Excercises „Methoden Moderner Röntgenphysik II“

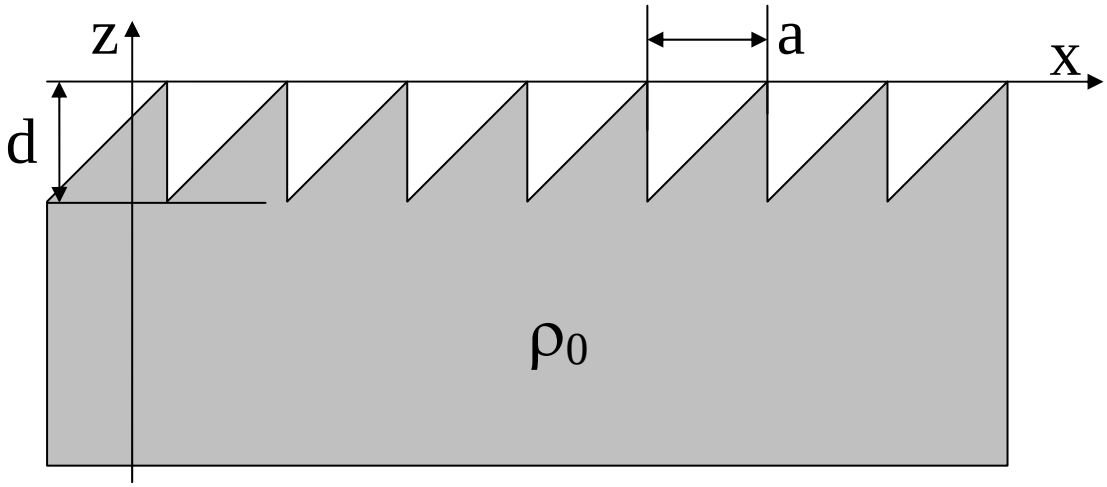
— X-Ray Reflectivity —

In Born-Approximation, the x-ray intensity reflected by a surface is given by

$$I(q_z) \propto \frac{1}{q_z^4}\left|\int_{-\infty}^{\infty}\frac{d\rho(z)}{dz}\exp(iq_z z)dz\right|^2$$

with the wave vector transfer q_z along z and the electron density profile $\rho(z)$ along z.

1) Calculate the reflected intensity of the following “sample surface with steps” by first generating the function $d\rho(z)/dz$ averaged over x.



2) Compare the result with a reflectivity of a one-layer system with substrate density ρ_0 , film density $\rho_0/2$ and film thickness d.

For a smooth single surface of a substrate with refractive index $n=1-\delta$ (no absorption) the exact Fresnel reflectivity $I(a)$ is given by

$$I(\alpha) = \left|\frac{k_{z1}-k_{z2}}{k_{z1}+k_{z2}}\right|^2 \quad \text{with} \quad k_{z1} = \frac{2\pi}{\lambda}\sin\alpha \quad \text{and} \quad k_{z2} = \frac{2\pi}{\lambda}\sqrt{n^2-\cos^2\alpha}$$

and the incident angle a .

3) Show that the Fresnel reflectivity is identical to the Born approximation for a smooth surface for $\sin a >> 2\delta$ **.**