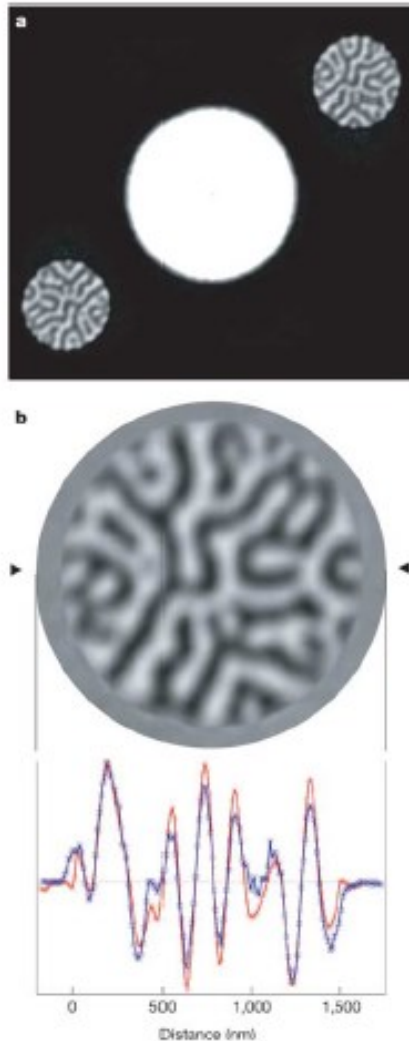
Fourier Transform Holography (1)



Random magnetic (stripe) domains in a [Co(4)Pt(7)]50 ML sample, illuminated together with a reference aperture (1.5 μm) at the Co LIII edge absorption edge with a 778 eV (1.59 nm) 20 μm coherent soft x-ray beam.

S. Eisebitt, J. Lüning, W.F. Schlotter, M. Lörger, O. Hellwig, W. Eberhardt and J. Stöhr,
NATURE, 432, 885 (2004)

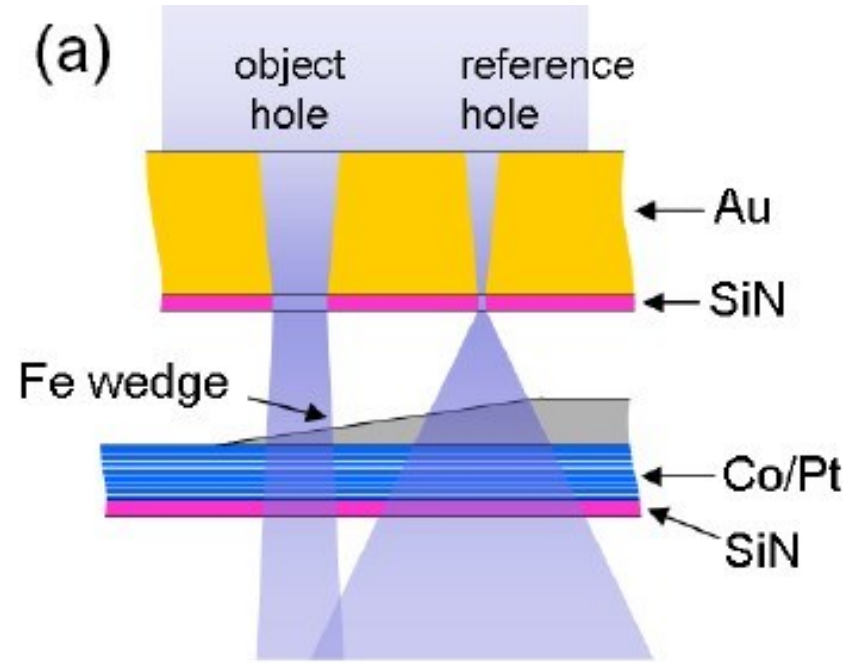
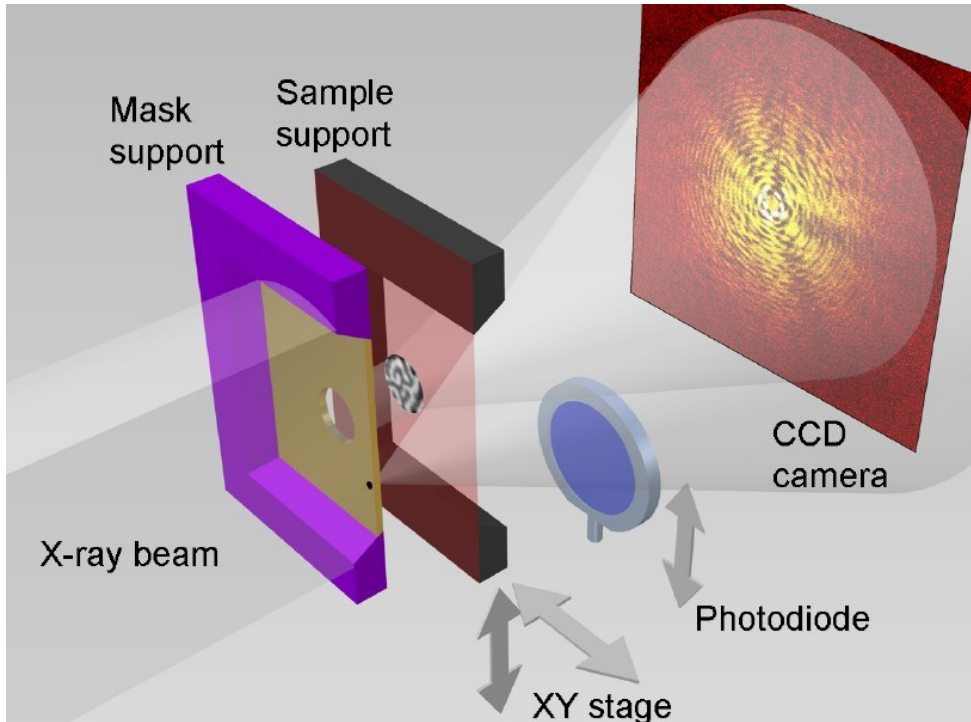
Fourier Transform Holography (2)



Reconstruction via direct 2-D Fourier transform of the [Co(4)Pt(7)]₅₀ speckle pattern (top). Difference pattern obtained from two reconstructed images recorded with different helicity (bottom) compared to line profiles through the difference pattern (red) and an STXM image (blue) revealing 50 nm resolution.

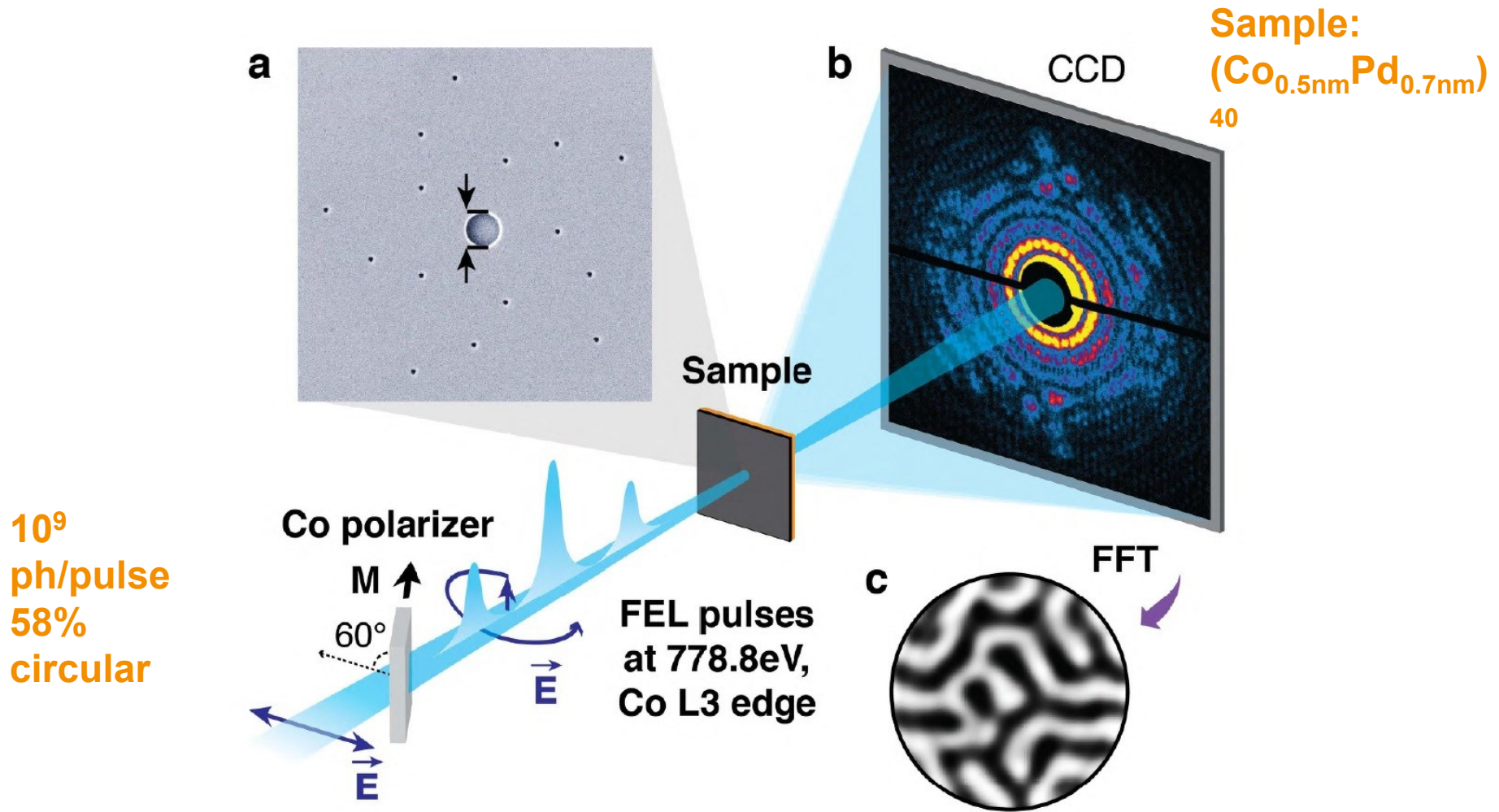
S. Eisebitt, J. Lüning, W.F. Schlotter, M. Lörger, O. Hellwig, W. Eberhardt and J. Stöhr, NATURE, 432, 885 (2004)

Soft X-ray holographic microscopy (I)



D. Stickler et al., APL **96**, 042501 (2010)

Resonant Magnetic Imaging at SXR (LCLS)



T. Wang et al., Phys. Rev. Lett. 108, 267403 (2012)

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X-Ray Photon Correlation Spectroscopy (XPCS)

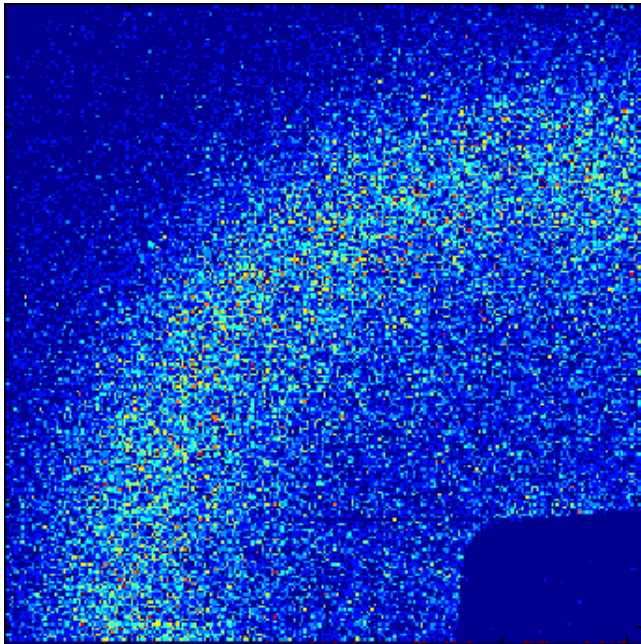
analogous to Dynamic Light Scattering (DLS) or Photon Correlation Spectroscopy (PCS) with visible light

- gives access to larger momentum transfers ($Q_{\max} = 2\pi \cdot \sin\theta / \lambda$) or shorter lengthscales
- not subject to multiple scattering
- can be combined with the surface sensitivity of X-rays

Introduction: Scattering with coherent X-rays

If coherent light is scattered from a disordered system it gives rise to a random (grainy) diffraction pattern, known as “speckle”. A speckle pattern is an interference pattern and related to the exact spatial arrangement of the scatterers in the disordered system.

$$I(Q,t) \sim S_c(Q,t) \sim \left| \sum e^{iQR_j(t)} \right|^2$$



j in coherence volume $c = \xi_t^2 \xi_l$

Incoherent Light:

$$S(Q,t) = \langle S_c(Q,t) \rangle_{V \gg c}$$

ensemble average

- quantify dynamics in terms of the intensity correlation function $g_2(\mathbf{Q},t)$:

$$I(\mathbf{Q},t) = |\mathbf{E}(\mathbf{Q},t)|^2 = \left| \sum b_n(\mathbf{Q}) \exp[i\mathbf{Q} \cdot \mathbf{r}_n(t)] \right|^2$$

Note: $\mathbf{E}(\mathbf{Q},t) = \int d\mathbf{r}' \rho(\mathbf{r}') \exp [i\mathbf{Q} \cdot \mathbf{r}'(t)]$ $\rho(\mathbf{r}')$: charge density

$$g_2(\mathbf{Q},t) = \langle I(\mathbf{Q},0) \cdot I(\mathbf{Q},t) \rangle / \langle I(\mathbf{Q}) \rangle^2$$

if $\mathbf{E}(\mathbf{Q},t)$ is a zero mean, complex gaussian variable:

$$g_2(\mathbf{Q},t) = 1 + \beta(\mathbf{Q}) \langle \mathbf{E}(\mathbf{Q},0) \mathbf{E}^*(\mathbf{Q},t) \rangle^2 / \langle I(\mathbf{Q}) \rangle^2 \quad \langle \rangle \text{ ensemble av.}; \beta(\mathbf{Q}) \text{ contrast}$$

$$g_2(\mathbf{Q},t) = 1 + \beta(\mathbf{Q}) |f(\mathbf{Q},t)|^2 \quad \text{with} \quad f(\mathbf{Q},t) = F(\mathbf{Q},t) / F(\mathbf{Q},0)$$

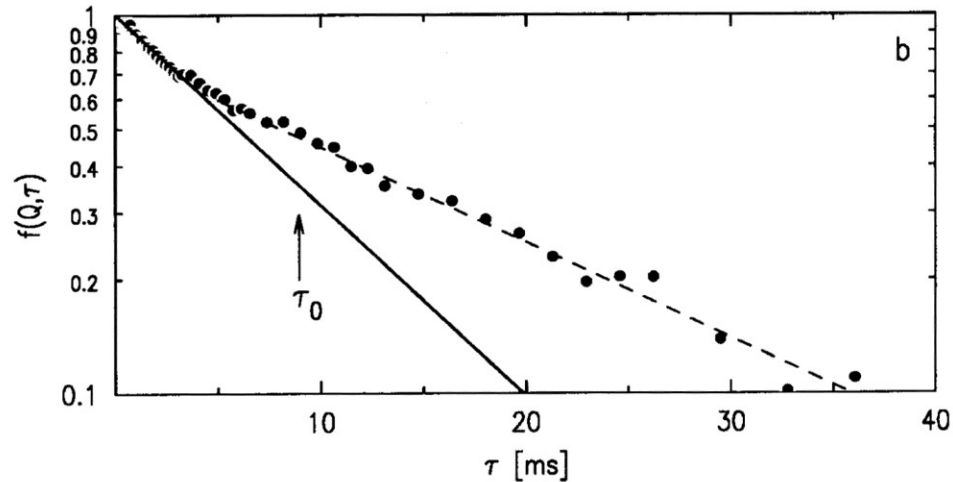
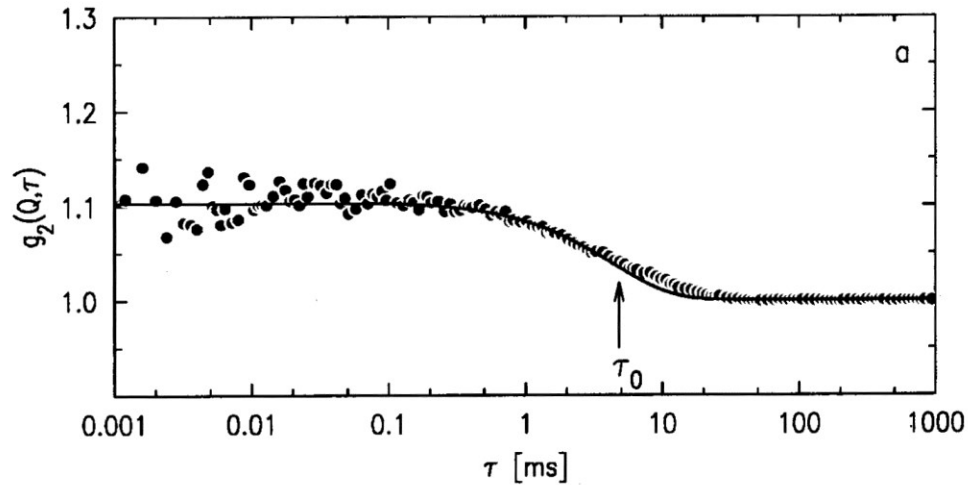
$F(\mathbf{Q},0)$: static structure factor N : number of scatterers

$$F(\mathbf{Q},t) = [1/N \{b^2(\mathbf{Q})\}] \left| \sum_{m=1}^N \sum_{n=1}^N \langle b_n(\mathbf{Q}) b_m(\mathbf{Q}) \cdot \exp\{i\mathbf{Q}[\mathbf{r}_n(0) - \mathbf{r}_m(t)]\} \rangle \right|$$

- A time correlation function $g_2(\mathbf{Q}, \tau)$

$$g_2(\mathbf{Q}, t) = 1 + \beta(\mathbf{Q}) |f(\mathbf{Q}, t)|^2 \quad \text{and} \quad f(\mathbf{Q}, t) = \exp(-\Gamma t) = \exp(-t/\tau)$$

$\beta(\mathbf{Q})$ \updownarrow



▪

Examples:

Dynamics of interacting colloidal particles

Dynamics in heterodyne mode detection

Dynamics at surfaces/interfaces

Dynamics (of a liquid crystalline system) near a phase transition

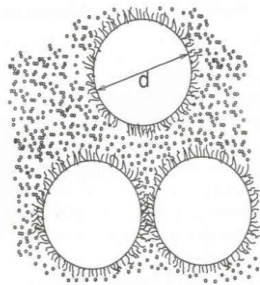
Non-equilibrium dynamics

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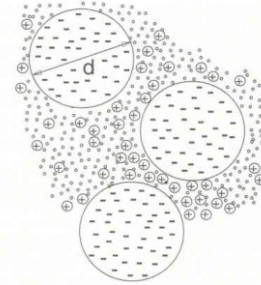
Dynamics of interacting Colloidal Particles

Colloidal particles (paints, ink, clays,silica,.....) suspended in a solvent (molecular fluid, ...). Stabilised against van der Waals“ attraction.

„Hard Spheres“



„Soft Spheres“



Structure: $S(Q) = 1 + 4\pi\rho \int [g\{r\} - 1] (\sin(Qr)/Qr) r^2 dr$; $g\{r\} = \exp[-V(r)/kT]$ ρ : number density

$\Phi \ll 1\%$:

$$S(Q) = 1$$

$$D(Q) = D_0 (=kT/6\pi\eta R_H)$$

$\Phi > 1\%$:

$$V(r) = \begin{cases} 0 & r \geq d \\ \infty & r < d \end{cases}$$

weak interaction: DLVO

$$V(r) \propto (eZ_{eff})^2/r \exp(-\kappa r)$$

$$S = S(Q, \Phi) \text{ (Percus-Yevick)}$$

$$S = S(Q, \Phi, Z_{eff}, \kappa) \text{ (MSA, RMSA)}$$

Dynamics: Interaction: colloid-solvent, colloid-colloid, hydrodynamics

Smoluchowski (many particle) diffusion equation:

$$D_{short}(Q) = D_0/S(Q) * H(Q) \quad t \ll R^2/D_0$$

$H(Q)$ (Beenakker and Mazur)

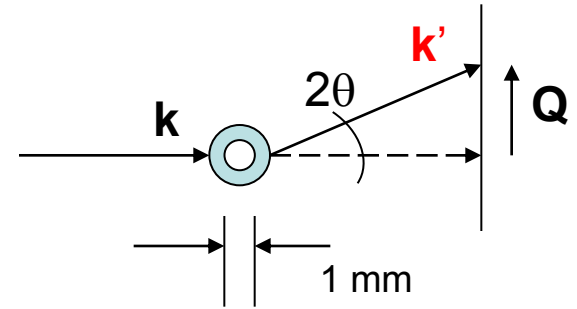
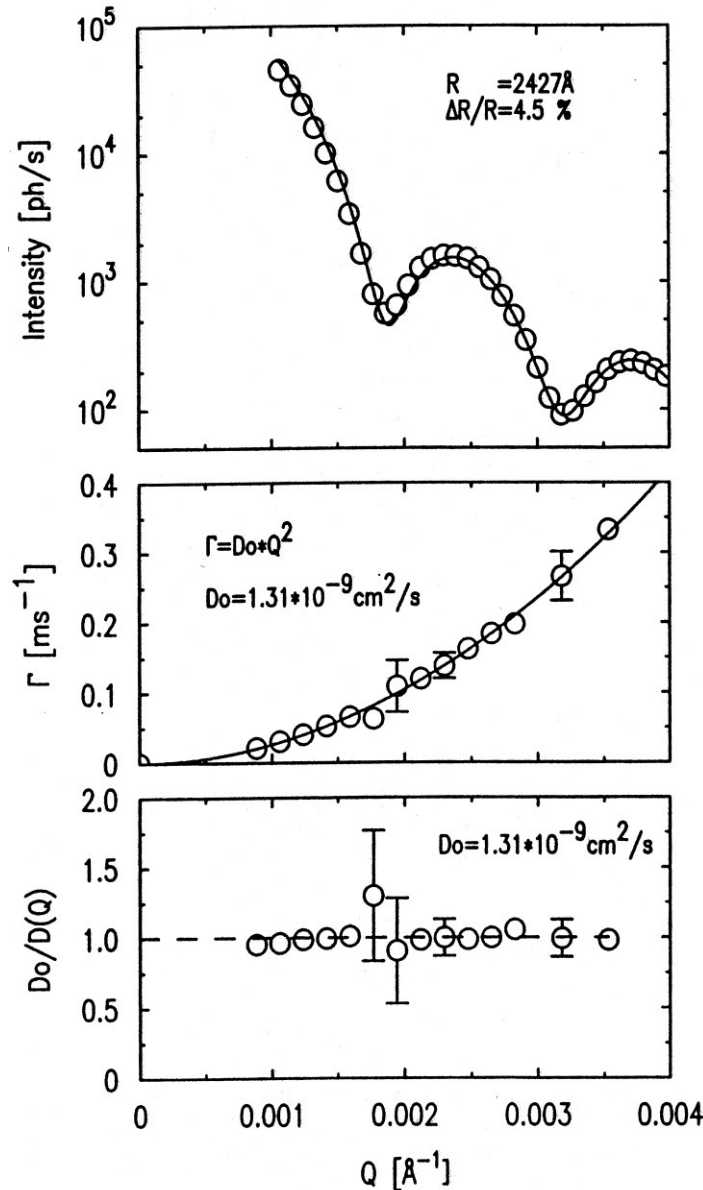
$H(Q)$ (via $D_0, S(Q), D(Q)$)

The dilute case

$$I \sim |F(Q)|^2 S(Q)$$

$$\sim [(\sin QR - QR \cos QR) / (QR)^3]^2$$

$$\Gamma = D_0 Q^2$$



$$Q = k' - k$$

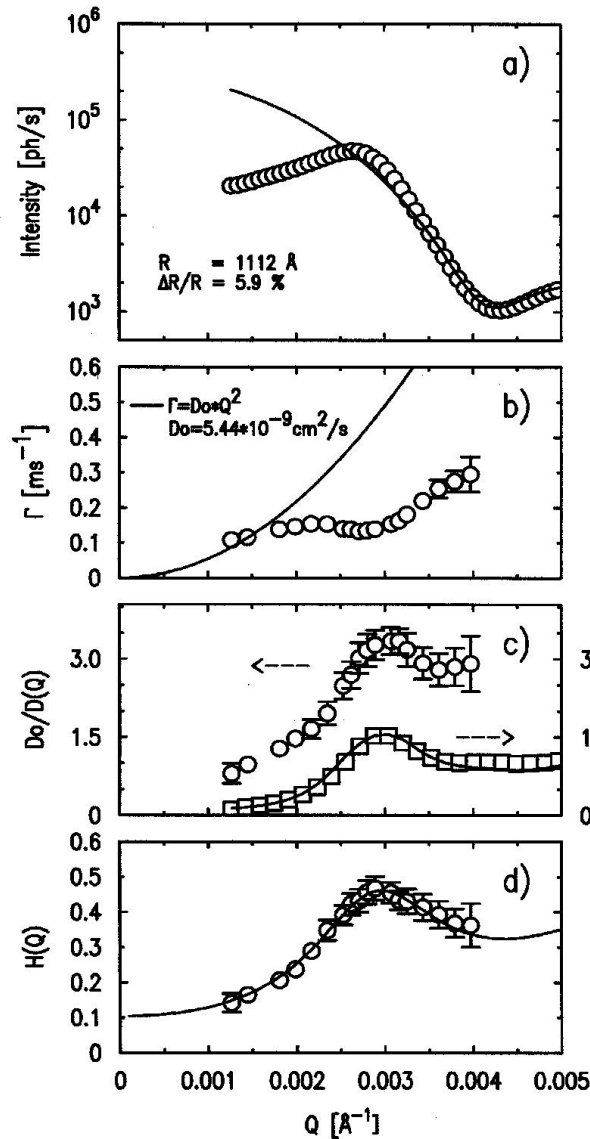
$$Q = 2k \sin \theta$$

$$k = 2\pi/\lambda$$

G. Grübel, A. Robert, D. Abernathy
 8th Tohwa University International
 Symposium on "Slow Dynamics in
 Complex Systems", 1998, Fukuoka, Japan

▪ The Hard Sphere Case

Poly-
methylmetacrylate
37% volume fraction
in cis-decaline
sterically stabilized
(**hard-spheres**)



--- $F(Q)$

--- $\Gamma = D_0 Q^2$

$S(Q) = I(Q)/F(Q)$

caging (deGennes
narrowing)

$H(Q) = S(Q)/[D_0/D(Q)]$

--- δ - γ expansion

▪

Examples:

Dynamics of interacting colloidal particles

Dynamics in heterodyne mode detection

Dynamics at surfaces/interfaces

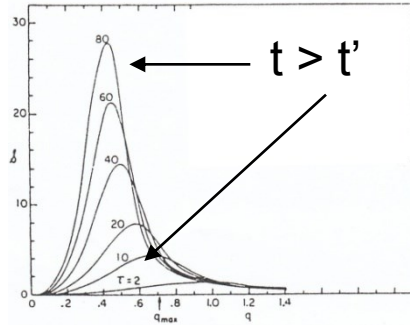
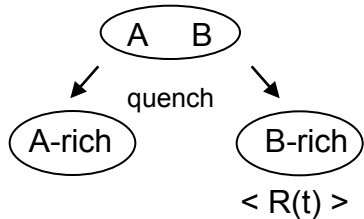
Dynamics (of a liquid crystalline system) near a phase transition

Non-equilibrium dynamics

...

Non-Equilibrium Dynamics

Domain coarsening in phase separating systems (glasses, alloys,...) e.g. after quenching, aging...



Phase – separating Glass

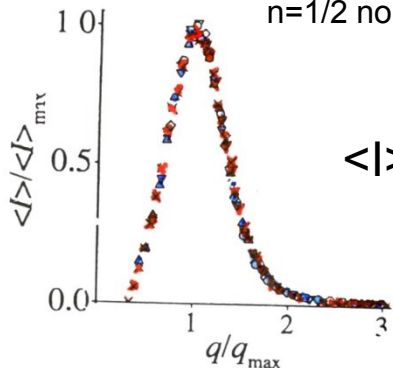
Malik *et al.*, PRL 81, 5832, 1998



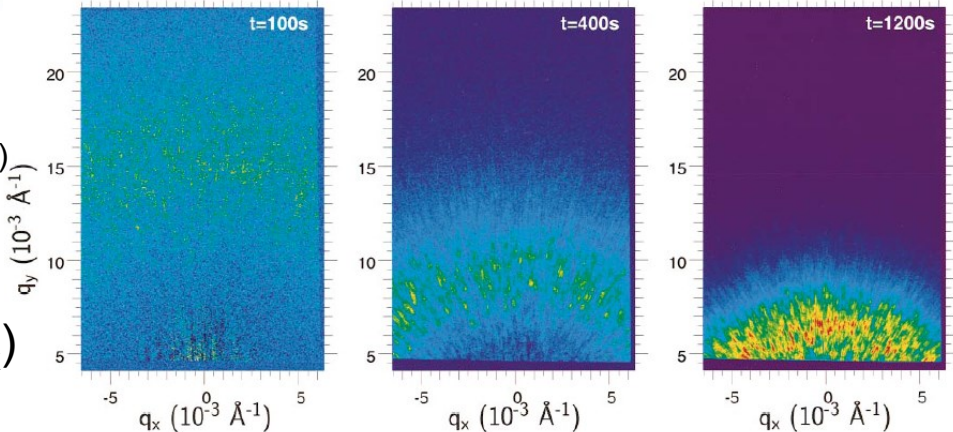
quench
 (B_2O_3) -rich (SiO_2) -rich $943\text{K} < T < 963\text{K}$

Dynamic Scaling: $\langle R(t) \rangle \sim t^n$

$n=1/3$ conserved order parameter (model B)
 $n=1/2$ non-cons. order parameter (model A)



$$\langle I \rangle(q,t) / \langle I \rangle_{\text{max}} = F(q/q_{\text{max}})$$



$$\langle R(t) \rangle \sim t^n \quad n=1/3$$

XPCS: investigate fluctuations around the average scaling behaviour: $\tau = \tau(q,t)$

Phase-Ordering in Cu_3Au

high T: fcc sites occupied by either Cu or Au

$T \leq T_c = 383 \text{ C}$: ordering with Au on corner and Cu on face sites

4-fold degenerate ground-state

can be chosen in 4 different ways)

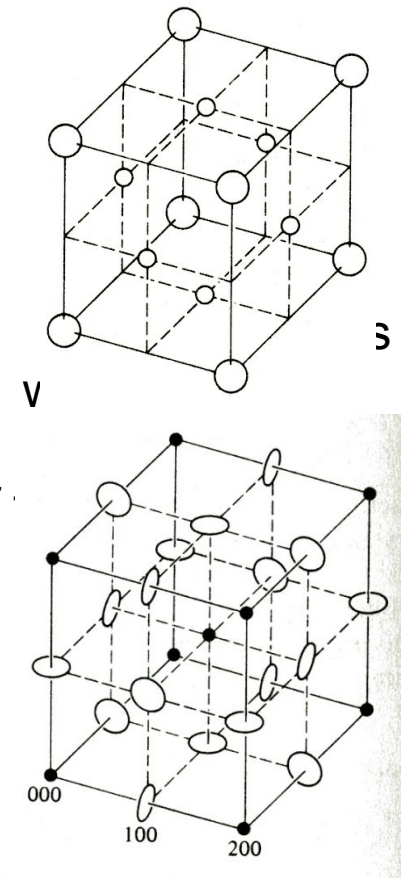
groundstates separated by domain walls

walls give rise to ellipse shaped superlattice

reflections of

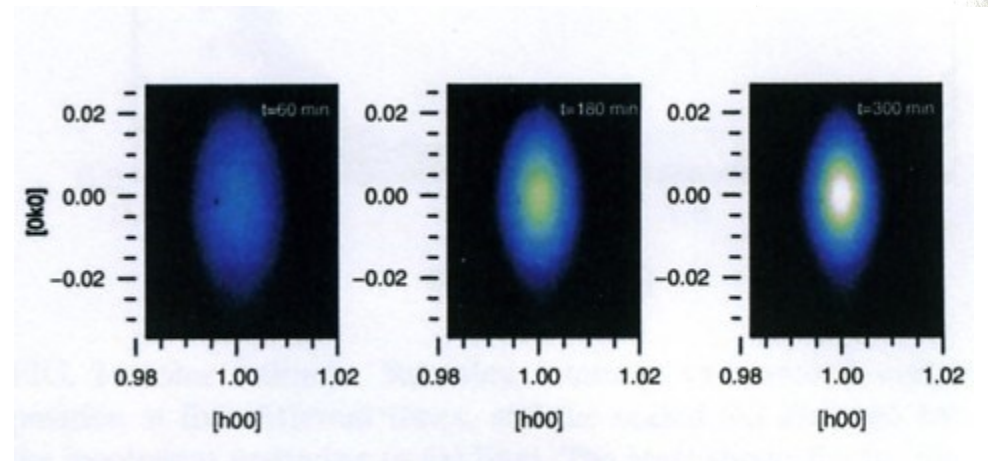
quench: domain formation and growth in disordered phase

coarsening with $R \sim t^{1/2}$



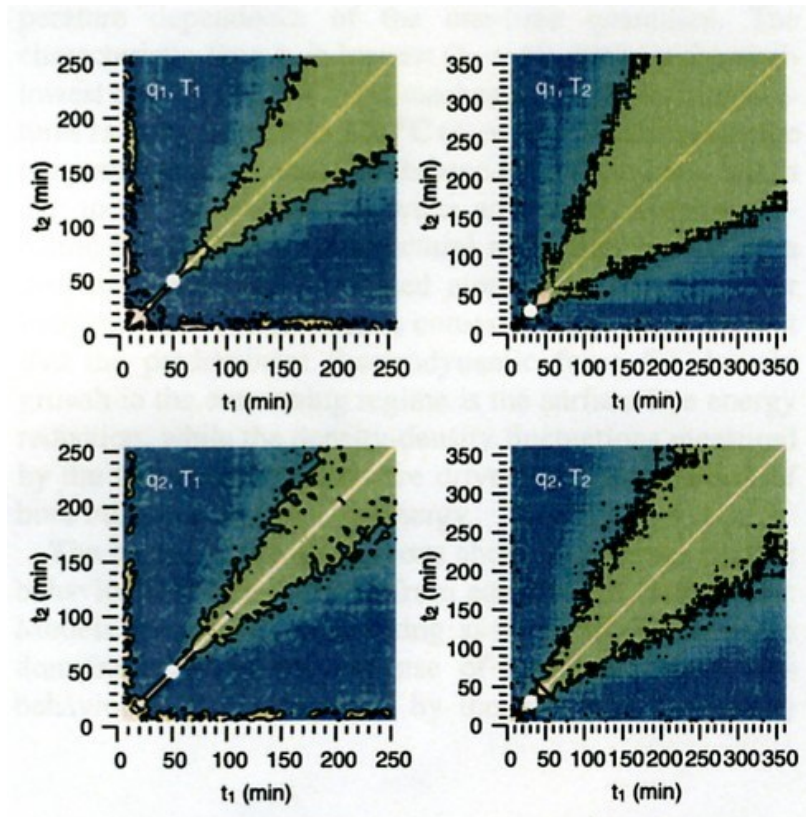
[100] superlattice reflection
after quench from 425 C to
370 C

Fluerasu&Sutton, PRL94(2005)55501



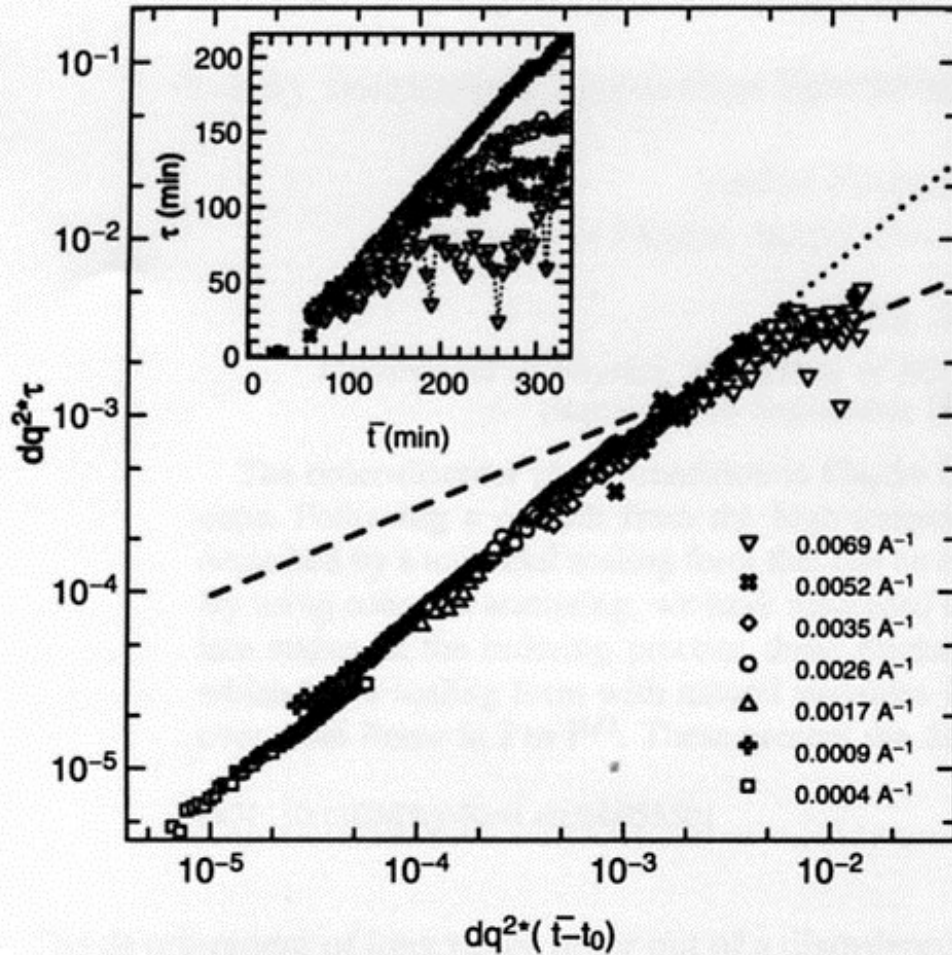
- Study fluctuations about the average behaviour: XPCS characterize by two-time correlation function

$$C(q, t_1, t_2) = \frac{\langle I(t_1) I(t_2) \rangle - \langle I(t_1) \rangle \langle I(t_2) \rangle}{[\langle I^2(t_1) \rangle - \langle I(t_1) \rangle^2]^{1/2} [\langle I^2(t_2) \rangle - \langle I(t_2) \rangle^2]^{1/2}}$$



rescaled correlation time $\tau \sim t_{\text{mean}}$ in the low t_{mean} limit

$\sim t_{\text{mean}}^{1/2}$ in the high t_{mean} limit



= Q - [100]

▪

XPCS at a FEL source

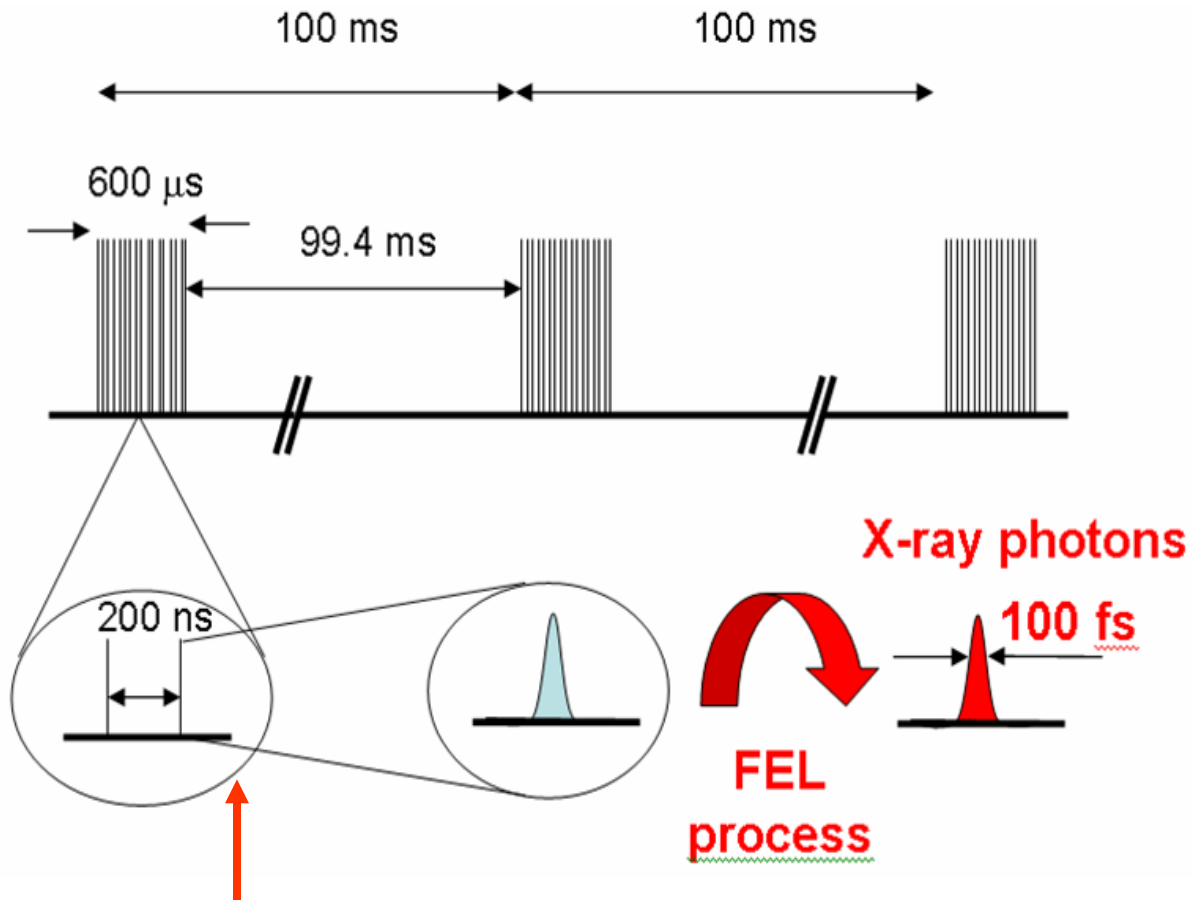
Magnetization Dynamics

Ferroelectrics

Ultrafast dynamics at surfaces and interfaces of liquids

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XPCS at a XFEL source:



$t > 0.1$ s
 $200\text{ns} < t < 600$
 μs :

” movie” mode

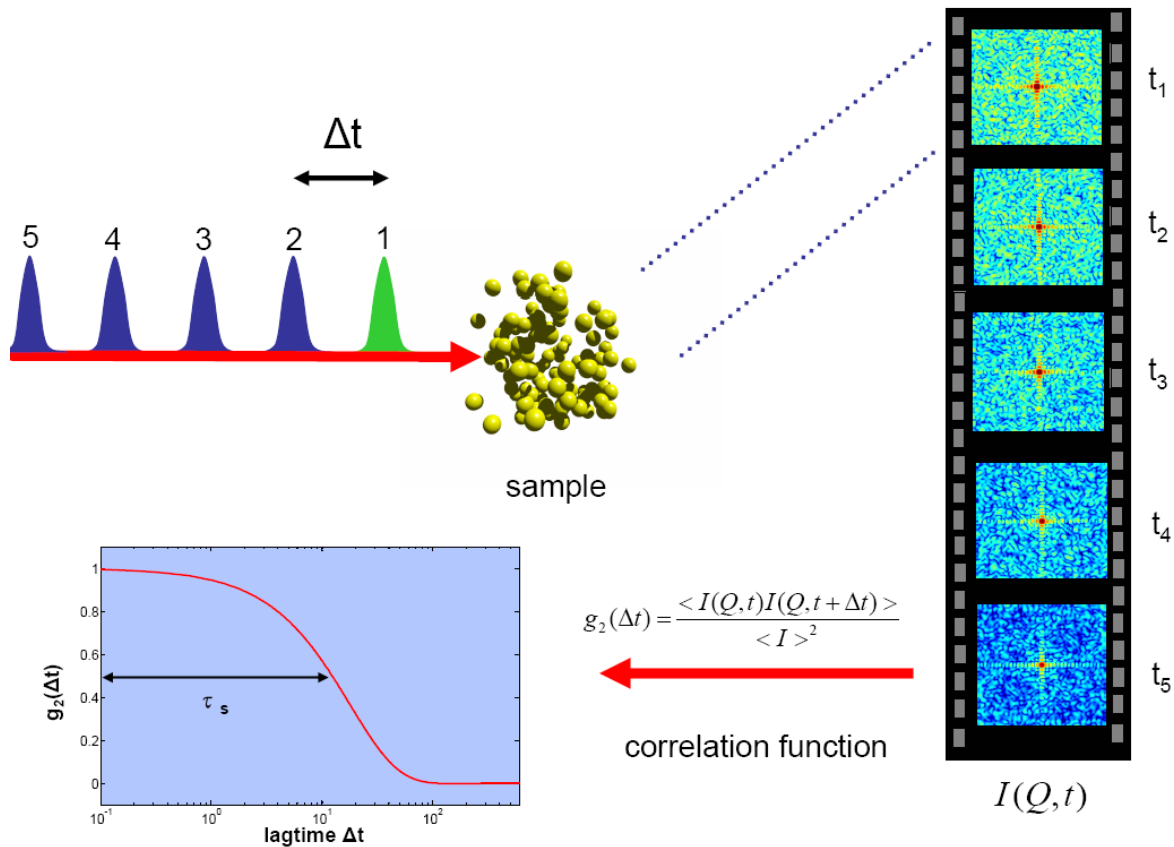
$1\text{ps} < t < 10$ ns:

for “all” times:

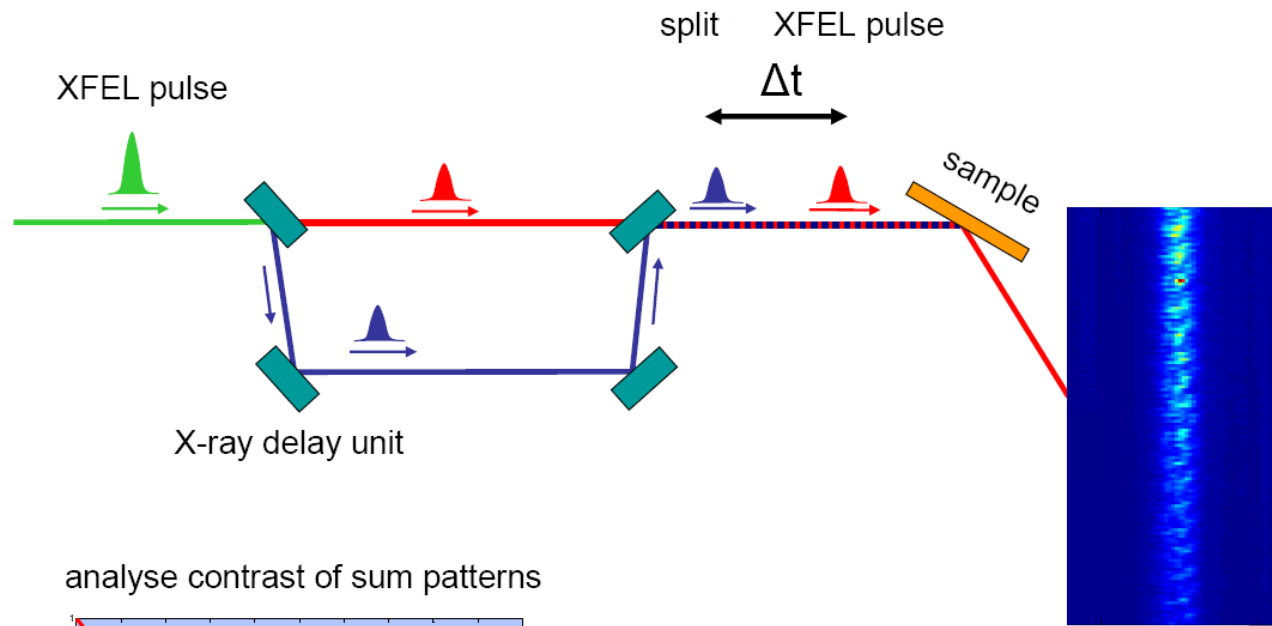
“delay-line” mode

“pump-probe” mode

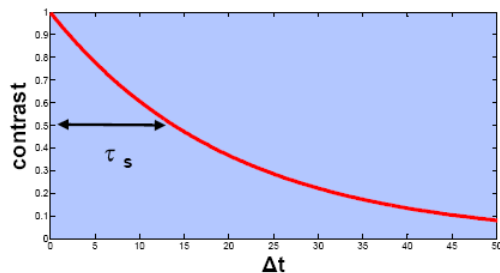
- XPCS at a FEL source: Movie Mode



Delay Line Mode



analyse contrast of sum patterns



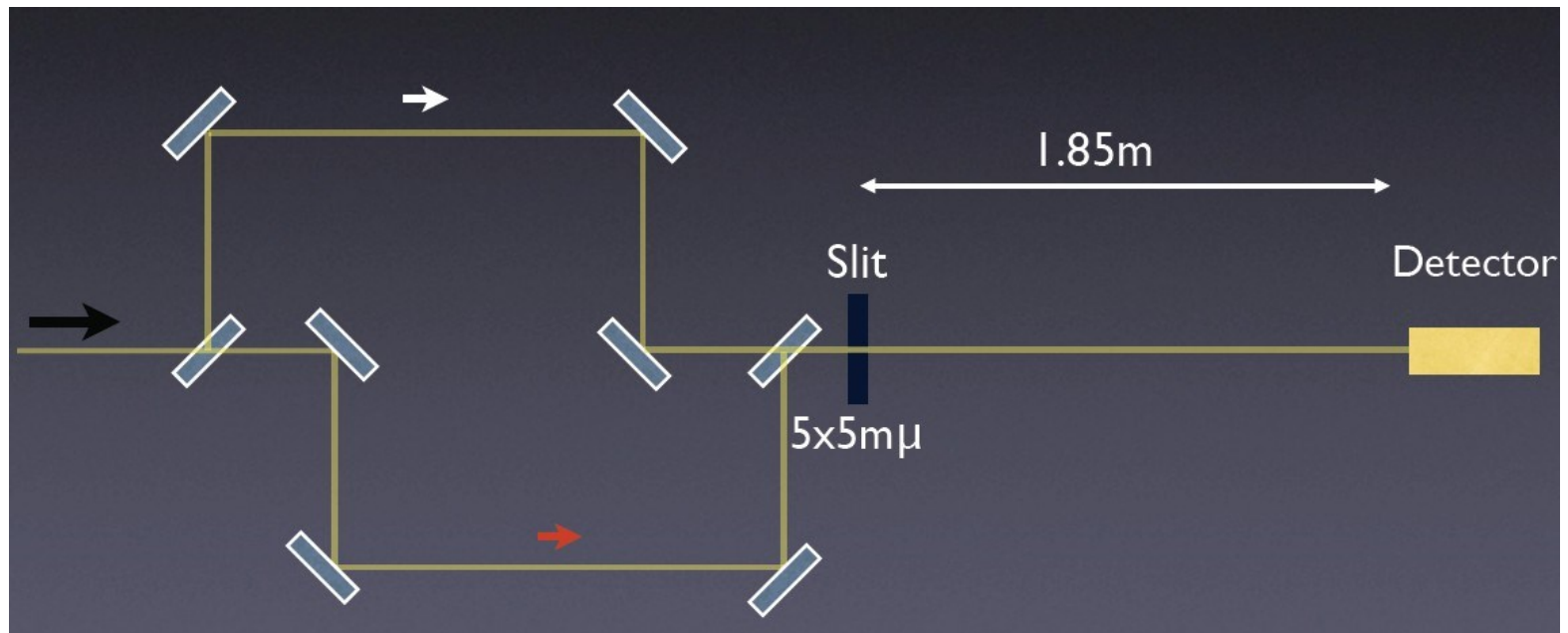
sum of speckle patterns
from prompt and delayed
pulse recorded on CCD

“Delay Line” Mode: $1\text{ ps} < \Delta t < 10\text{ ns}$ ($1\text{ ps} \Leftrightarrow 0.3\text{ mm}$; $1\text{ ns} \Leftrightarrow 300\text{ mm}$) “luminosity limited”.

Delay Line for “hard” X-Rays

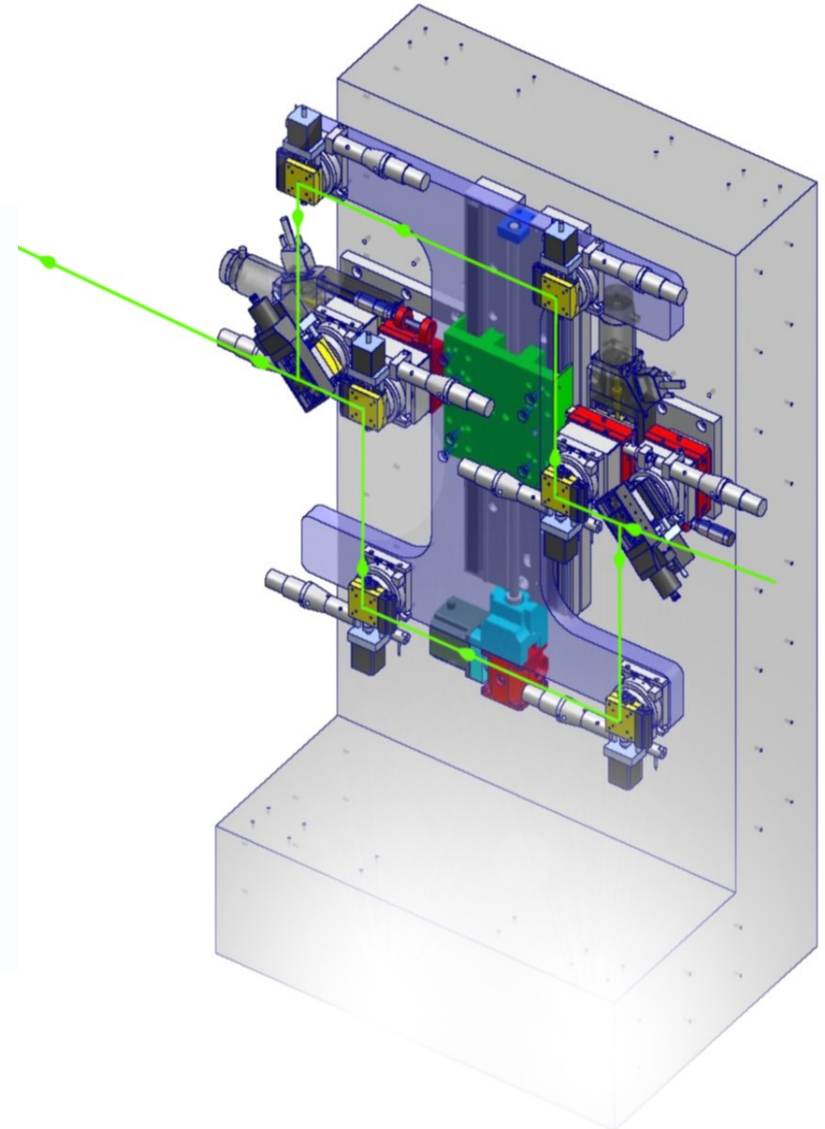
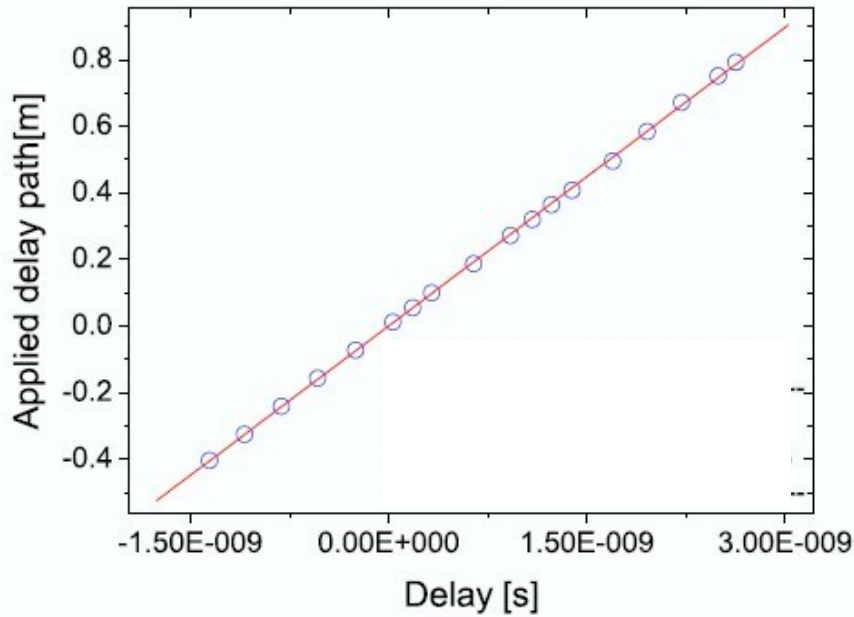
Si(511) at 8.38 keV

$\Delta t_{\max} = 2.8 \text{ ns}$

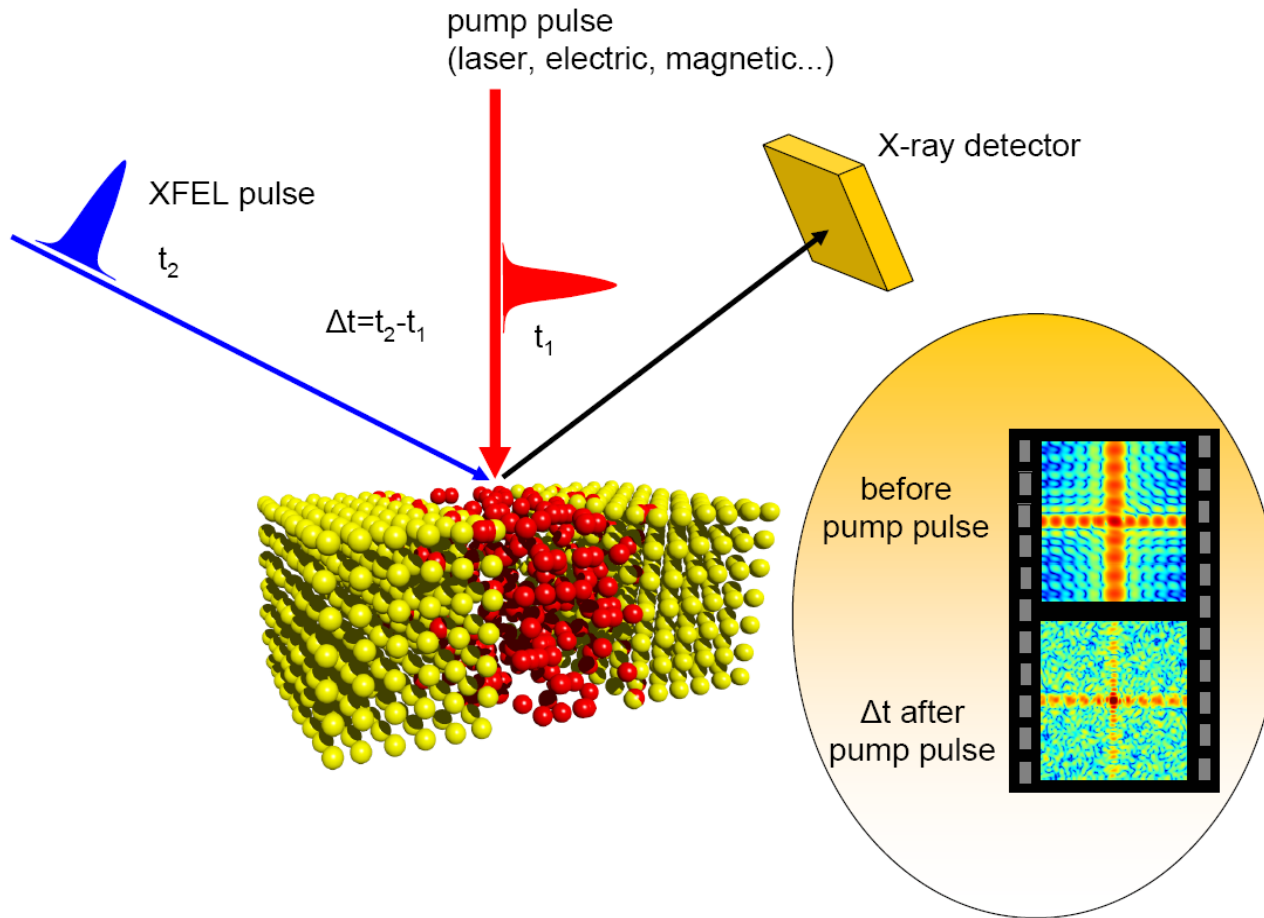


W. Roseker, H. Schulte-Schrepping,
A. Ehnes, H. Franz, O. Leupold and
G. Grübel

- X-Ray delay line



XPCS at a FEL source: pump-probe mode



The XFEL

www.xfel.eu



← 3.4km →

