Methoden moderner Röntgenphysik I

Coherence based techniques

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Outline

18.12. 2008

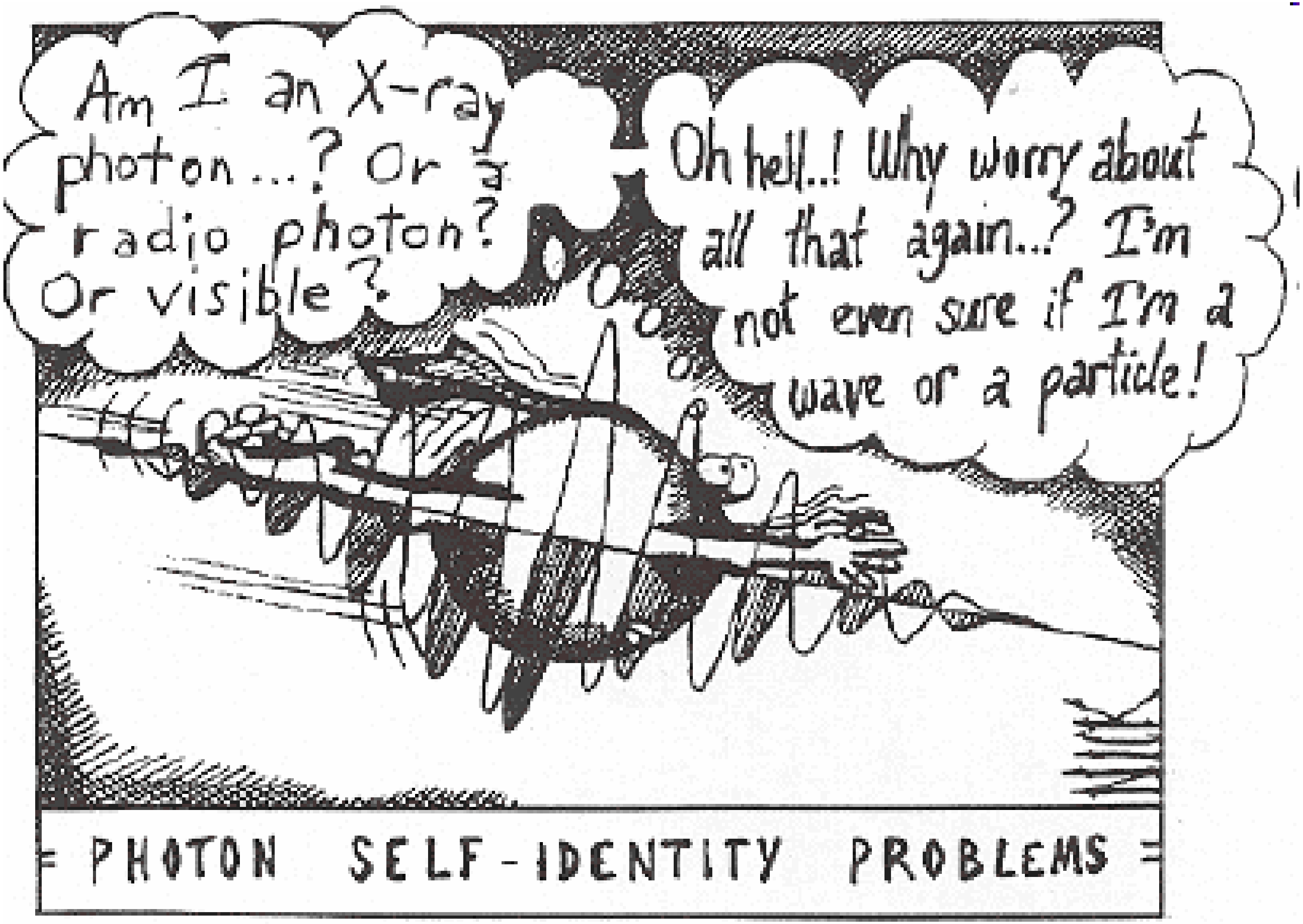
- **Introduction to Correlations**
- **Coherence Lengths**
- **van Zittert-Cernike Theorem**
- **Examples**
- **Speckle Pattern**

8.01. 2009

Coherent Diffractive Imaging

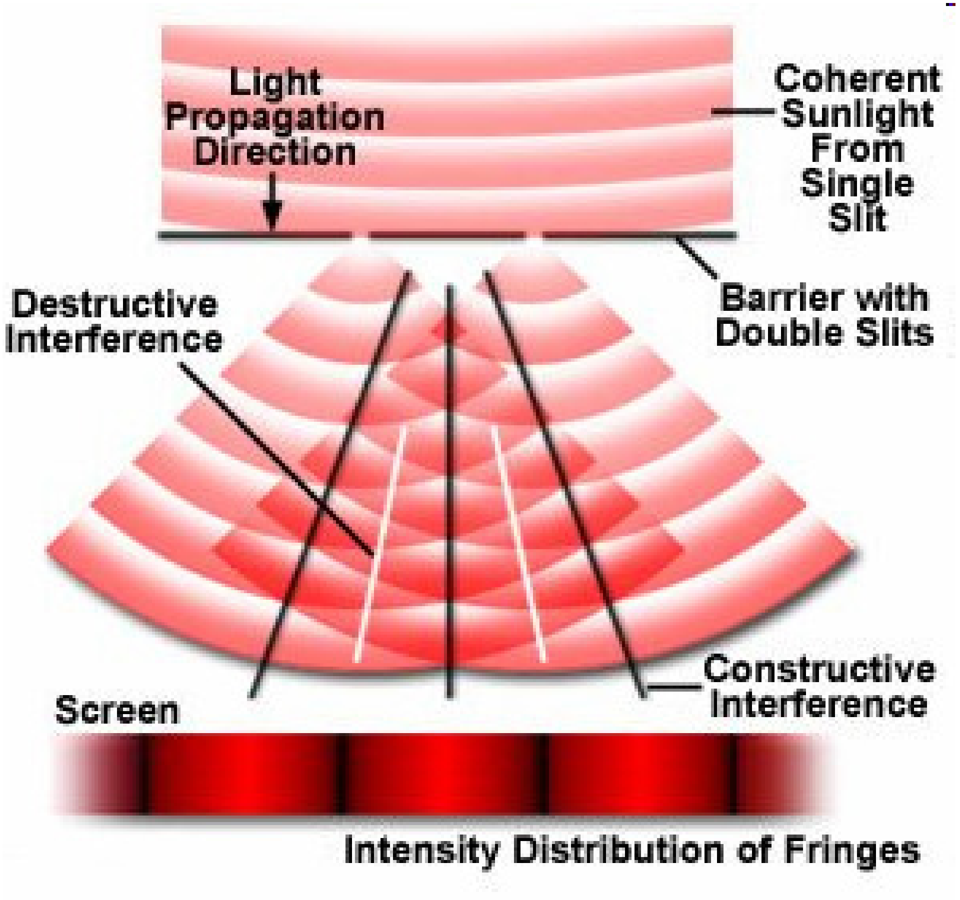
15.01.2009

Correlation Spectroscopy



www.etresoi.ch/Denis/quantum.html

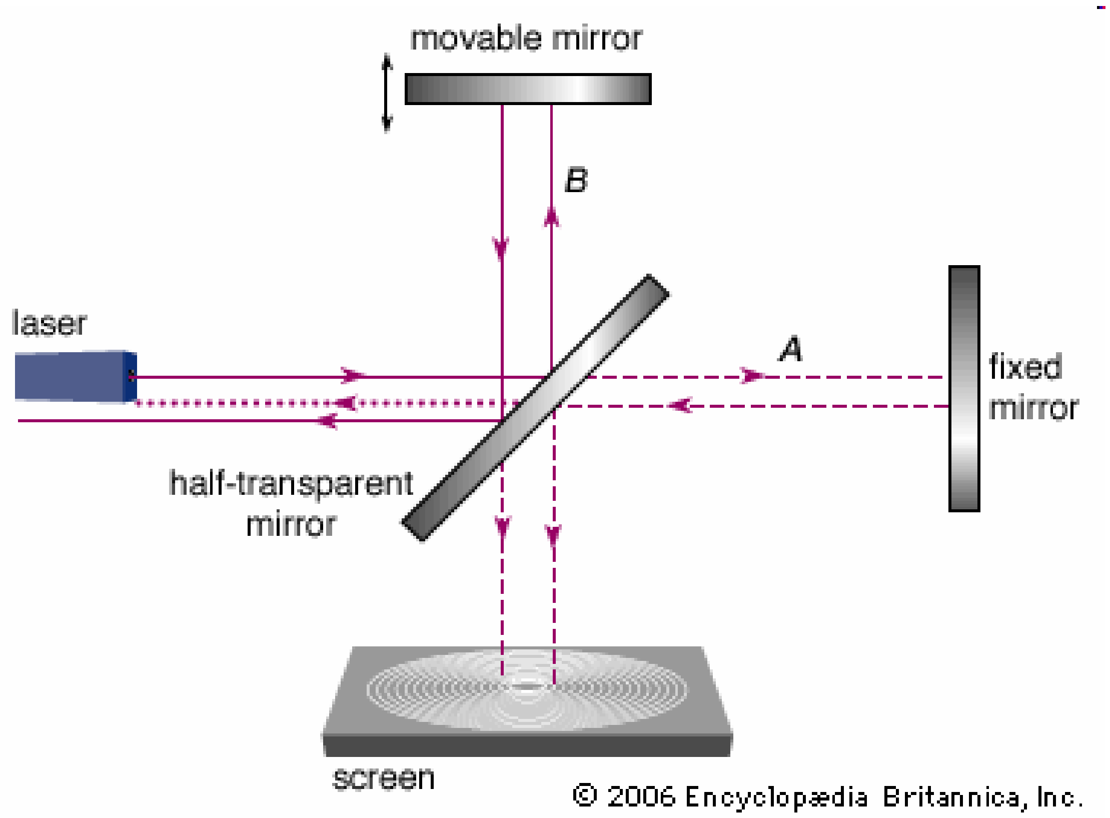
Young's Double Slit Experiment



Thomas Young, 1773-1829

Light is a wave

Michelson Interferometer

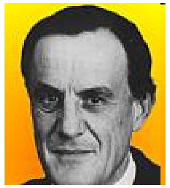


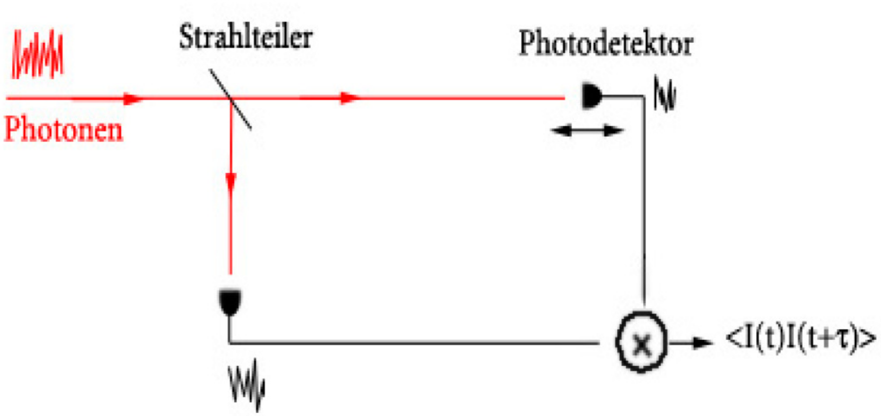
Albert Abraham Michelson
1852-1931

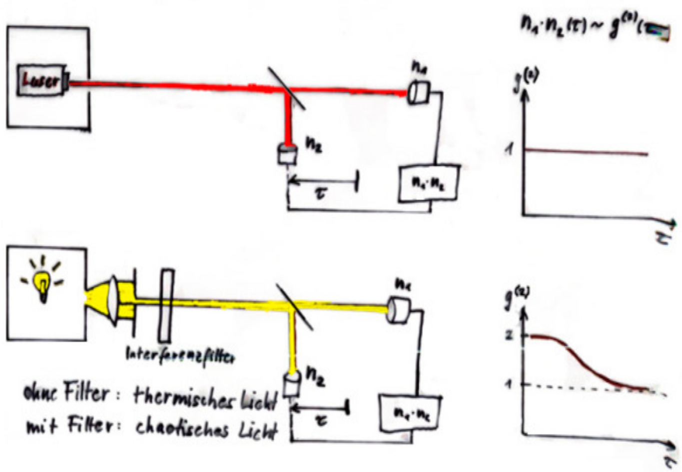
no Äther
-> Einstein

Intensity Correlations
2nd order correlations

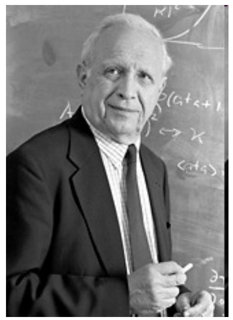
Robert Hanbury Brown
Richard Q. Twiss





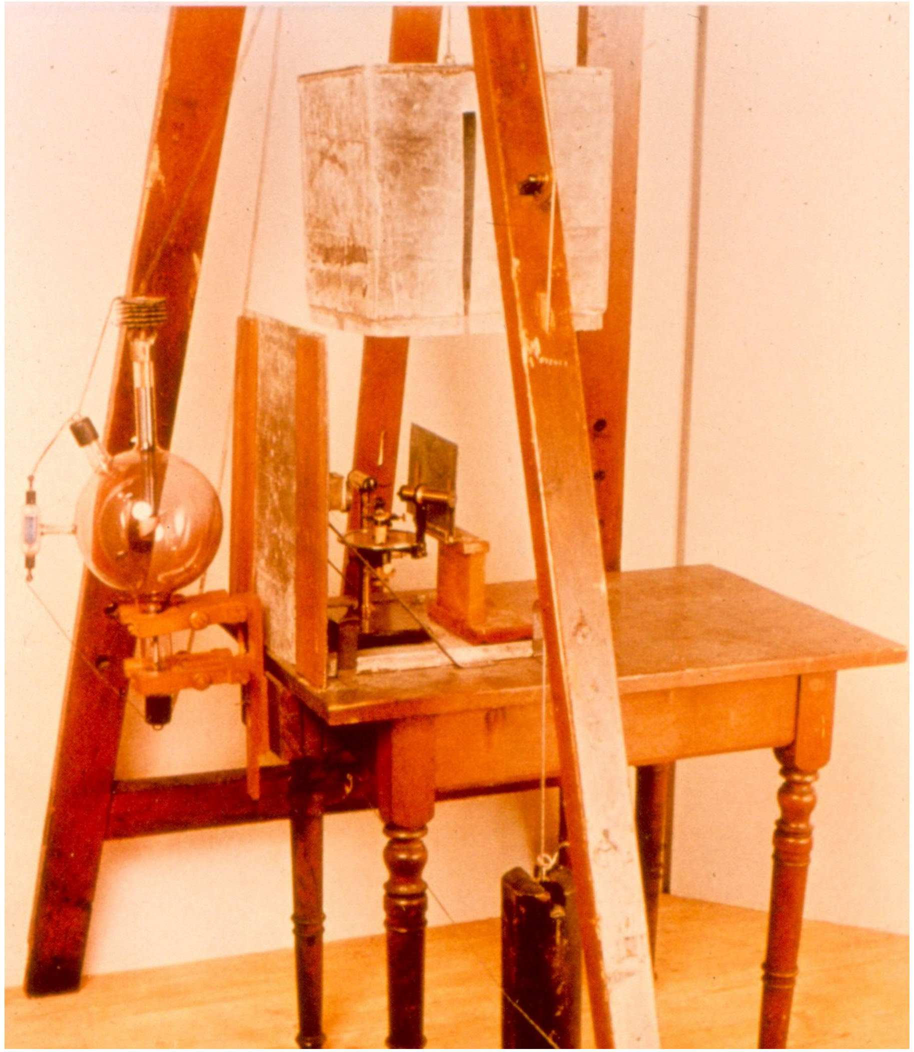


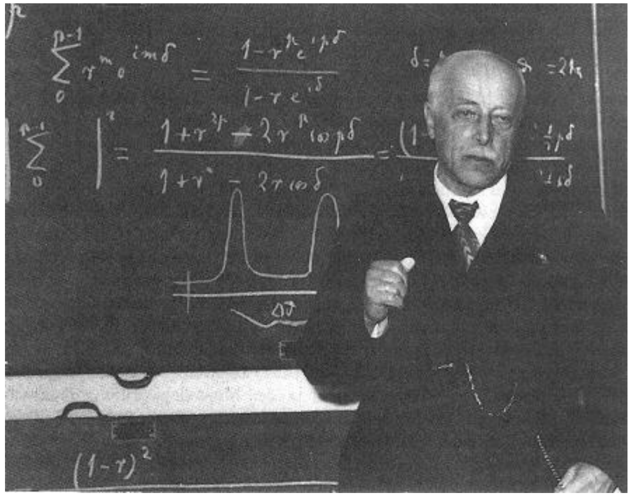
Roy Glauber- Nobel price in Physics 2005
Theory of quantum coherence and optics



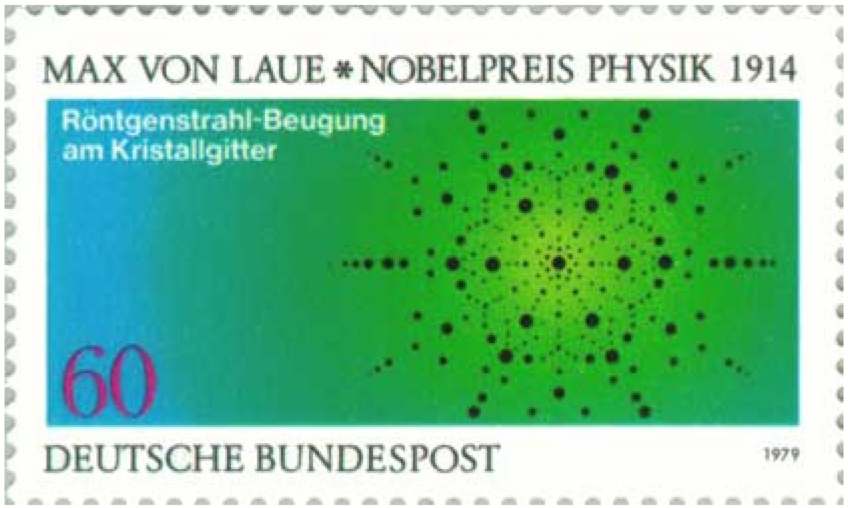
Interference from a crystal lattice

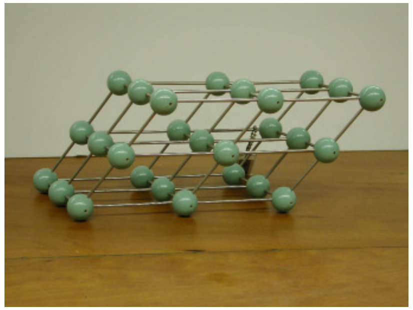
Laue, Friedrich and Knipping 1912

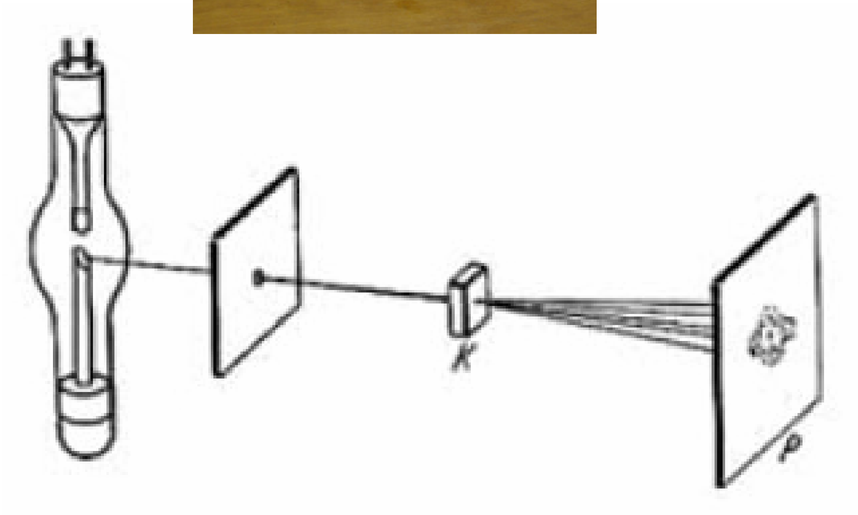




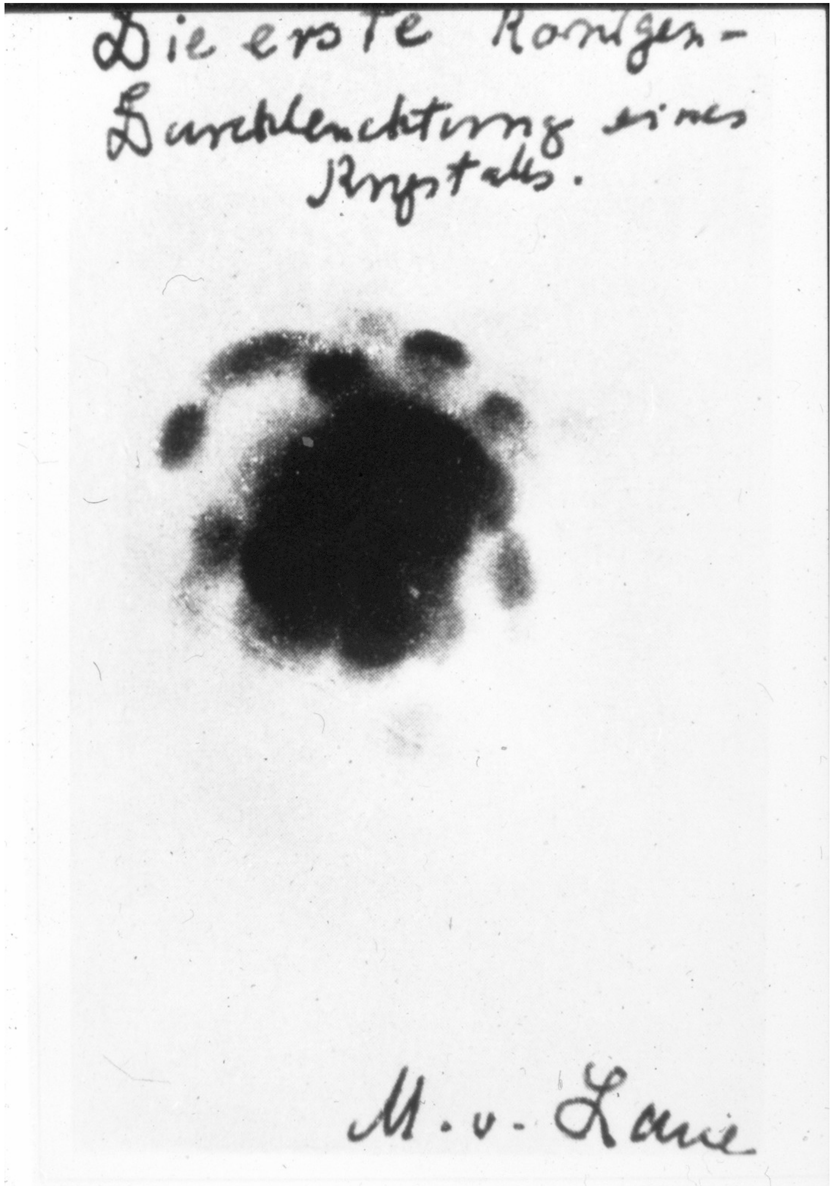
LAUE hielt seine Idee keineswegs geheim, sondern diskutierte sie sowohl mit SOMMERFELD als auch in der berühmten Runde junger Physiker, die sich täglich im Cafe Lutz traf. Aber weder SOMMERFELD noch die Runde waren überzeugt: Die Temperaturbewegung der Atome würde wohl die vermuteten Interferenzen hoffnungslos zerstören. Auch WIEN und MIE, mit denen sich SOMMERFELD zum Schilaufen traf, verwarfen die Idee. Nur LAUE insistierte weiter, und so erklärte sich schließlich WALTHER FRIEDRICH, der als Schüler RÖNTGENs SOMMERFELDs experimenteller Assistent wurde, zu dem Experiment bereit. Allerdings stimmte SOMMERFELD nicht zu. Erst als ein weiterer Doktorand RÖNTGENs, PAUL KNIPPING, seine Mithilfe anbot, wagte FRIEDRICH den Aufbau einer ersten, noch primitiven Apparatur. Das erste Ergebnis war negativ. Aber KNIPPING bestand auf einer anderen Aufstellung der Photoplatte, und nun zeigte die Durchstrahlungsaufnahme des Kupfersulfats einen Kranz abgebeugter Spektren um den direkten Strahl herum. LAUE schreibt dazu: "Tief in Gedanken ging ich durch die Leopoldstraße nach Haus, als mir FRIEDRICH die Aufnahme gezeigt hatte. Und schon nahe meiner Wohnung... kam mir der Gedanke für die mathematische Theorie der Erscheinung". Die dreifache Anwendung der einfachen Beugungsbedingung des eindimensionalen Gitters erklärte die neue Entdeckung.





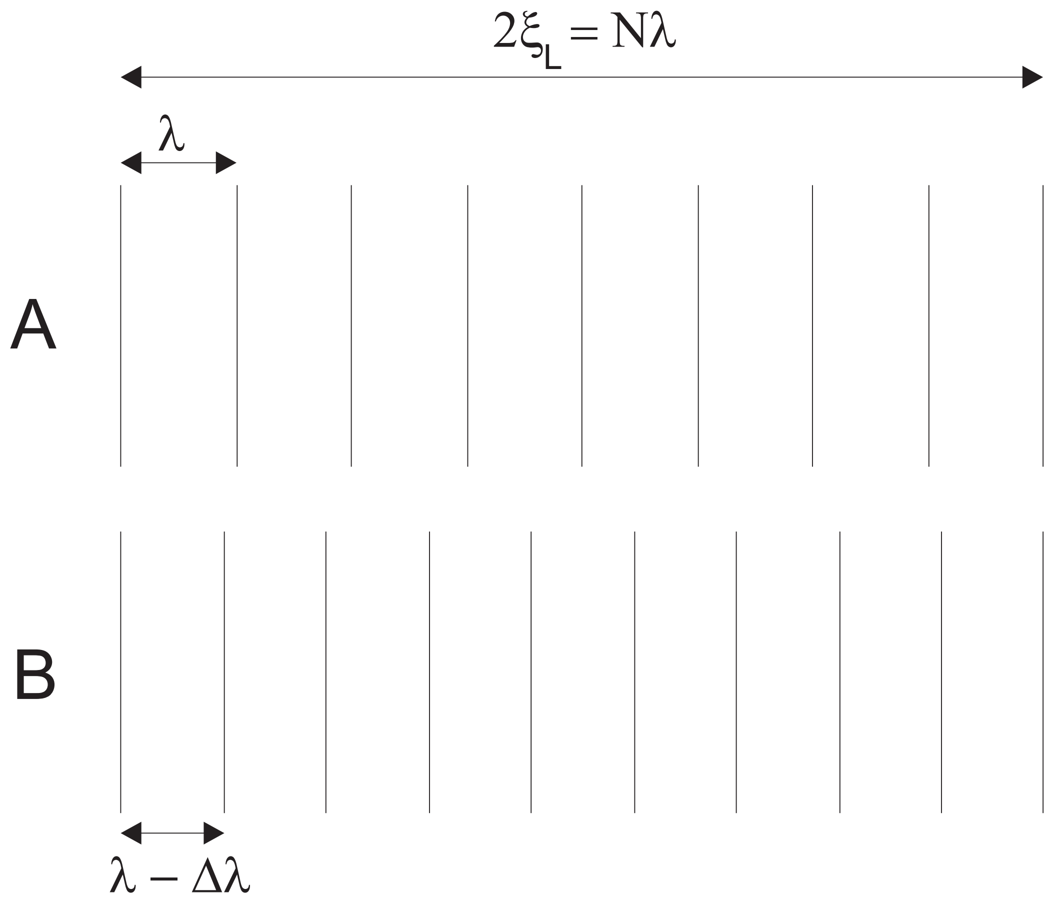


Kupfersulfat
$CuSO_4 \cdot 5 H_2O$



exposure time: many hourse

Longitudinal coherence



$$N\lambda = (N+1)(\lambda - \Delta\lambda)$$

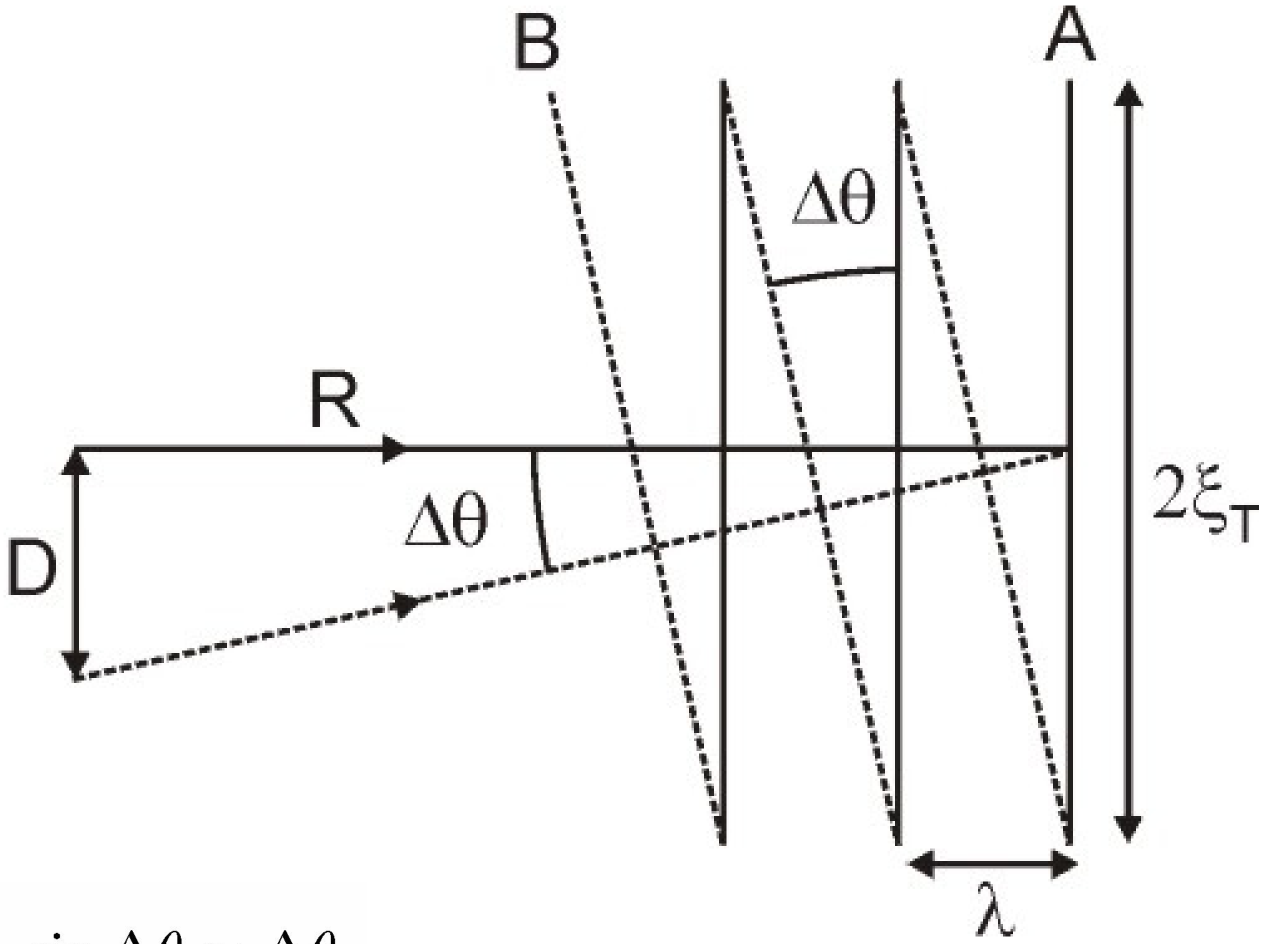
$$(N+1)\Delta\lambda = \lambda$$

$$N \approx \frac{\lambda}{\Delta\lambda}$$

$$\xi_L = \frac{N\lambda}{2} = \frac{1}{2}\frac{\lambda^2}{\Delta\lambda}$$

longitudinal coherence depends on bandwidth

Transverse coherence



$$\frac{\lambda}{2\xi_T} = \sin\Delta\theta \approx \Delta\theta$$

$$\frac{D}{R} = \tan\Delta\theta \approx \Delta\theta$$

$$\xi_T \approx \frac{\lambda}{2}\frac{R}{D}$$

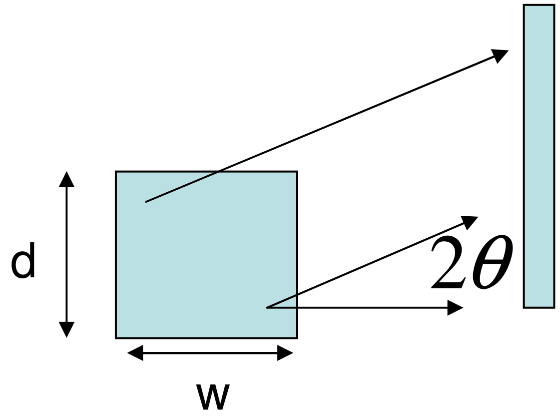
transverse coherence depends on distance and source size

Path length difference and coherence lengths

Interference is only possible when the path length differences (PLD) involved are of the same order as the coherence length.

beam size d

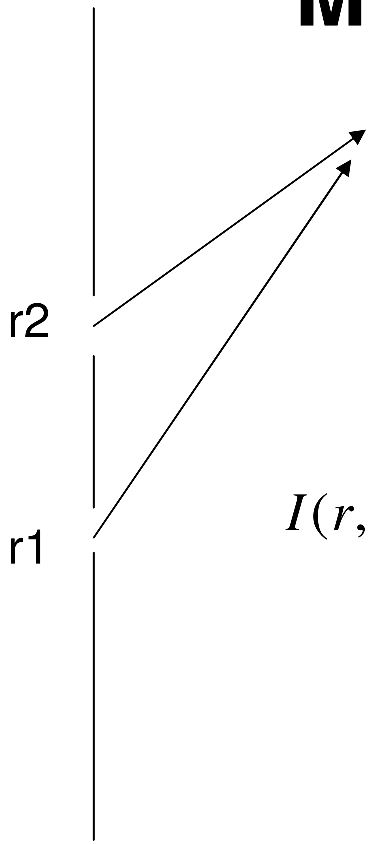
sample thickness w



$$\mathrm{PLD} \approx 2\mathrm{w}\sin^2(\theta) + \mathrm{d}\sin(2\theta)$$

for x-rays typically 1 micron

Mutual Coherence Function



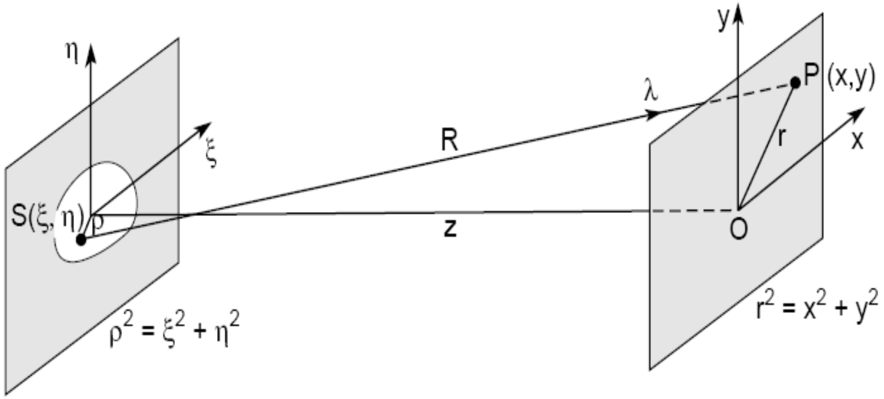
$$V(r,t) = A_1 V(r_1,t) + A_2 V(r_2,t)$$ Field amplitude

$$I(r,t) = A_1^2 \left|V(r_1,t)\right|^2 + A_2^2 \left|V(r_2,t)\right|^2 + 2\,\mathrm{Re}(A_1^* A_2 \Gamma(r_1,r_2,\tau))$$ Intensity

$$\Gamma(r_1,r_2,\tau) = \langle V^*(r_1,t) V(r_2,t) \rangle$$ Mutual Coherence Function (MCF)

$$\mu(r_1,r_2,\tau) = \frac{\langle V^*(r_1,t) V(r_2,t) \rangle}{\sqrt{I(r_1,t) I(r_2,t)}}$$ Degree of coherence

van Cittert – Zernike Theorem



$$p = \frac{X}{R}, \quad q = \frac{Y}{R}, \quad \psi = \frac{k(X^2 + Y^2)}{R}$$

$$\theta = \frac{r}{z}$$

complex degree of coherence

$$\mu(0,P) = \frac{e^{-i\psi} \iint_S I(\xi,\eta) e^{ik(p\xi + q\eta)} d\xi d\eta}{\iint_S I(\xi,\eta) e^{ik(p\xi + q\eta)} d\xi d\eta}$$

Fourier Transform of the source intensity distribution!

Axial symmetry

$$\mu(0,P) = \frac{e^{i\psi} \int_0^\infty I(\rho) J_0(k\rho\theta) \rho \, d\rho}{\int_0^\infty I(\rho) \rho \, d\rho}$$

Applications van Cittert – Zernike Theorem (1)

point source $$I(\rho) = \frac{I_0 \delta(\rho)}{2\pi\rho}$$

$$\mu(0,P) = \frac{e^{-i\psi} \int_0^\infty \delta(\rho) J_0(k\rho\theta)\, d\rho}{\int_0^\infty \delta(\rho)\, d\rho} = e^{-i\psi} J_0(0)$$

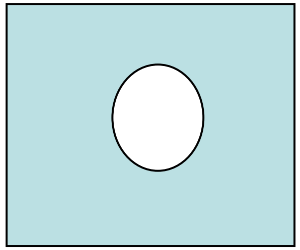
$$\left|\mu(0,P)\right| = 1$$

For a point source the normalized absolute degree of coherence is 1,
i.e. it is a fully coherent source!

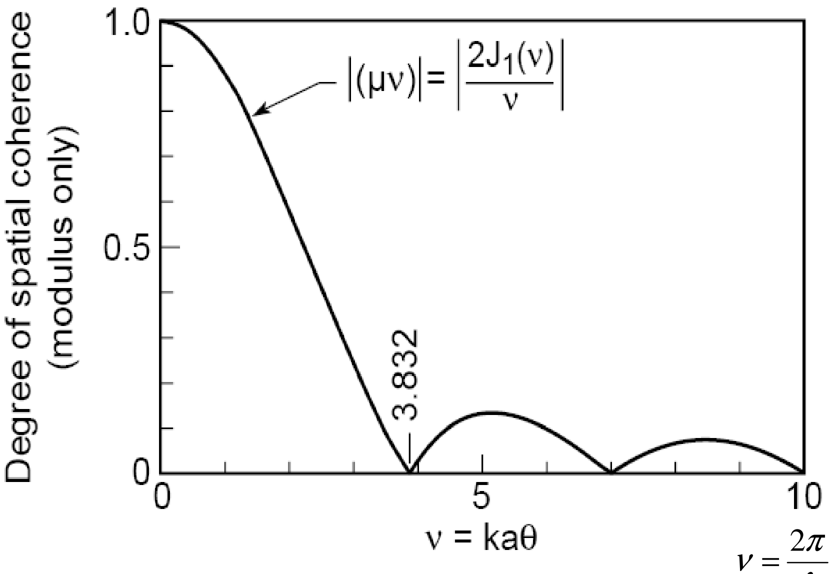
Applications van Cittert – Zernike Theorem (2)

pinhole diameter 2a

$$I(\rho) = 1 \text{ for } \rho \le \mathrm{a}$$
$$I(\rho) = 0 \text{ for } \rho > \mathrm{a}$$



$$\mu(0,P) = \frac{e^{-i\psi}\int_0^a J_0(k\rho\theta)\,\rho d\rho}{\int_0^a \rho\, d\rho} = \frac{8e^{-i\psi}}{d^2}\int_0^a J_0(k\rho\theta)\,\rho\, d\rho = e^{-i\psi}\,\frac{2J_1(ka\theta)}{ka\theta}$$



$$\nu = \frac{2\pi}{\lambda} a \frac{r}{z}$$

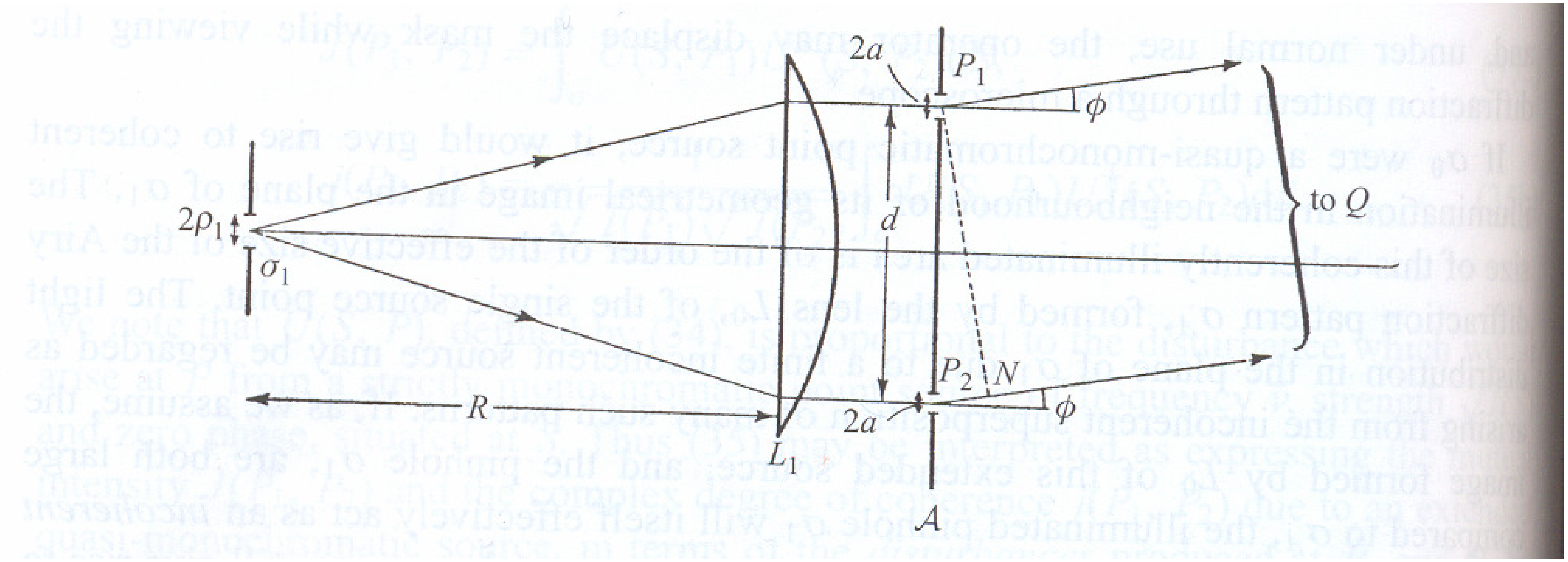
$$|\mu(0,P)| = 0.88$$

$$r = 0.16\frac{z\lambda}{a}$$

Sun: r=19 microns

x-rays at synchrotron r=10-100 microns

Revisiting Young's Double-Slit experiment



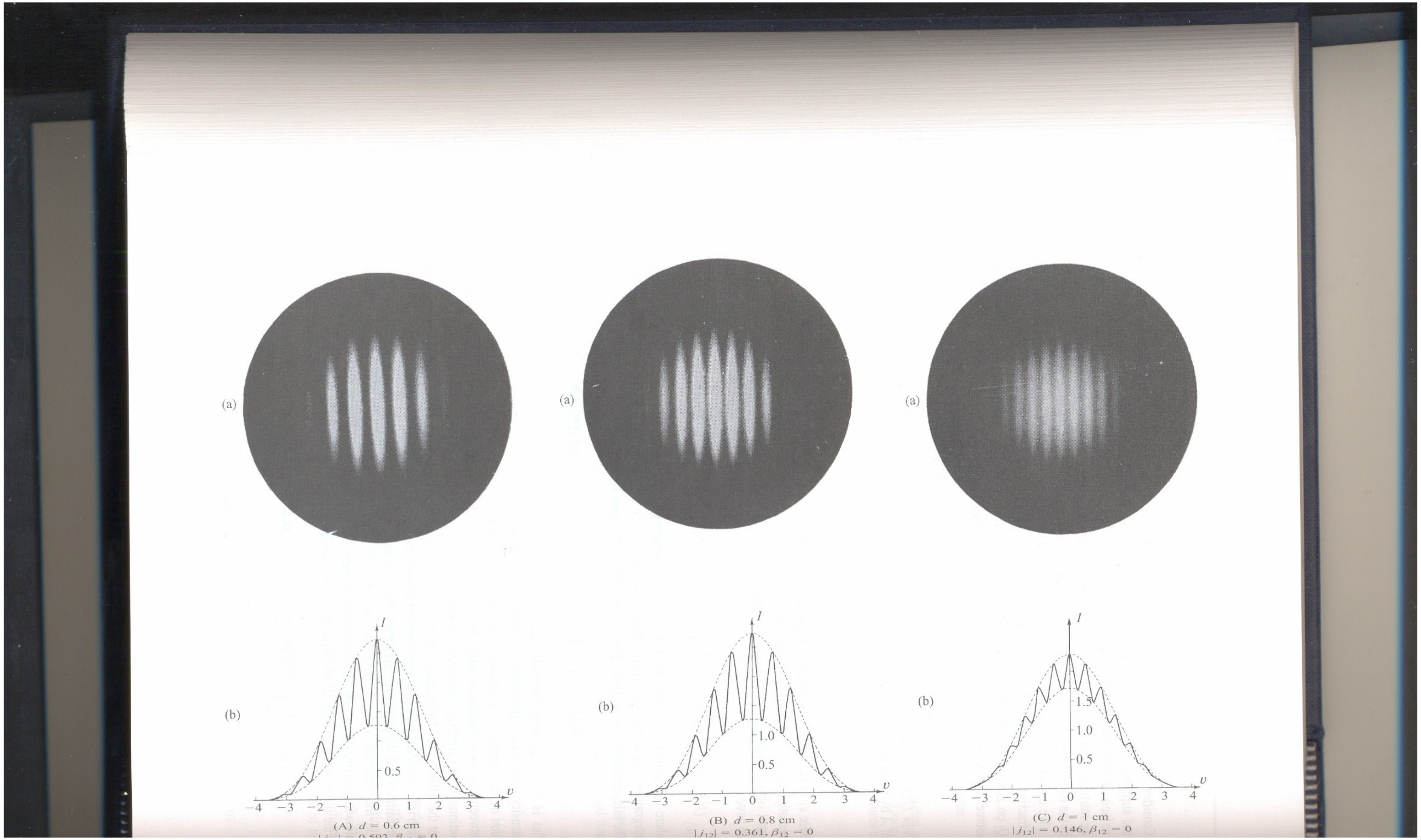
$$I(\phi,d) = 2\left(\frac{2J_1(u)}{u}\right)^2\left(1+\left|\frac{2J_1(v)}{v}\right|\cos(\beta(v)-Cuv)\right)$$

$$u = ka\sin(\phi),\ \ v = \frac{k\rho_1 d}{R},\ C = k\frac{R}{\rho_1 a}$$

$\beta = 0$ when $J_1(v) > 0$

$\beta = \pi$ else

Measuring the Degree of Coherence (1)



Born and Wolf, Optics

Measuring the Degree of Coherence (1)

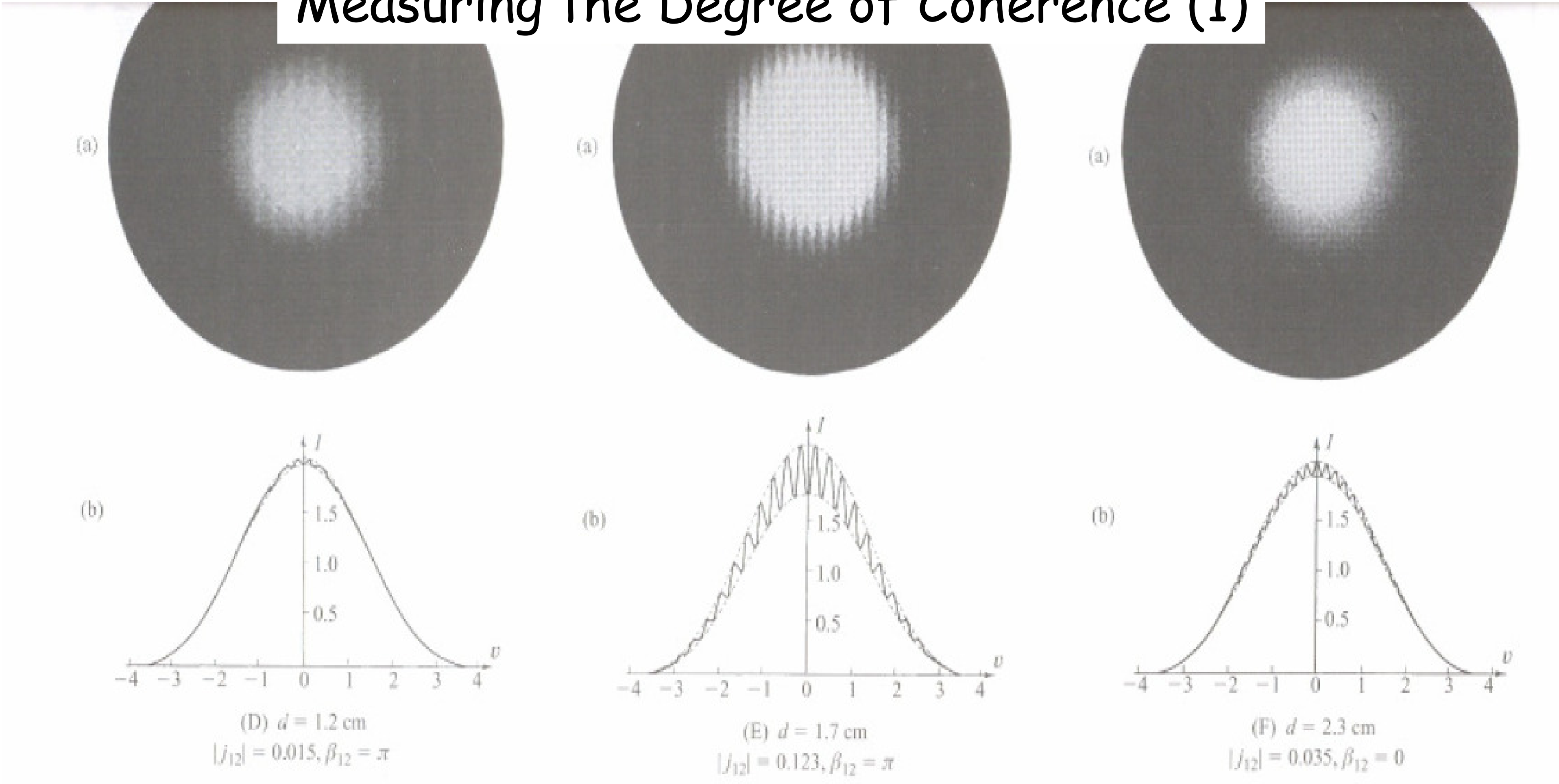
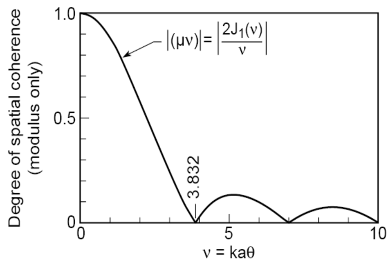


Fig. 10.6 Two-beam interference with partially coherent light. (a) Observed patterns, (b) theoretical intensity curves. Focal length of lenses L_0



Born and Wolf, Optics

Coherence

Measure of coherence : Degeneracy parameter Δ_c
- number of photons per mode

$$\Delta_c := \mathrm{Br} \cdot \tau_0 \cdot \delta\Omega \cdot \frac{\Delta\nu}{\nu} = \mathrm{Br} \cdot \frac{\lambda^3}{\pi c}$$

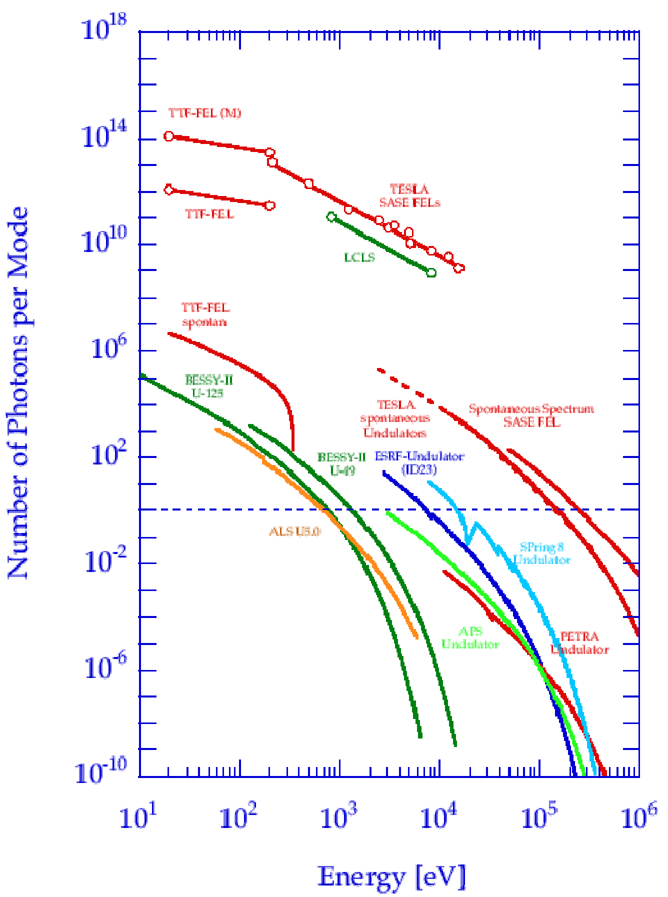
Br.: Brilliance $[\text{photons}/(s \cdot mm^2 \cdot mrad^2 \cdot 10^{-3} BW)]$

Quelle	Energie	Leistung	Br.	Δ_c
Quecksilber-Lampe	$4.9\,eV$	$1\,W$	$2 \cdot 10^{11}$	$3 \cdot 10^{-3}$
ESRF high β Undulator	$6.4\,keV$	einige kW	$2 \cdot 10^{20}$	$2 \cdot 10^{-3}$
HeNe Laser	$1.96\,eV$	$1\,mW$	$8 \cdot 10^{19}$	$2 \cdot 10^{7}$
ILL neutrons	$25\,meV$		$5 \cdot 10^{2}$	$3 \cdot 10^{-15}$

Brilliance of X-rays Sources

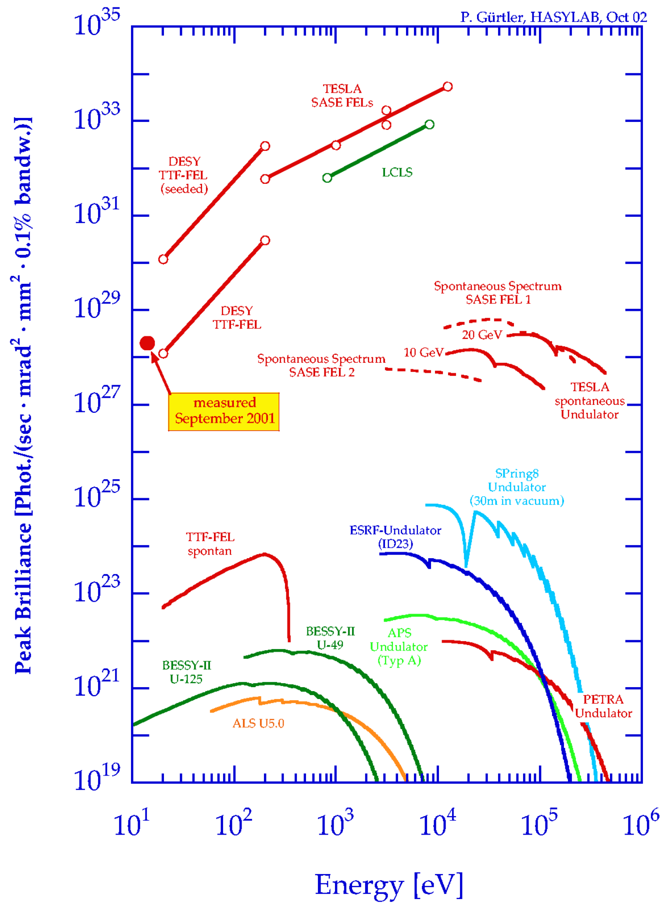
Number of photons per mode

$$N_C = B\,\lambda^3/(16\pi c)$$



Coherent Flux: $F_0 = B\,\lambda^2\,(\Delta\lambda/\lambda)$

(ESRF: ID10A $F_0 \sim 10^{10}$ ph/s)



Coherent X-rays

third generation synchrotron sources
(ESRF, APS, Spring-8,...)
(and soon : Diamond, Soleil, SLS, Petra-III)

Transverse coherence lengths ξ_T

$$\xi_T = \frac{\lambda}{2} \cdot \frac{R}{s} = 155(V) \times 3.8(H)\ \mu m^2$$

Troika I Beamline parameters

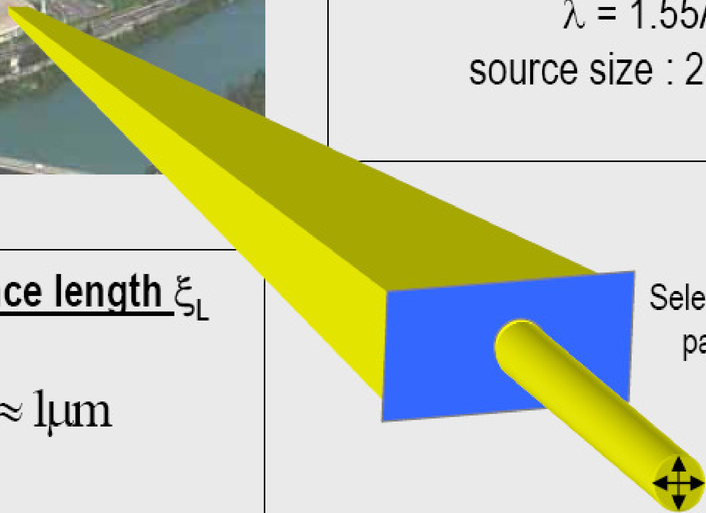
λ = 1.55Å, R=46m,

source size : 23(V)×928(H)µm

Longitudinal coherence length ξ_L

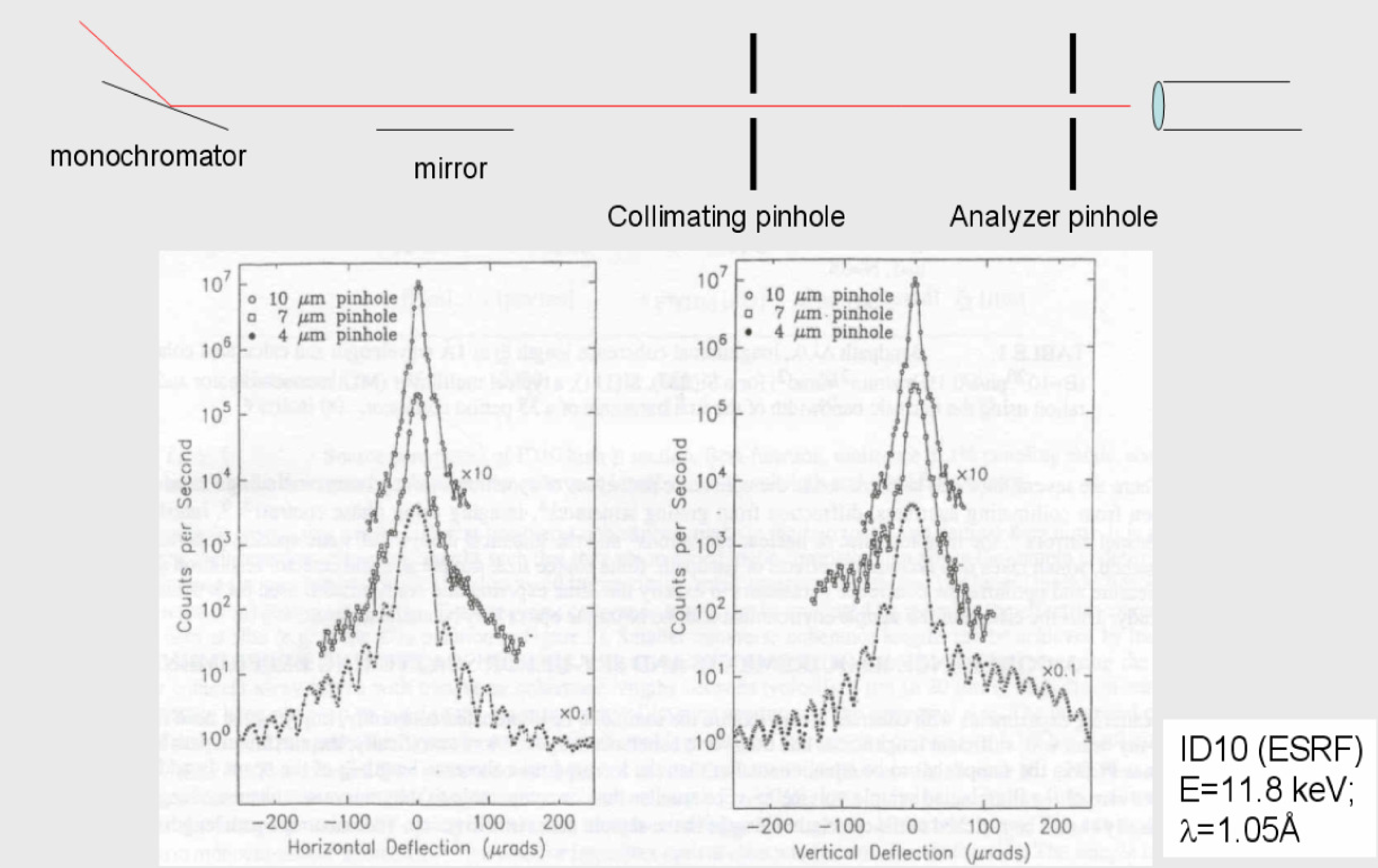
$$\xi_L = \lambda \cdot \left(\frac{\lambda}{\Delta\lambda} \right) \approx 1 \mu m$$

λ = 1.55Å , Si(111) $\Delta\lambda/\lambda = 1.4 \cdot 10^{-4}$



A. Robert, SLAC

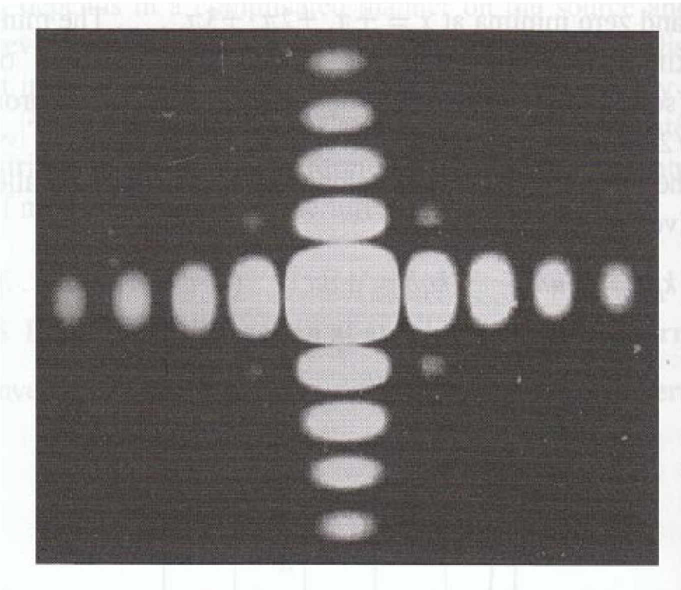
Fraunhofer Diffraction (λ=0.1nm)



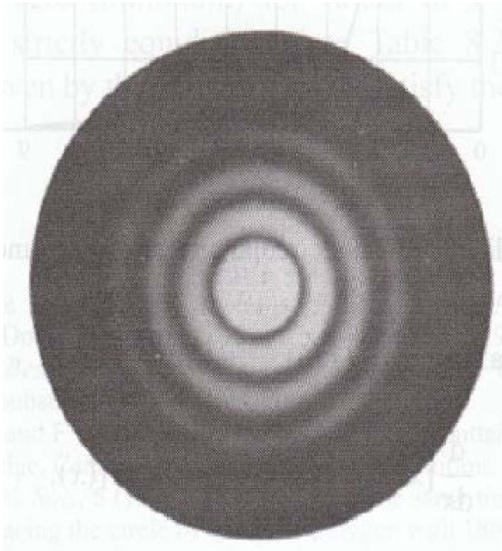
ID10 (ESRF)
E=11.8 keV;
λ=1.05Å

Fraunhofer Diffraction

coherent illumination of apertures

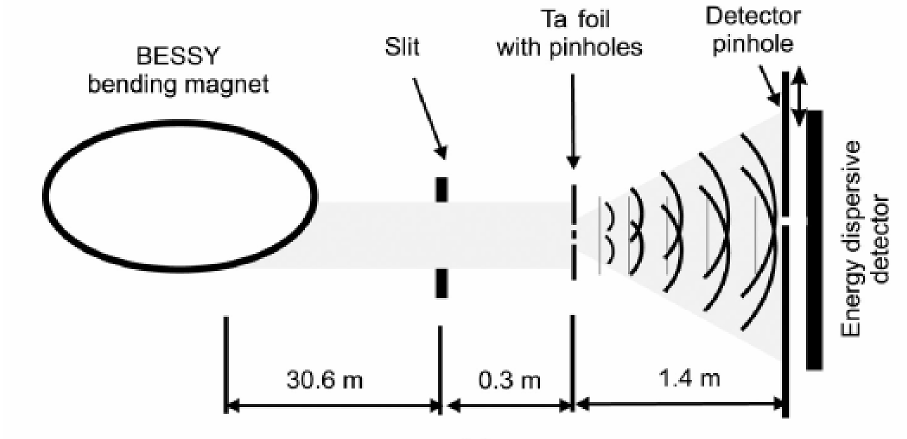


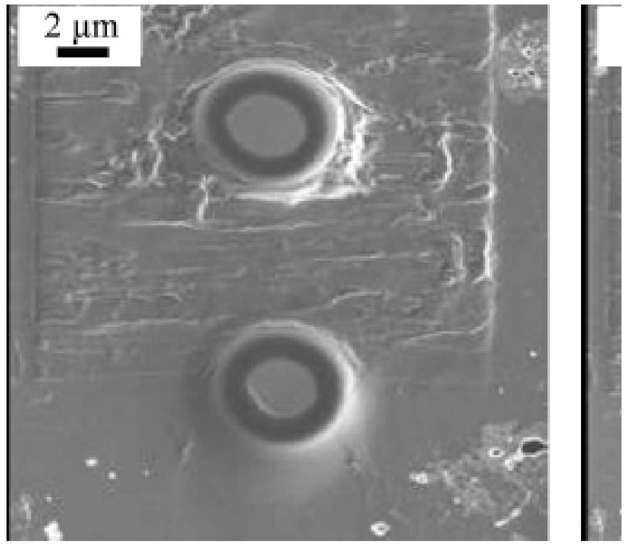
Fraunhofer diffraction of a rectangular aperture 8 x 7 mm^2, taken with mercury light λ=579nm
(from Born&Wolf, chap. 8)



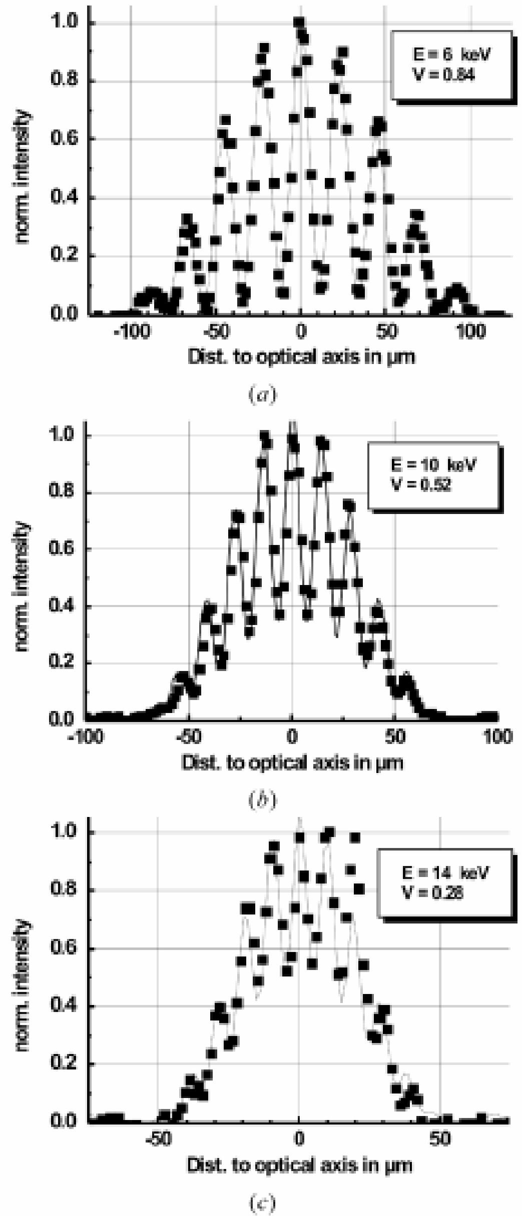
Fraunhofer diffraction of a circular aperture, taken with mercury light λ=579nm
(from Born&Wolf, chap. 8)

Young's experiment with X-rays





(a)



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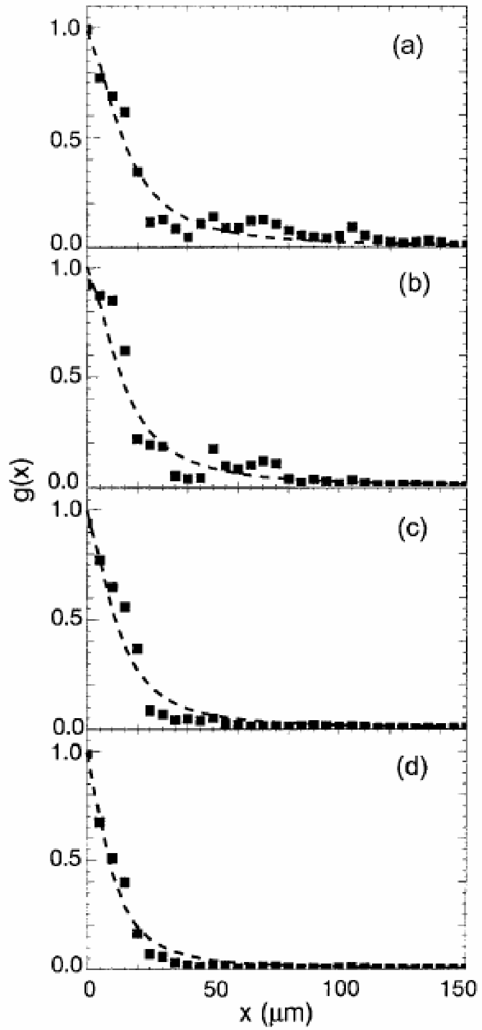
Leitenberger et al. J. Synchrotron Rad. 11, 190 (2004)

Measuring the absolute degree of coherence for x-rays



10 μm pinhole

50 μm pinhole

90 μm pinhole

100 μm pinhole

J. Lin et al. Phys.Rev.Lett. 90, 074801

Speckle Pattern

"Everything interferes with everything"

Coherent scattering

perfect coherence

$$I(q) = \left| \int_{-A}^{A} \rho(r) \exp(iqr) dr \right|^2$$

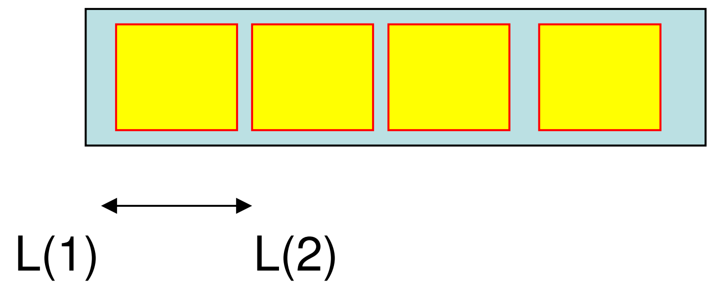
- absolute phase is lost
- relative phases are still in I(q)

A sample size

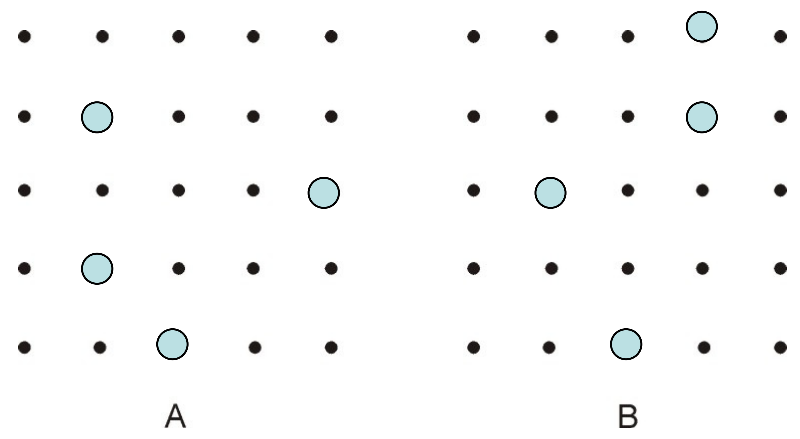
partial coherence

$$I(q) = \sum_i \left| \int_{-L(i)}^{L(i)} \rho(r) \exp(iqr) dr \right|^2$$

- absolute phase is lost
- relative phases are lost to a large extent

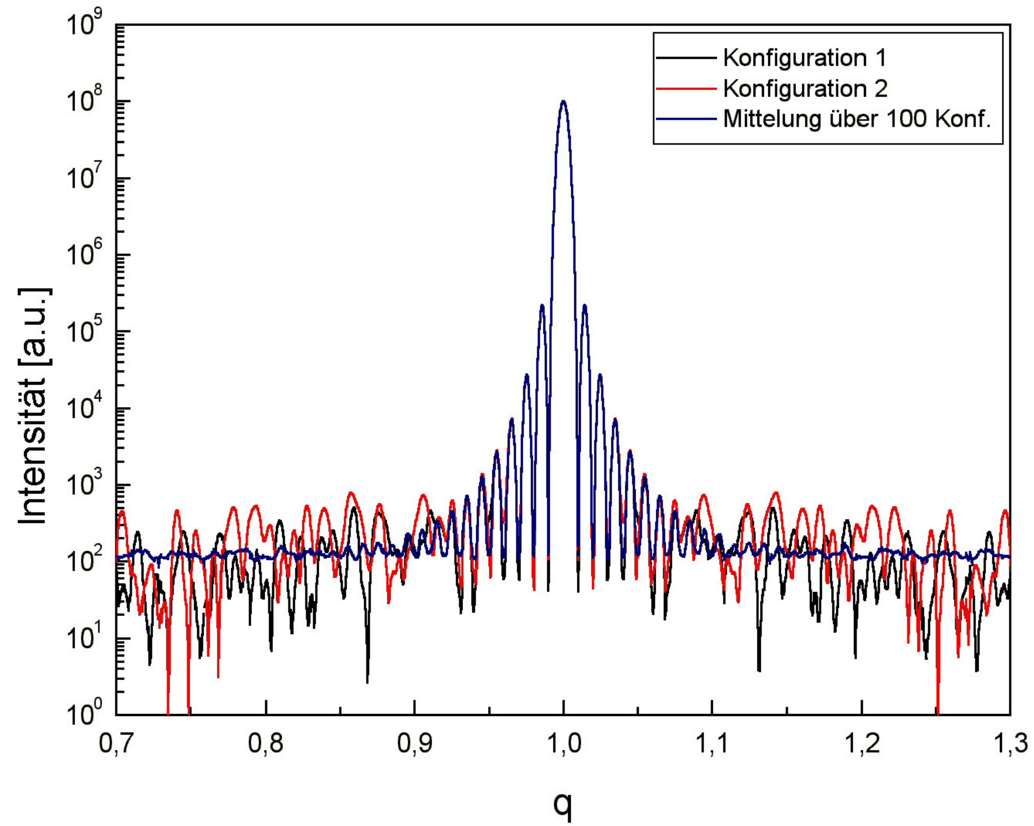


Simulation of coherence scattering from a lattice:



● formfactor f_1

○ formfactor $f_2 = 1.5 * f_1$



simulation parameter:

lattice 100x100

99 % : ●

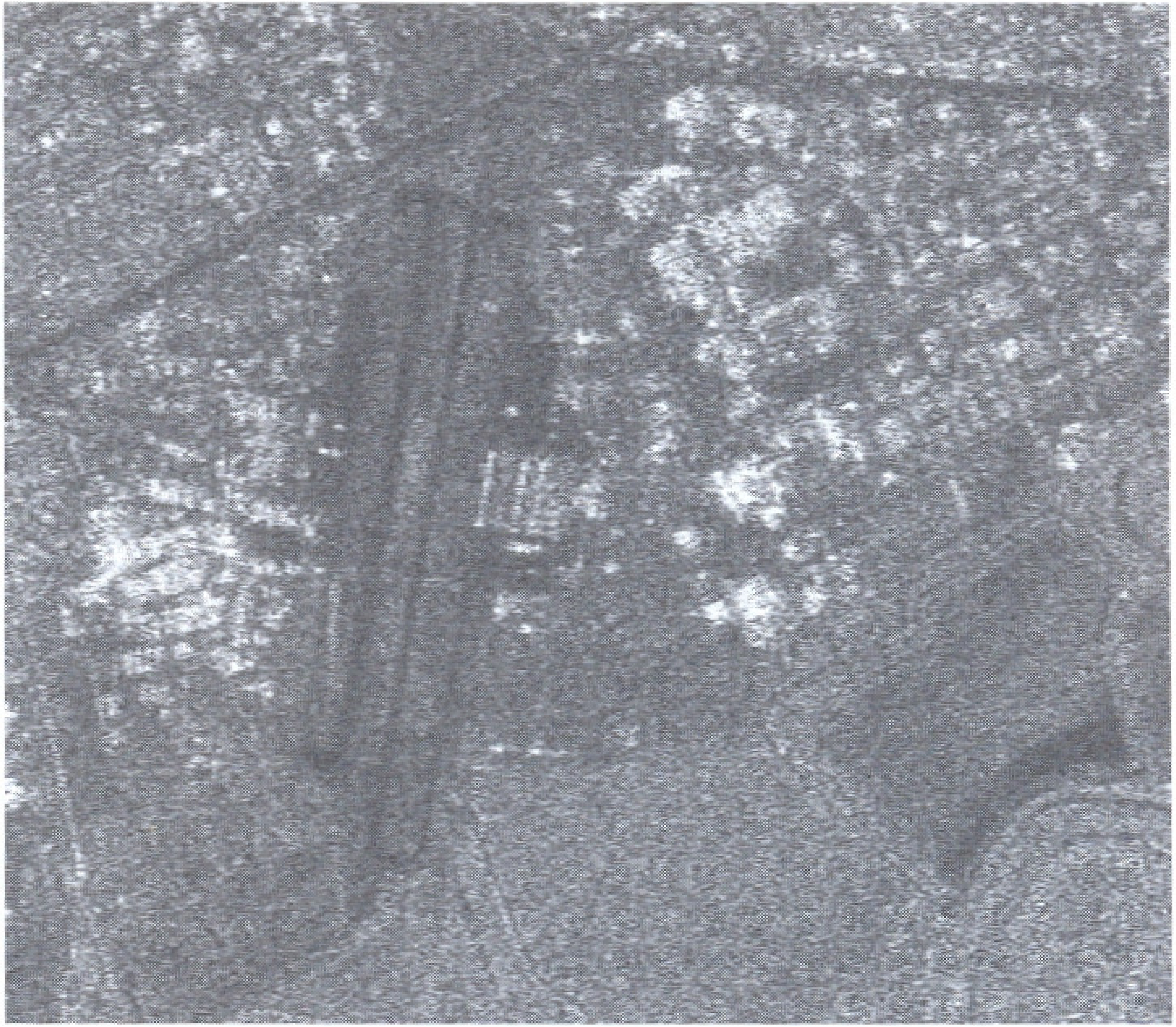
1 % : ○

incoherence scattering:
averaging over 100 configurations

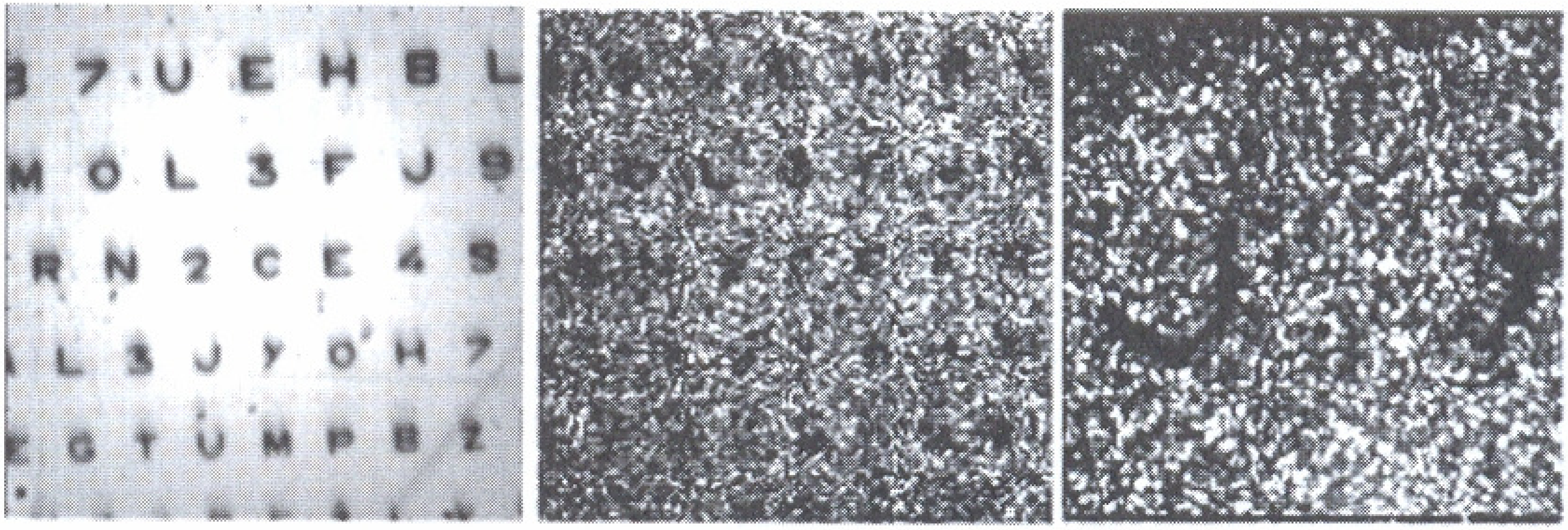
Moffet field, San Francisco, CA as shown on Google Earth

Moffet field, San Francisco, CA

Radar Speckles

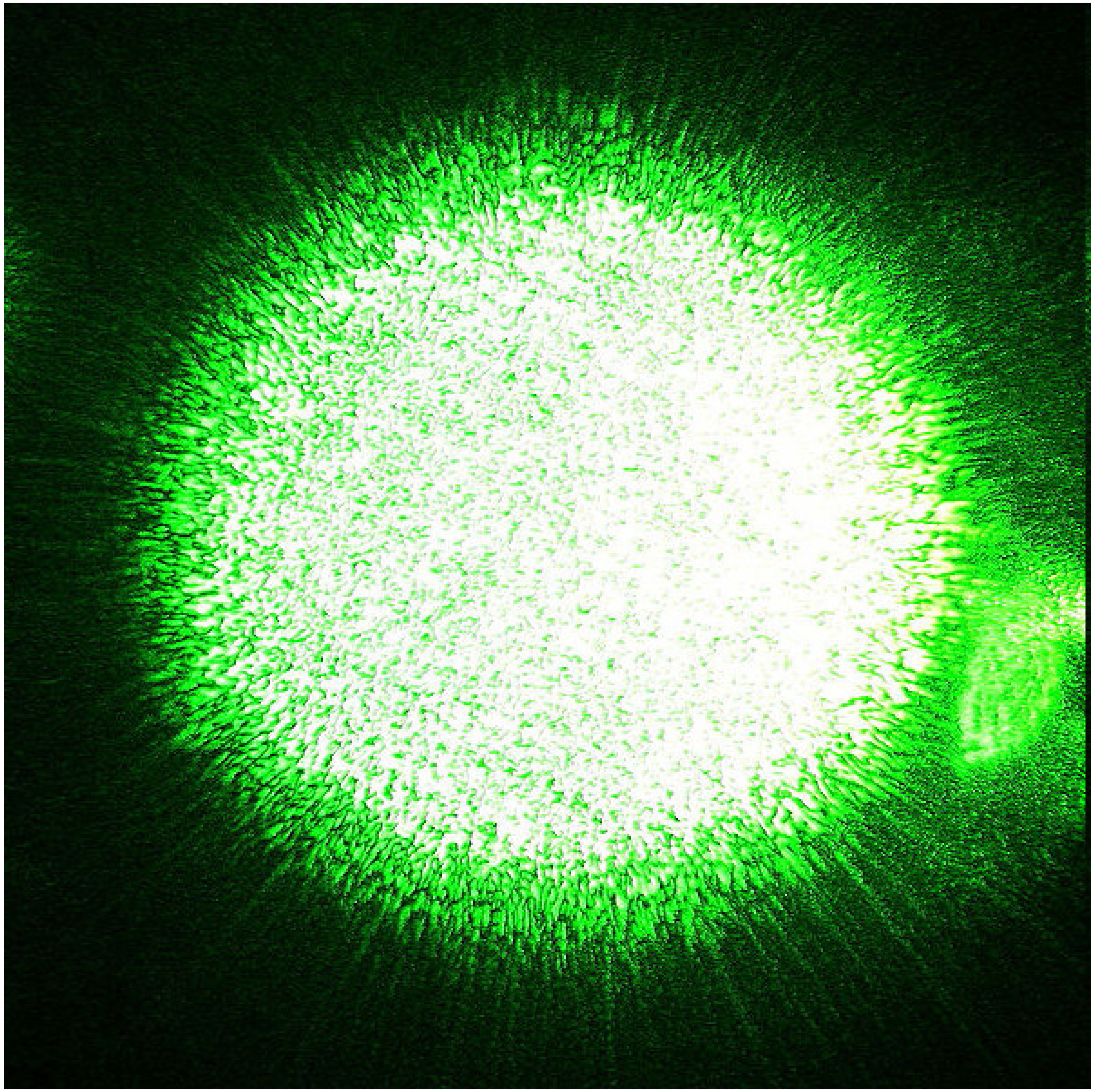


Optical Speckles



Incoherent light   Coherent light   Close up

Laser Speckles



X-ray speckle pattern

Aerogel sample

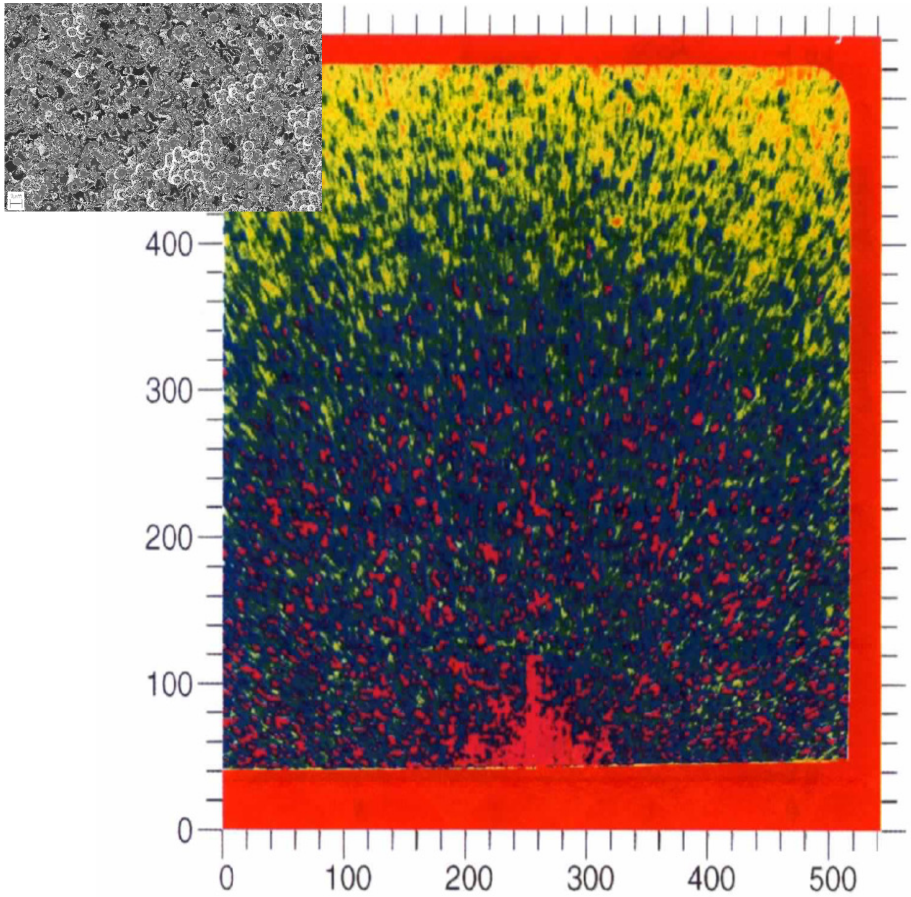


Figure 1
Aerogel speckle pattern.

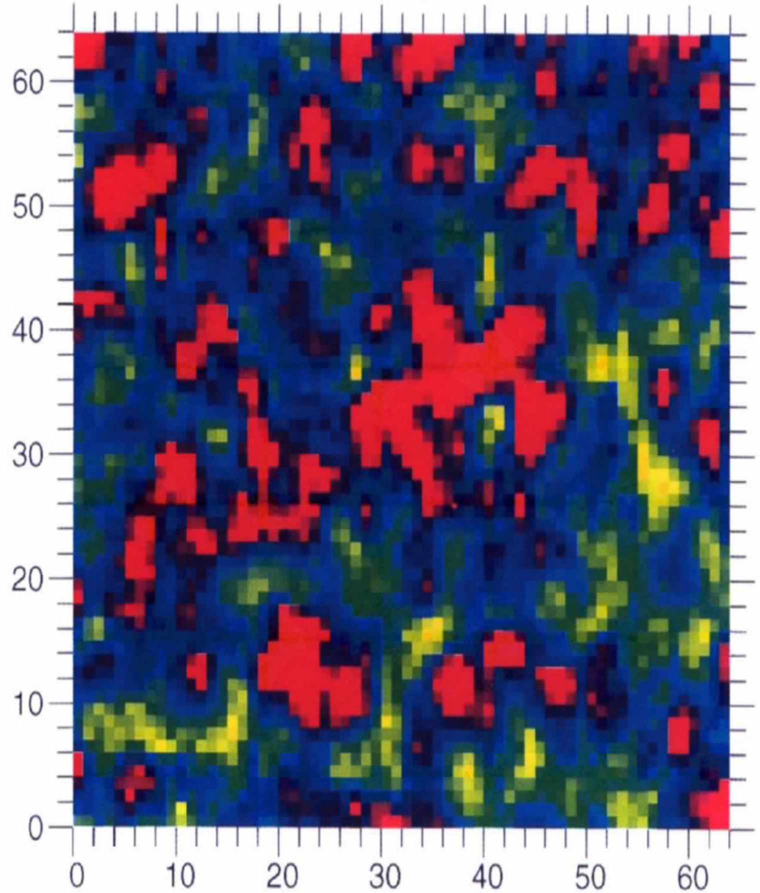


Figure 2
Detail of the aerogel speckle pattern at 0.006 $Å^{-1}$.

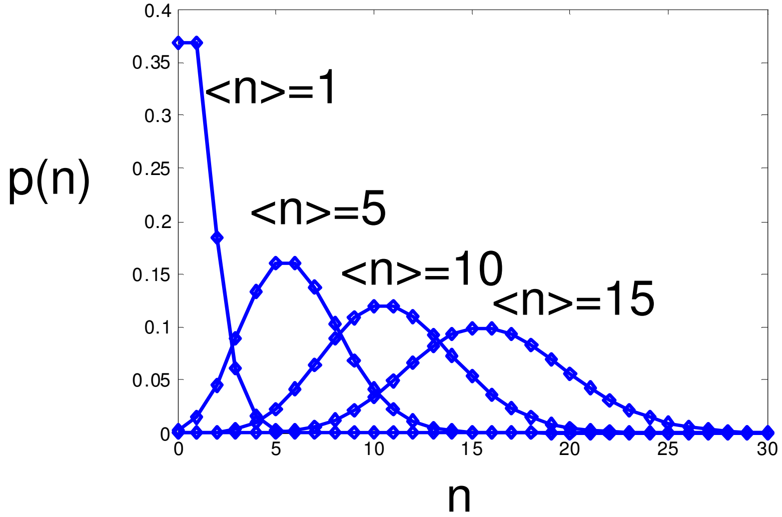
O.K.C. Tsui et al., Statistical Analysis of X-ray Speckle at the NSLS
J. Synchrotron Rad. (1998),5, 30

Analysis of Speckle Pattern in terms of counting statistics (1)

incoherent light

$$p(n) = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{n!}$$

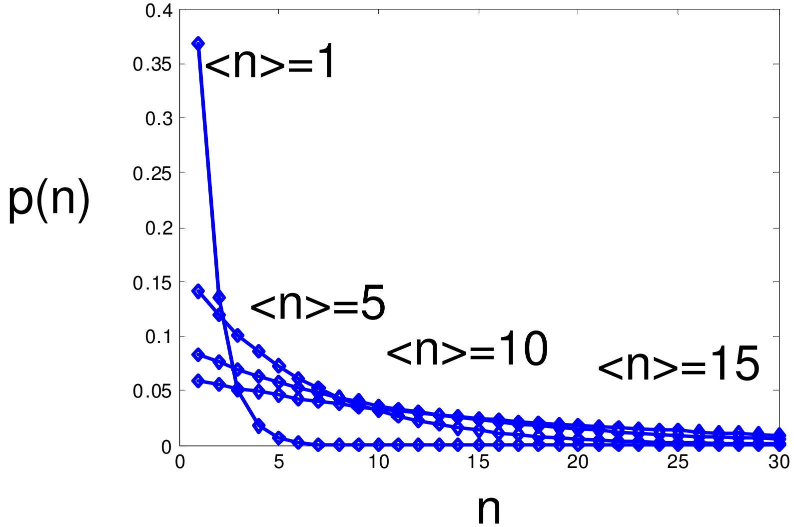
Poisson distribution



coherent light

$$p(n) = \frac{e^{-n/\langle n \rangle}}{\langle n \rangle}$$

exponential distribution



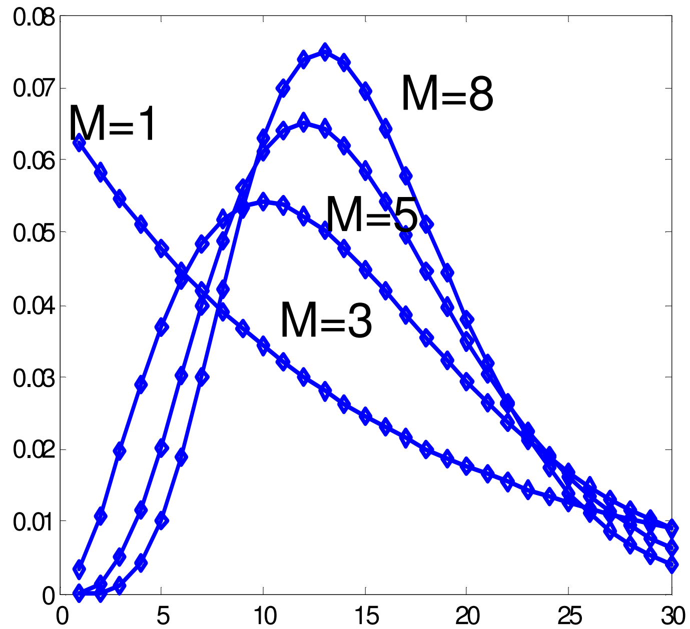
Analysis of Speckle Pattern in terms of counting statistics (2)

Approximation partial coherent light

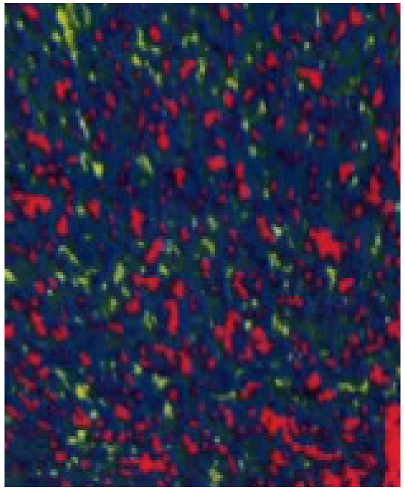
$$p(n) = M^M \left(\frac{n}{\langle n \rangle} \right)^{M-1} \frac{e^{-Mn/\langle n \rangle}}{\Gamma(M)\langle n \rangle}$$

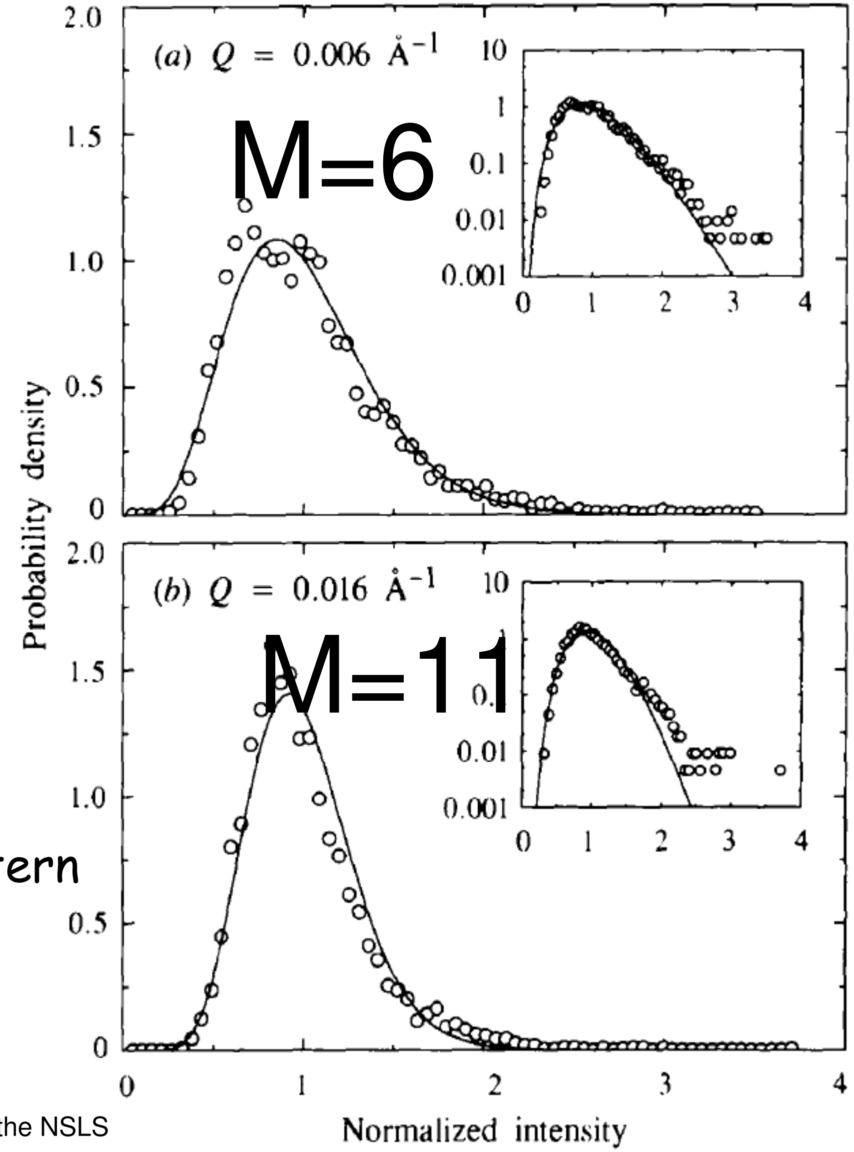
degree of coherence 1/M

<n>=15



X-ray speckle pattern at synchrotron





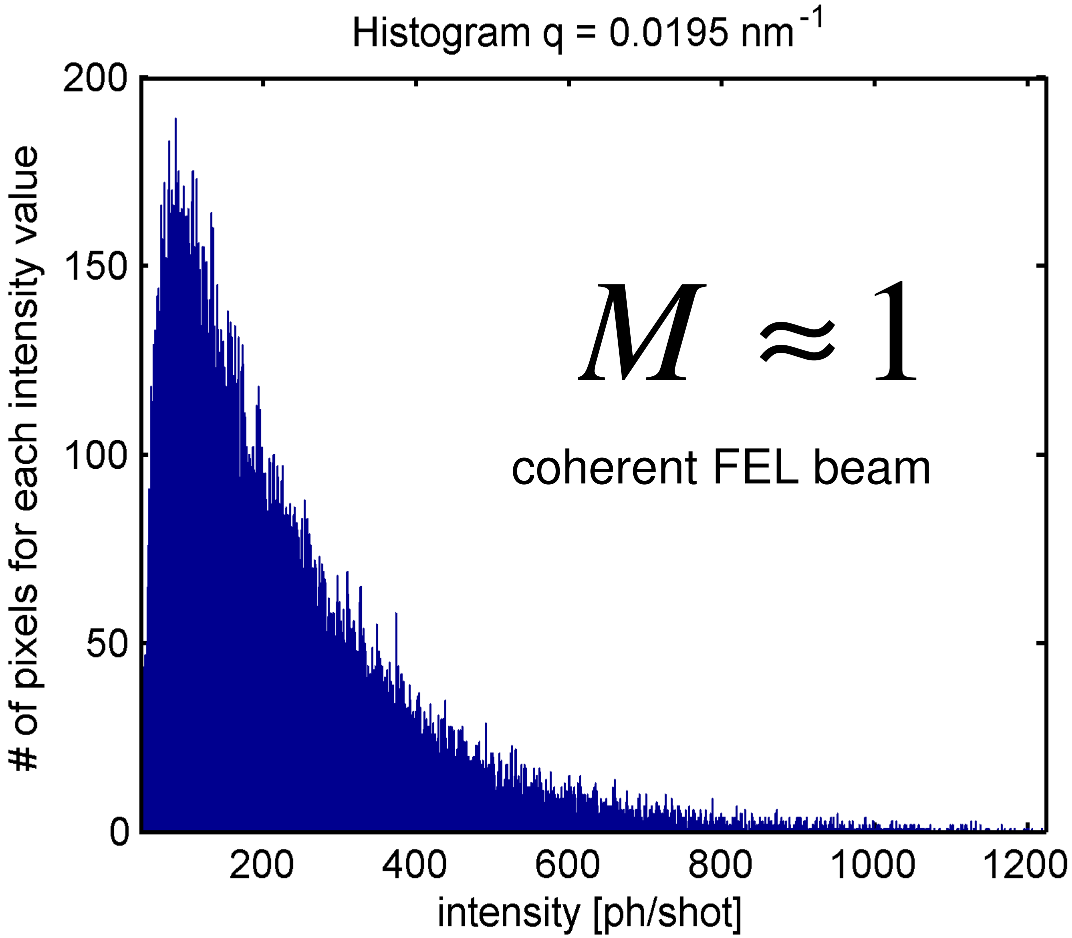
$$p(n) = M^M \left(\frac{n}{\langle n \rangle}\right)^{M-1} \frac{e^{-Mn/\langle n \rangle}}{\Gamma(M)\langle n \rangle}$$

Contrast ('Visibility') of speckle pattern

$$C = \frac{\sigma}{\langle n \rangle} \propto \frac{1}{\sqrt{M}}$$

O.K.C. Tsui et al., Statistical Analysis of X-ray Speckle at the NSLS
J. Synchrotron Rad. (1998),5, 30

Soft X-ray speckle pattern at FLASH



$$p(n) = M^M \left(\frac{n}{\langle n \rangle} \right)^{M-1} \frac{e^{-Mn/\langle n \rangle}}{\Gamma(M)\langle n \rangle}$$

Applications
Coherent X-ray Scattering

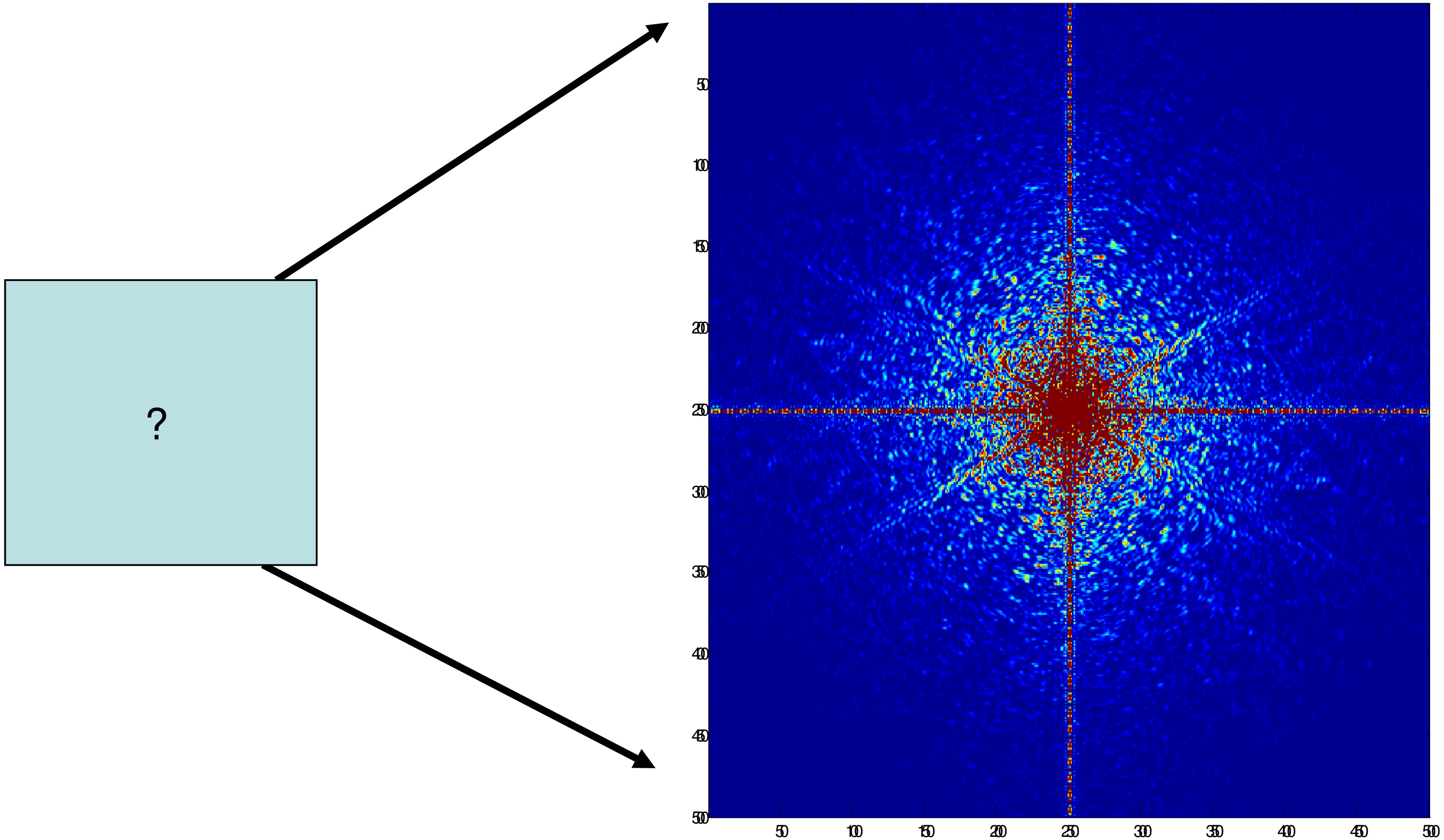
Structure

- Coherent Diffractive Imaging
- Holography

Dynamics

- X-Ray Photon Correlation Spectroscopy

Speckle Pattern for Coherent Diffractive Imaging



Happy X-mas – see you 08.01.2009