

Methoden moderner Röntgenphysik I + II: Struktur und Dynamik kondensierter Materie

Vorlesung zum Haupt/Masterstudiengang Physik SS 2009
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Materials Science

7. 5. Martin v. Zimmermann	correlated electron materials – structural properties
12. 5. Hermann Franz	glasses I
14. 5. Hermann Franz	glasses II
19. 5. Martin v. Zimmermann	correlated electron materials – magnetic properties

correlated electron materials: overview

- phase transitions
- structural phase transition of SrTiO_3
- x-ray diffraction to investigate phase transitions
- structural aspects of transition metal oxides
- orbital and charge order in $\text{La}_{1-x}\text{Ca}_x\text{MnO}_3$
- resonant scattering to study orbital/charge order

- magnetic properties of transition metal oxides
- magnetic scattering
- resonant magnetic scattering

Phase transitions

examples:

- solid - liquid - gas
- structural phase transition (SrTiO_3)
- magnetic phase transition
- Mott-metal-insulator transition
- macroscopic quantum phenomena (superconductivity, suprafluidity)
- quantum phase transitions (at zero temperature, driven by pressure, magnetic field)

- glass transitions (amorphous solids, spin-glasses, quasi-crystals)
(non-equilibrium states)

classification of phase transitions

Ehrenfest classification:

smoothness of the chemical potential μ

First order if the entropy $s = -\partial\mu/\partial T$ is discontinuous at the transition.

Problem: derivatives of μ can diverge as a transition is approached.

Modern classification:

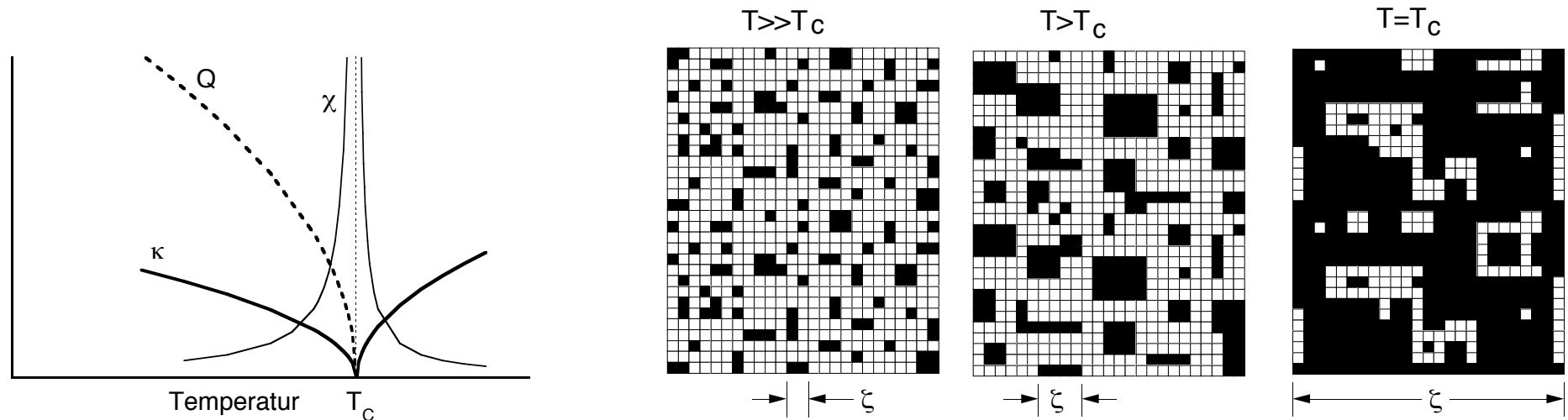
First order transitions have non-zero latent heat.

Are also called discontinuous.

All other transitions are continuous phase transitions.

structural phase transition of SrTiO₃

- phase transition - breaking of symmetry
- stable structure at temperature T determined by minimum of the free energy $F = U - T \cdot S$
- Orderparameter Q
 - $Q=0$ in the disordered phase
 - $Q=1$ in the completely ordered phase
- Phase transition at temperature T_c
- For continuous phase transitions, ordered and disordered regions form at T_c with out energy cost - critical fluctuations



Landau theory

phenomenological description of phase transitions

$$F(Q, T) = 1/2aQ^2 + 1/3bQ^3 + 1/4cQ^4 + \dots$$

$$\left. \frac{\partial F}{\partial Q} \right|_{Q_o} = 0 \quad \text{und} \quad \left. \frac{\partial^2 F}{\partial^2 Q} \right|_{Q_o} > 0$$

$$a > 0 : \quad a = a'(T - T_c) \quad b=0$$

$$F(Q, T) = 1/2a'(T - T_c)Q^2 + 1/4cQ^4$$

$$Q_o^2(T) = \begin{cases} 0 & \forall T > T_c \\ \frac{a'}{c}(T_c - T) & \forall T < T_c \end{cases}$$

$$\Rightarrow Q_o(T) \sim (T_c - T)^\beta \quad \text{mit } \beta = 0.5$$

β kritischer Exponent

Suszeptibility - korrelation function

$$\mathcal{F} = \left. \frac{\partial F}{\partial Q} \right|_T \quad \chi(T) = \left. \frac{\partial Q}{\partial \mathcal{F}} \right|_{\mathcal{F}=0}$$

$$\chi(T) = \begin{cases} \frac{1}{a'(T-T_c)} & \forall T > T_c \\ \frac{1}{2a'(T_c-T)} & \forall T < T_c \end{cases}$$

$$\implies \chi(T) \sim |T_c - T|^{-\gamma} \quad \text{mit } \gamma = 1$$

$$G(\vec{x}, T) = \langle Q(\vec{x}, T)Q(0, T) \rangle - \langle Q(T) \rangle^2 = k_B T \chi(\vec{x}, T)$$

$$\chi(\vec{q}, T) = \int d\vec{x} \exp(-i\vec{q}\vec{x}) \chi(\vec{x}, T) \sim \int d\vec{x} \exp(-i\vec{q}\vec{x}) G(\vec{x})$$

$$\text{mit } G(\vec{x}, T) \sim \frac{e^{-|\vec{x}|/\zeta}}{|\vec{x}|} \implies \chi(\vec{q}, T) \sim \frac{1}{\kappa^2 + q^2}.$$

Landau theory and beyond

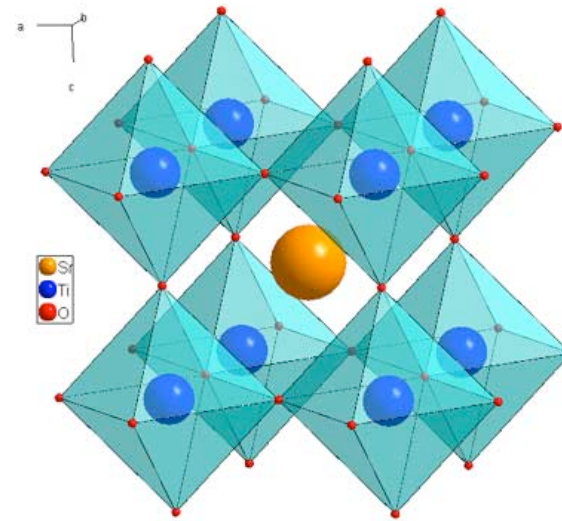
- Landau theory is independent of the dimension of the system and dimension of the orderparameter, fails to describe fluctuations around T_c , good approximation for $T \neq T_c$
- Landau-Ginzburg theory takes position dependent fields into account and describes behavior around T_c
- Renormalizing Group theory most complete theory to describe phase transitions. Results in proper values for critical exponents and could predict the scaling laws, the relation between different critical exponents.
- Predicts also the universality hypothesis, that the behavior at a phase transition is given only by the dimension of the system and the dimension of the orderparameter, but not the specific interactions.

example: structural phase transition in SrTiO₃

perovskite structure:

Pm3m (#221)

lattice parameter a_c



below 105 K:

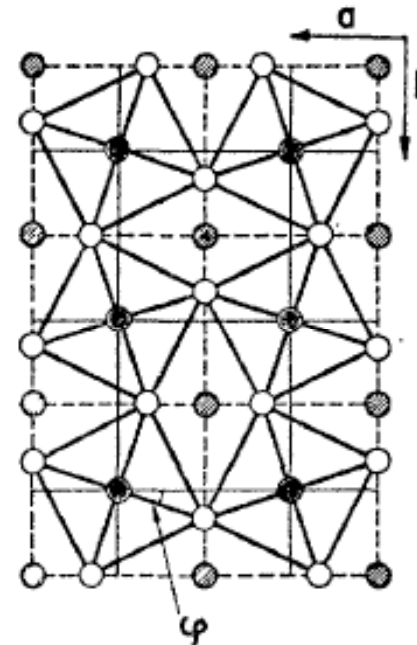
I4/mcm (#140)

$$a_t = \sqrt{2} a_c, c_t$$

orderparameter: spontaneous strain

$$\varphi^2 = c_t(T)/a_0(T) - 1$$

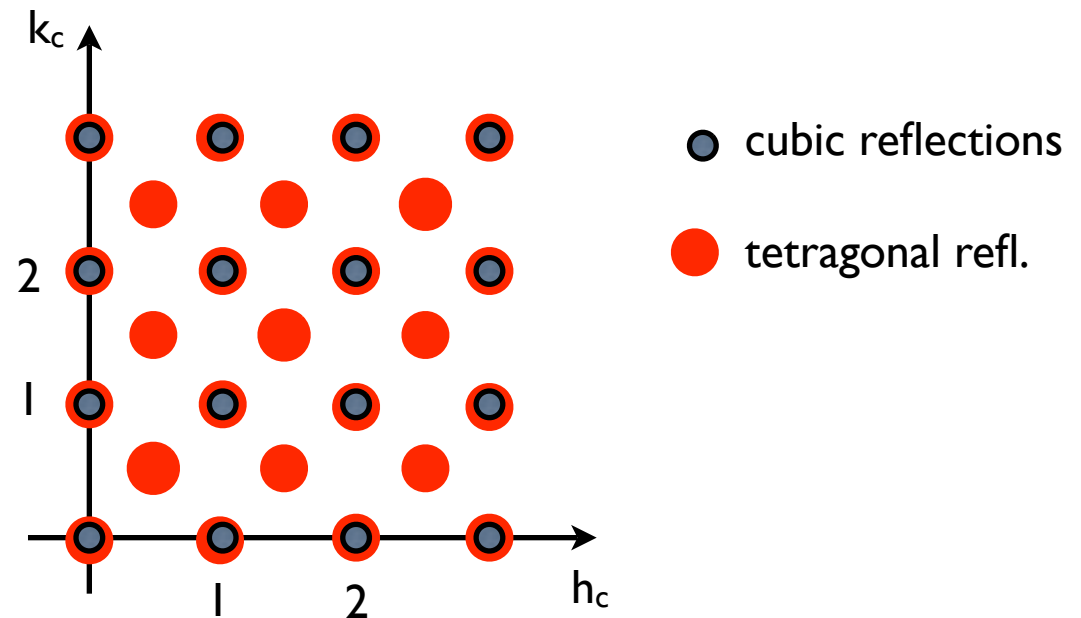
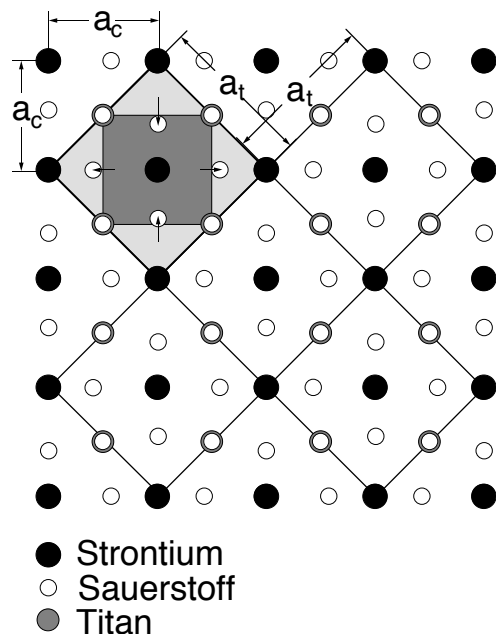
$$a_0(T) = 2/3 a(T) + 1/3 c(T)$$



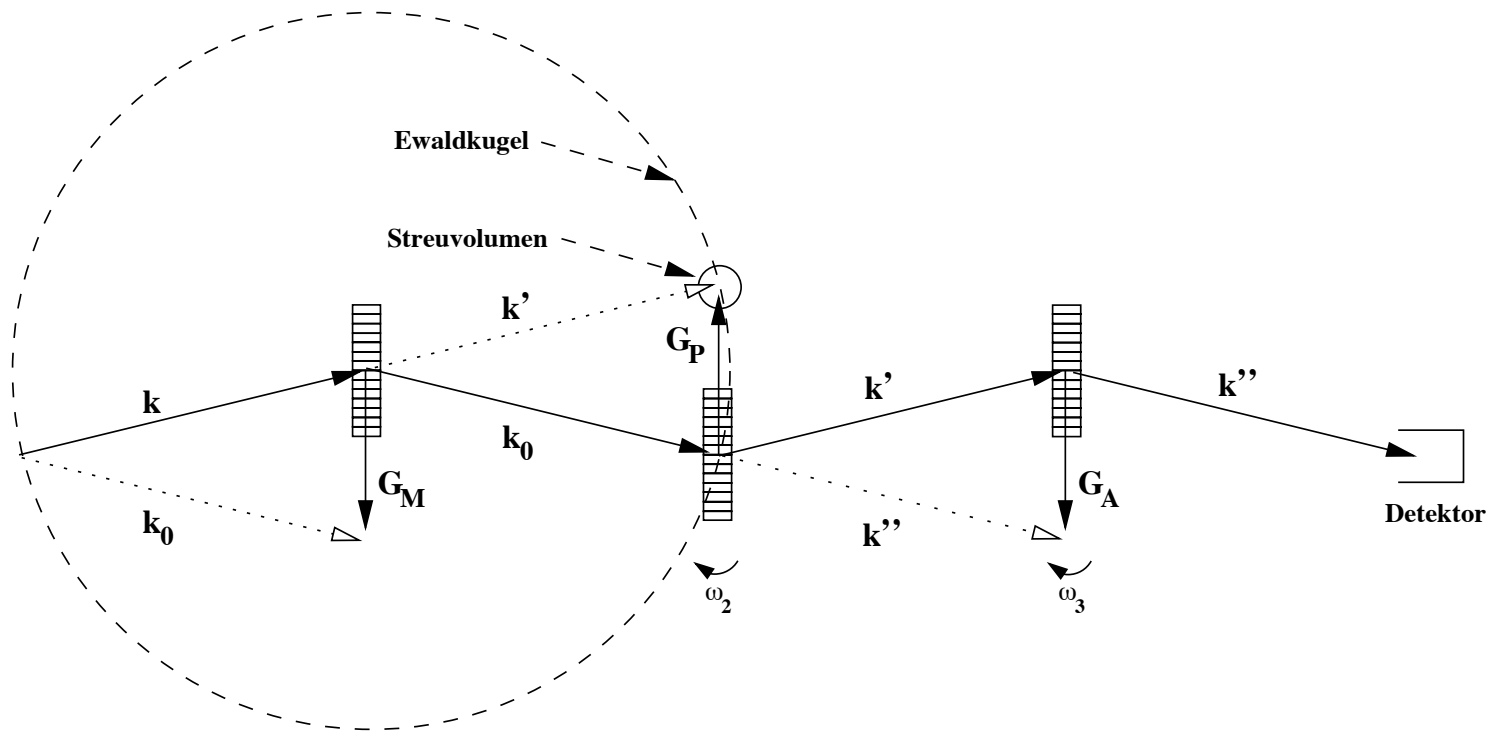
investigation of structural phase transitions by x-ray diffraction

1st approach: determination of lattice parameters

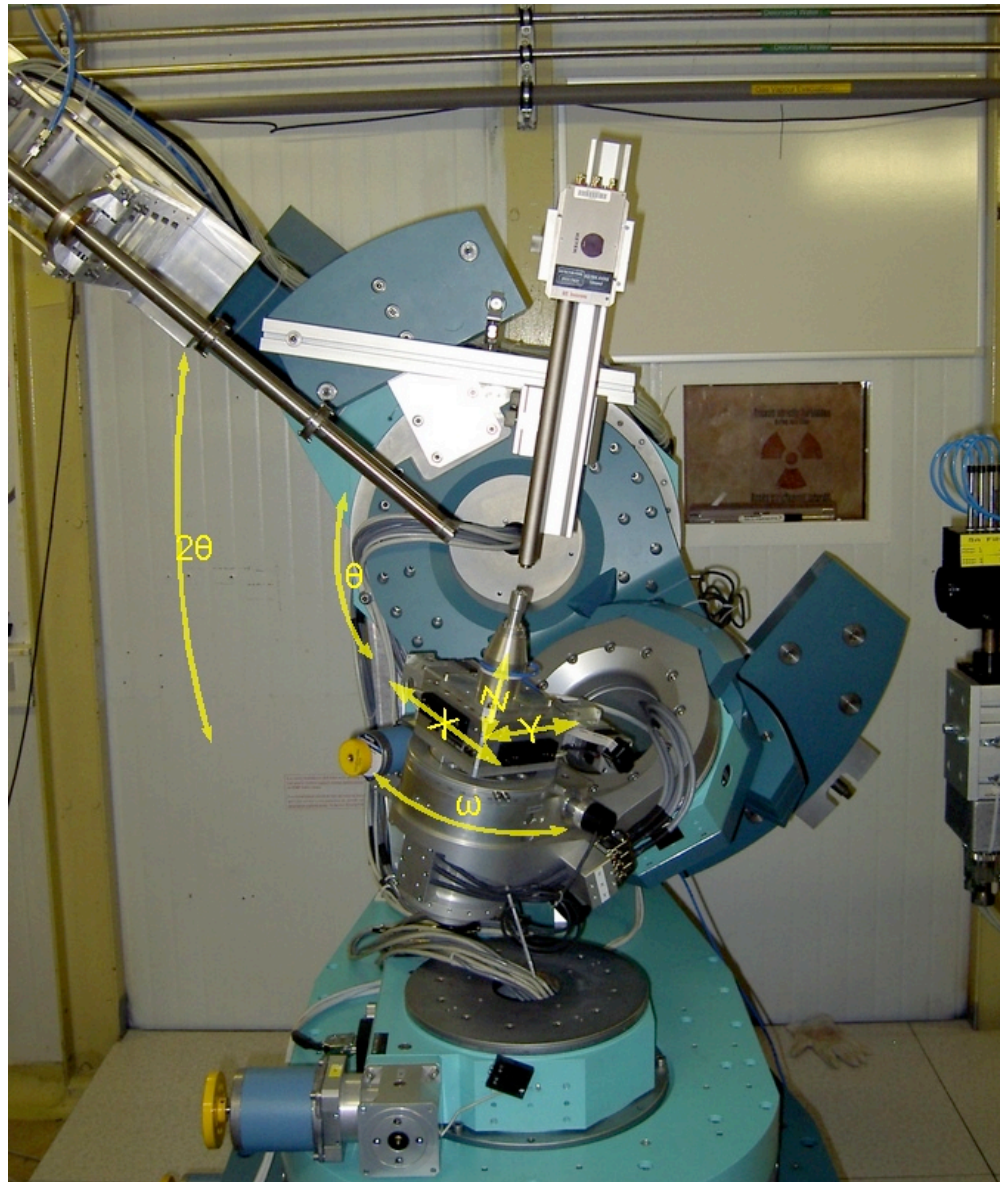
2nd approach: determinations of intensity of high-temperature phase
“forbidden” reflections



3-axis diffractometer



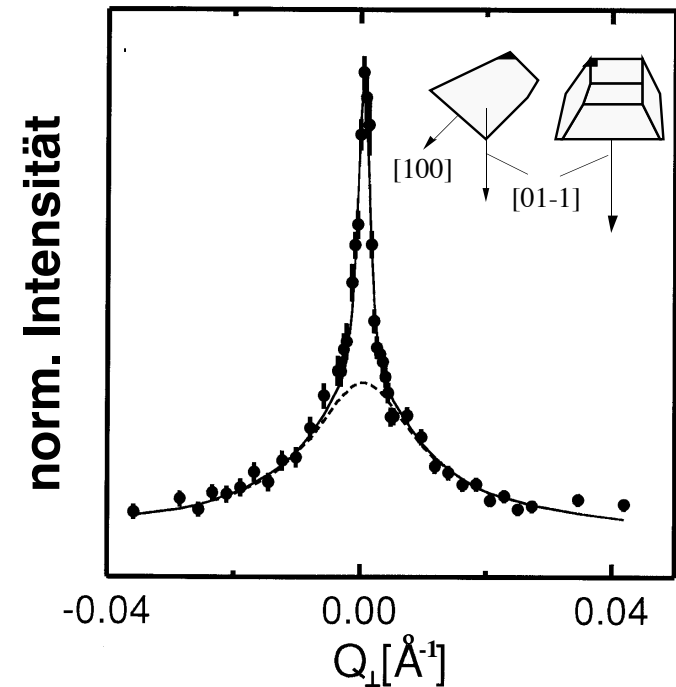
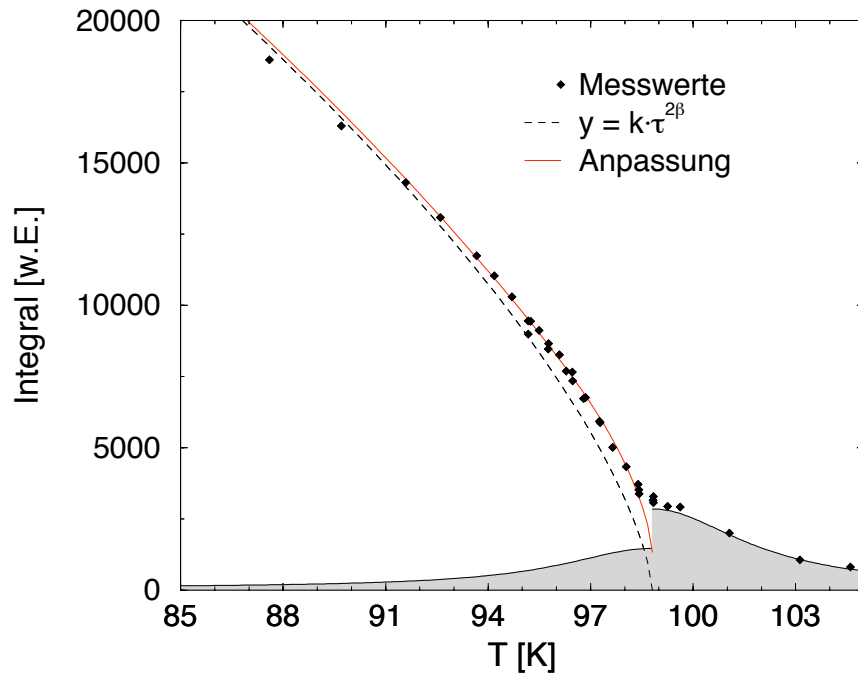
diffractometer



investigation of structural phase transitions by x-ray diffraction

$$I_{Bragg} \sim |F_{hkl}|^2 \sim Q_o^2 \sim (T_c - T)^{2\beta}$$

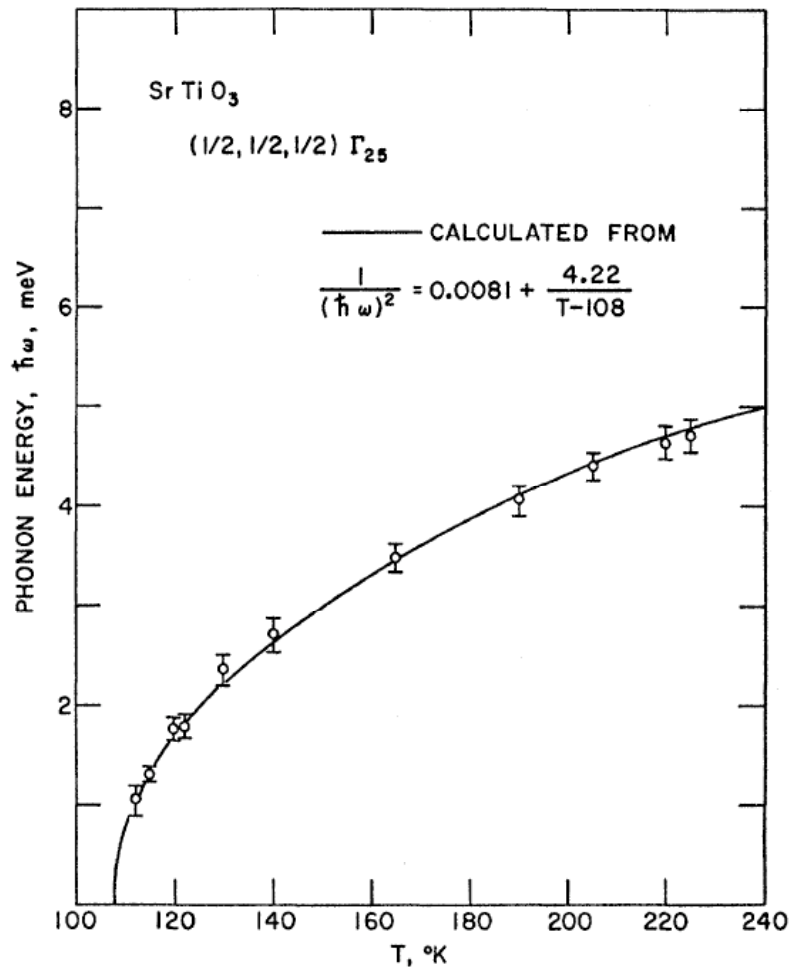
$$I_{Fl}(\vec{q}, T) \sim \chi(\vec{q}, T) \sim \frac{1}{\kappa^2 + q^2}$$



soft mode transition

phonon energy:
inelastic neutron scattering

static lattice distortion:
x-ray diffraction



correlated electron materials: transition metal oxides

- physical properties determined by interplay of charge, orbital, spin and lattice degrees of freedom
- high T_c superconductivity
- colossal magnetoresistance
- multiferroic behavior

Periodic Table of the Elements

1 H																	2 He																												
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne																												
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar																												
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr																												
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe																												
55 Cs	56 Ba	57 La	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn																												
87 Fr	88 Ra	89 Ac	104 Unq	105 Unp	106 Unh	107 Uns	108 Uno	109 Une	110 Uun																																				
<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>58 Ce</td><td>59 Pr</td><td>60 Nd</td><td>61 Pm</td><td>62 Sm</td><td>63 Eu</td><td>64 Gd</td><td>65 Tb</td><td>66 Dy</td><td>67 Ho</td><td>68 Er</td><td>69 Tm</td><td>70 Yb</td><td>71 Lu</td> </tr> <tr> <td>90 Th</td><td>91 Pa</td><td>92 U</td><td>93 Np</td><td>94 Pu</td><td>95 Am</td><td>96 Cm</td><td>97 Bk</td><td>98 Cf</td><td>99 Es</td><td>100 Fm</td><td>101 Md</td><td>102 No</td><td>103 Lr</td> </tr> </table>																		58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr
58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu																																
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Legend:

- hydrogen (black)
- alkali metals (yellow)
- alkali earth metals (orange)
- transition metals (purple)
- poor metals (green)
- nonmetals (blue)
- noble gases (pink)
- rare earth metals (teal)

correlated electron materials: transition metal oxides

3d electronic Eigenstates: $R_n * Y_m^2(\Theta, \phi)$

quantum numbers:

$n=3$ (radial)

$l=2$ (angular momentum)

$m= -2 \dots +2$ magnetic

5-fold degenerate

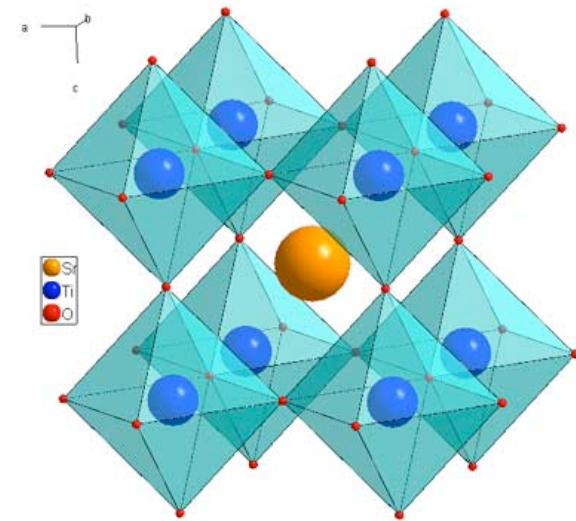
in a cubic crystal field: e.g. LaMnO_3

$$V(\mathbf{r}) = \sum Z_i e^2/|\mathbf{r} - \mathbf{R}_i| \quad \text{Madelung Potential}$$

in rectangular coordinates:

$$V_4(\mathbf{r}) = 5/2 V_{40} (x^4 + y^4 + z^4 - 3/5 r^4)$$

perovskite structure



cubic crystal field

Eigenstates in cubic crystal field:

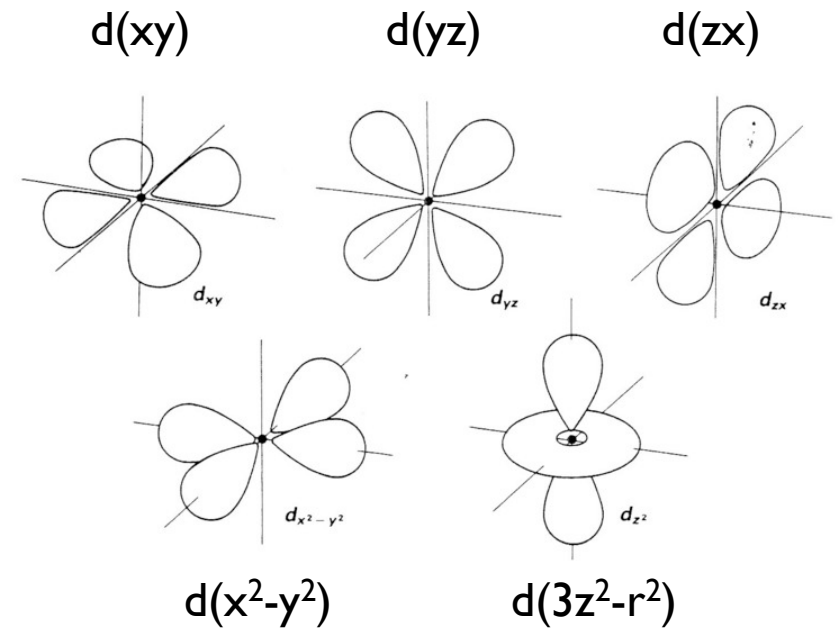
$$d(x^2-y^2) \propto \sqrt{2\pi/5} (Y_2^2 + Y_2^{-2}) = 1/2\sqrt{3} (x^2 - y^2)/r^2$$

$$d(3z^2-r^2) \propto \sqrt{4\pi/5} Y_2^0 = 1/2 (3z^2 - r^2)/r^2$$

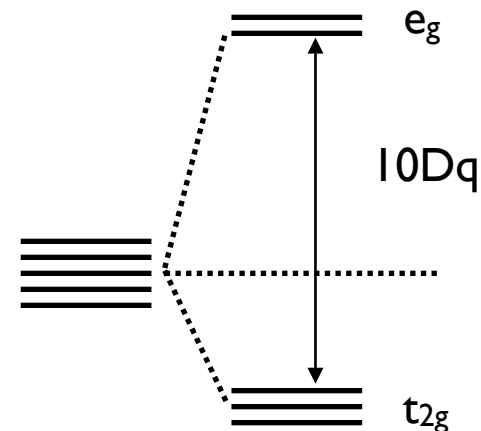
$$d(xy) \propto 1/i \sqrt{2\pi/5} (Y_2^2 - Y_2^{-2}) = \sqrt{3} (xy)/r^2$$

$$d(yz) \propto \sqrt{2\pi/5} (Y_2^{-1} + Y_2^1) = \sqrt{3} (yz)/r^2$$

$$d(zx) \propto 1/i \sqrt{2\pi/5} (Y_2^{-1} - Y_2^1) = \sqrt{3} (zx)/r^2$$



crystal field splitting:

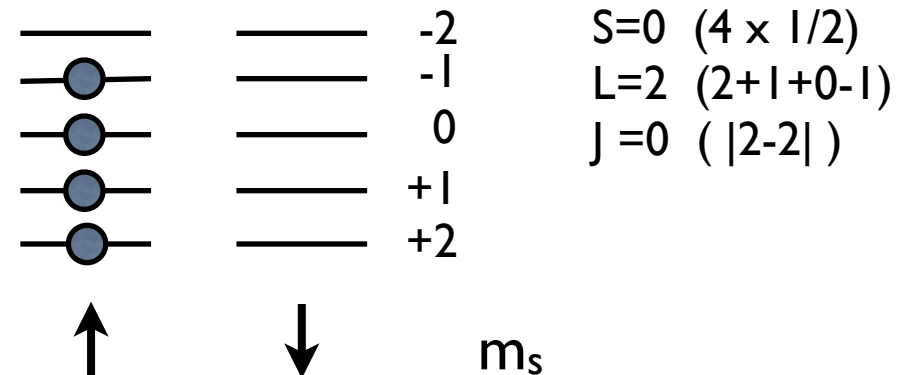


Hund's rules

electrons occupy orbitals such that the ground state is characterized by:

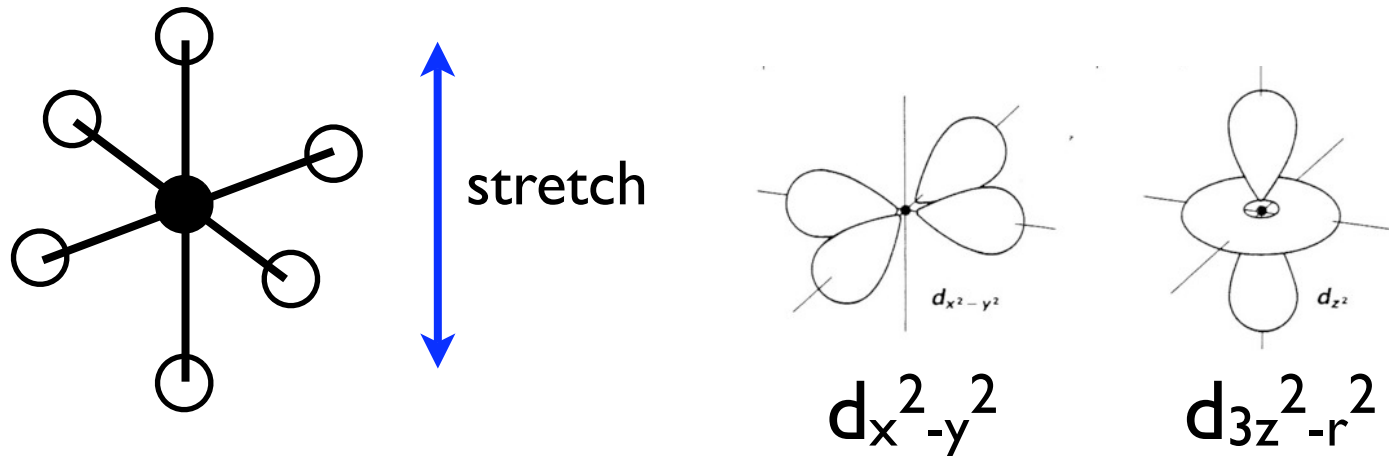
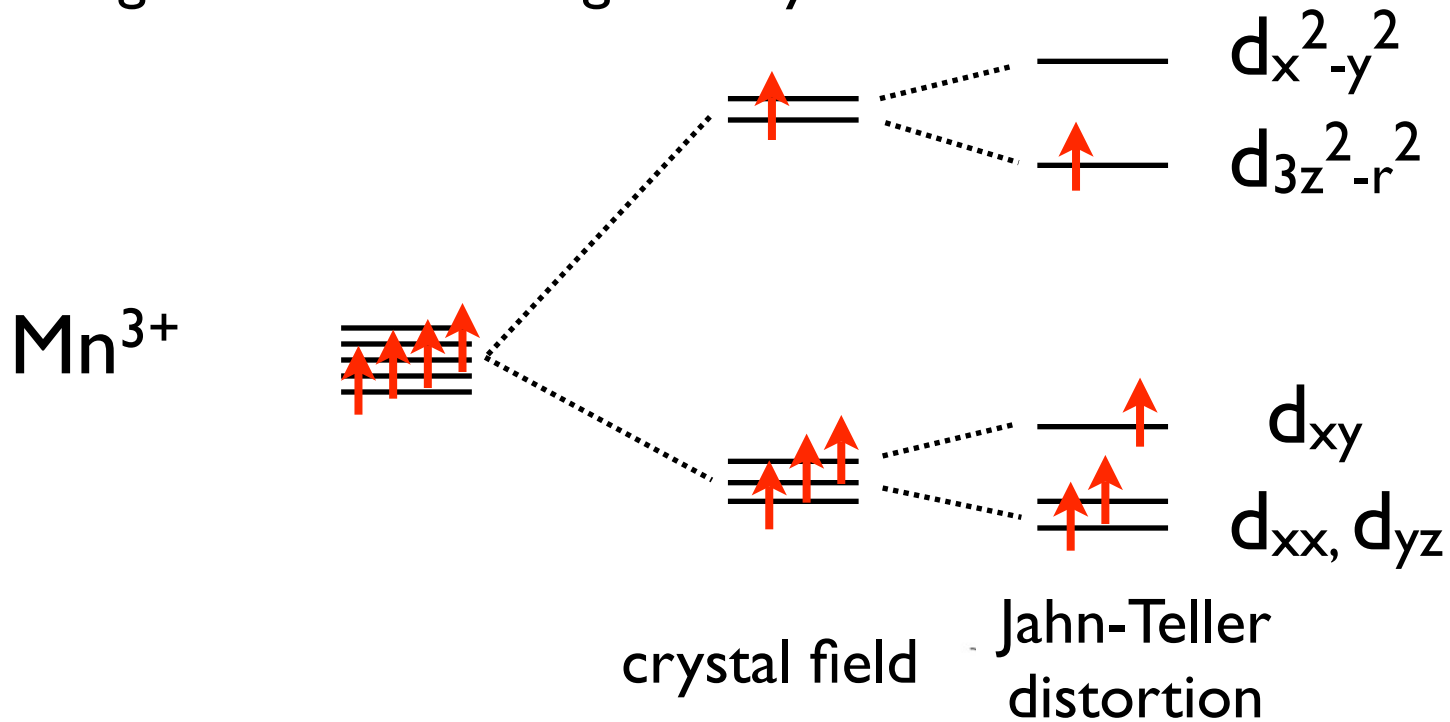
1. the maximum value of the total spin S allowed by the exclusion principle
2. the maximum value of orbital angular momentum L consistent with S
3. Spin-orbit interaction:
 $J = |L + S|$ for more than half filled shell
 $J = |L - S|$ for less than half filled shell

$\text{Mn}^{3+}: [\text{Ar}] 3d^4$



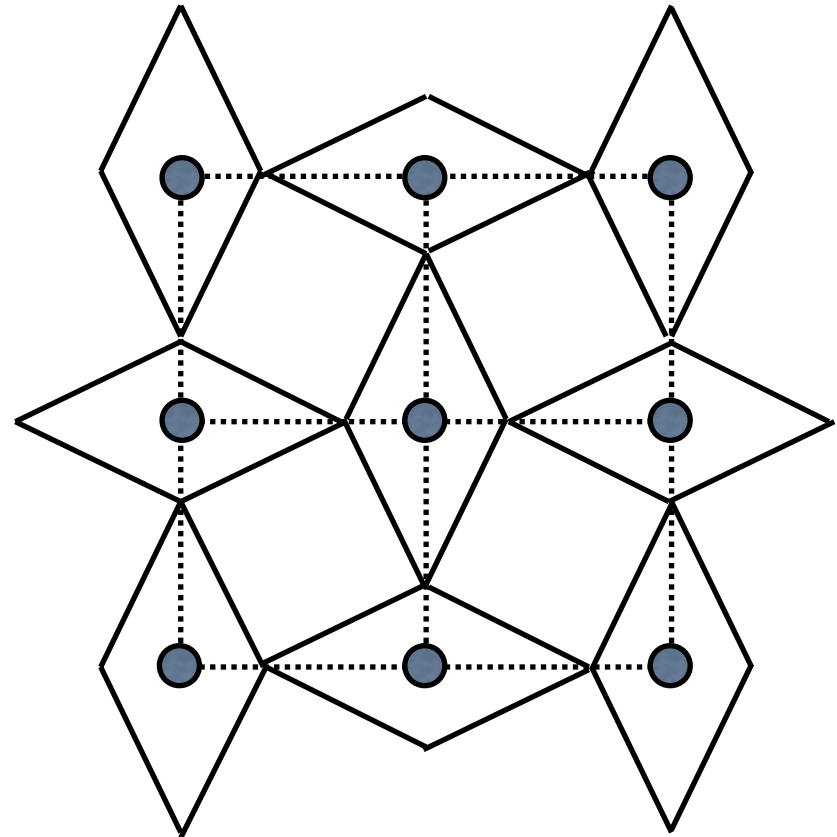
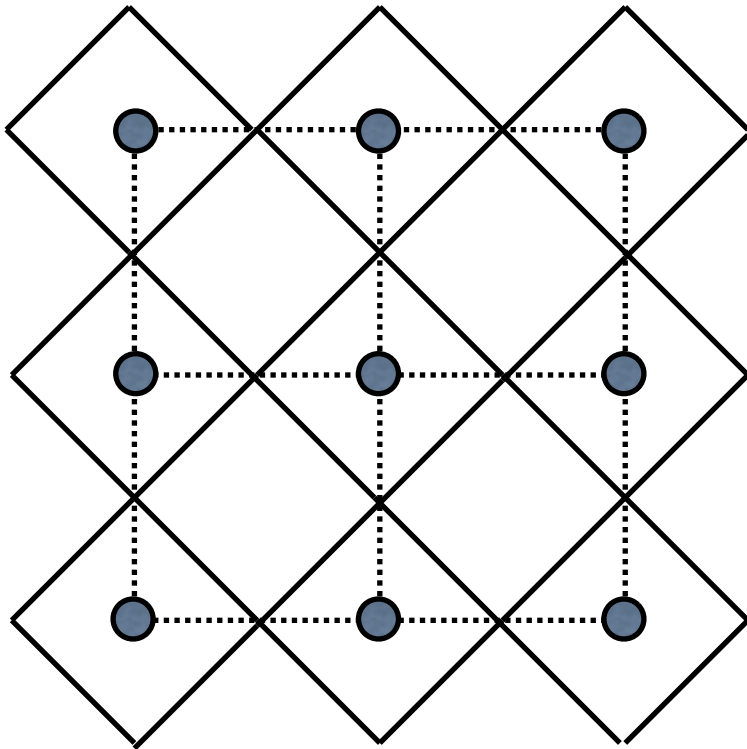
Jahn-Teller distortion

lifting of d-electron degeneracy



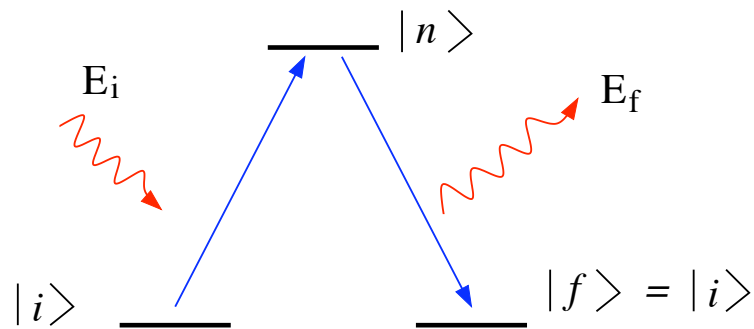
cooperative Jahn-Teller distortion - orbital order

e.g. LaMnO_3

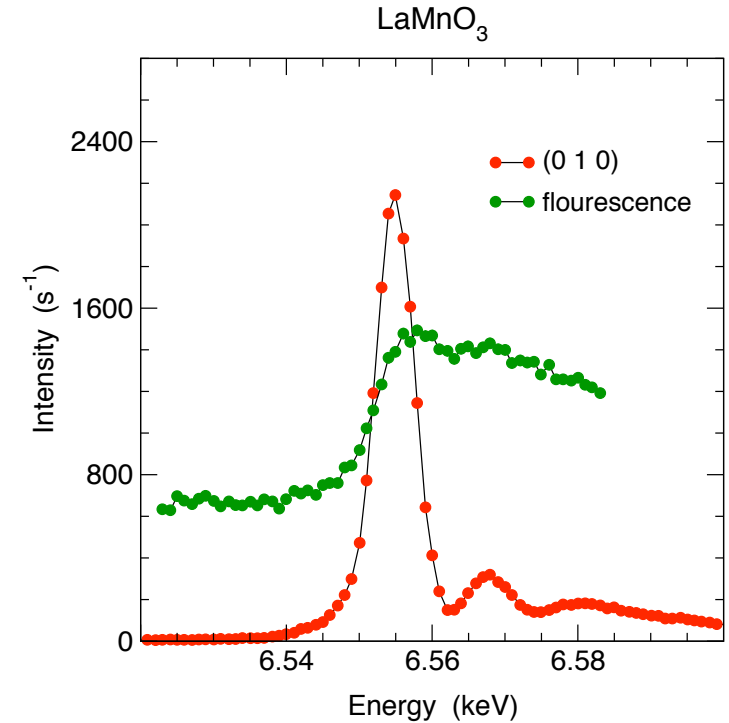


no change in lattice parameters

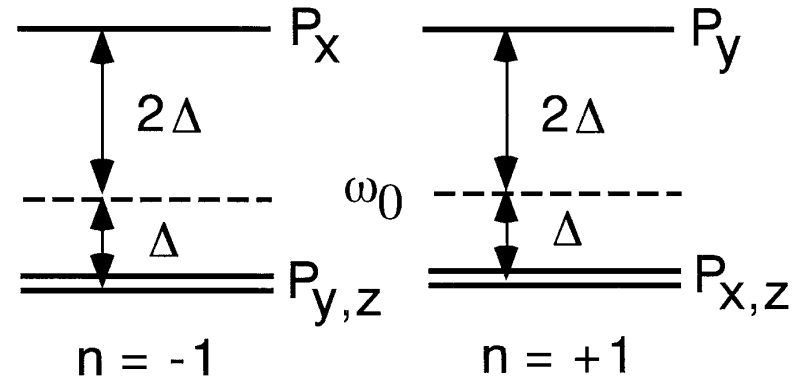
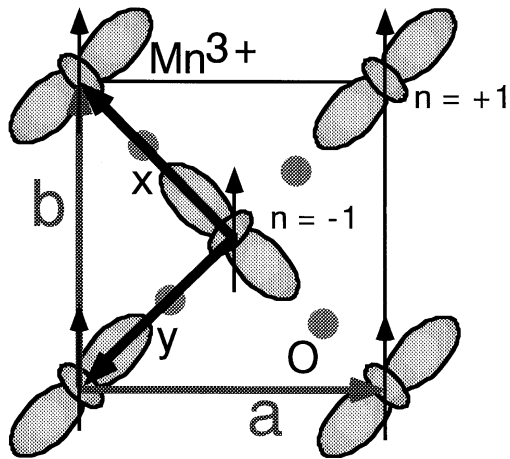
resonant x-ray scattering



at Mn K-edge: $|i\rangle = 1s$
 $|n\rangle = 4p$



resonant x-ray scattering



at the absorption edge the atom form factor depends on the incident and scattered polarization:

$$f = f_0 + \Delta f(\omega)$$

$$\Delta f(\omega) = \mathbf{e}_f \cdot \mathbf{f}'(\omega) \cdot \mathbf{e}_i$$

$$\mathbf{f}' = (f'_{\alpha,\beta}) = r_0/m \sum_j \frac{\langle 1s | P_\beta | 4p_j \rangle \langle 4p_j | P_\alpha | 1s \rangle}{E(4p_j) - E(1s) - \hbar\omega - i\Gamma/2}$$

$$f_{\parallel} = \frac{r_0}{m} \frac{|D|^2}{\hbar(\omega - \omega_0) + 2\Delta - i\Gamma/2}$$

$$f_{\perp} = \frac{r_0}{m} \frac{|D|^2}{\hbar(\omega - \omega_0) - \Delta - i\Gamma/2}$$

$$\text{with } \langle 1s | P_\alpha | 4p_j \rangle = D \delta_{\alpha j}$$

(3 0 0) Intensity

$$I(\mathbf{Q}) = I_0 \cdot |F(\mathbf{Q})|^2 = I_0 \cdot \left| \sum_l o_l f_l e^{i\mathbf{Q} \cdot \mathbf{b}_l} e^{-\mathbf{Q}^t \cdot U_l \cdot \mathbf{Q}} \right|^2$$

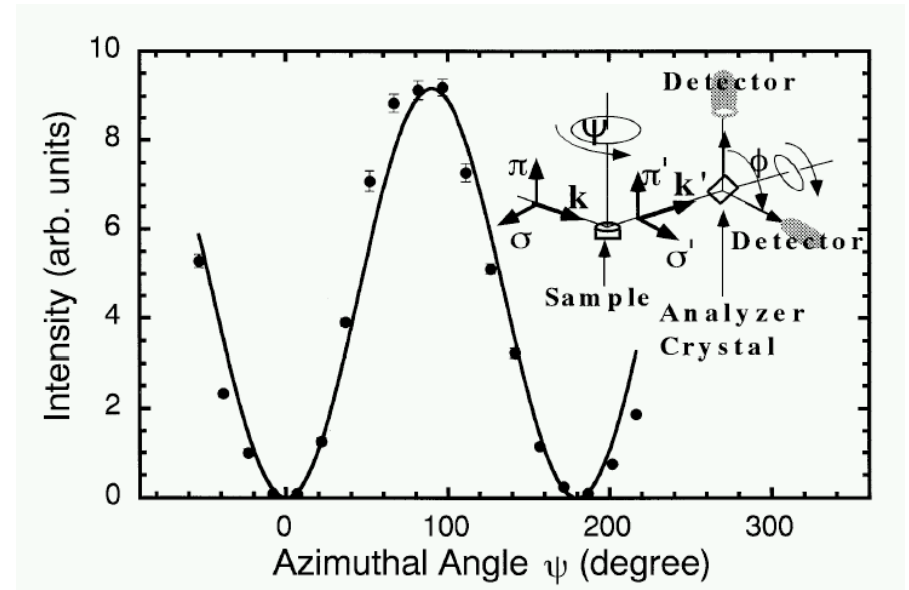
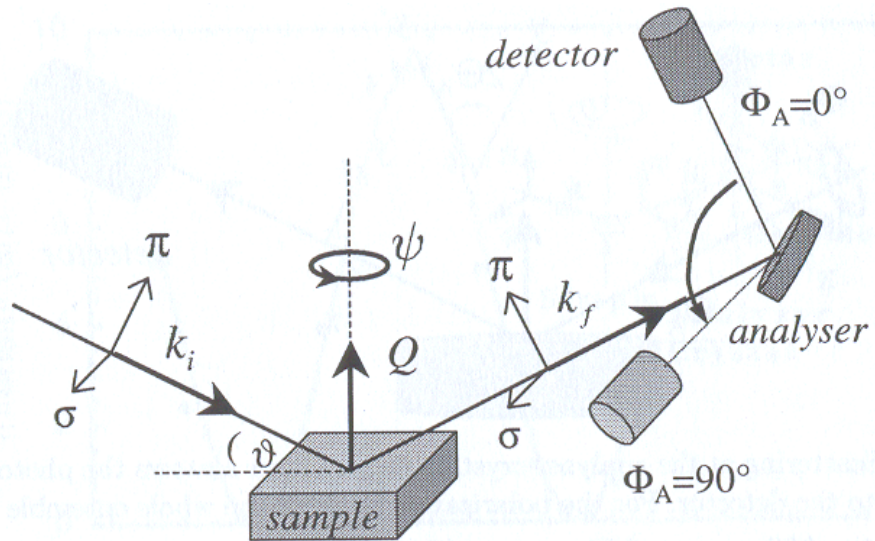
$$\begin{aligned} F(300) &= f_1(\omega, \mathbf{e}_i, \mathbf{e}_f) - f_2(\omega, \mathbf{e}_i, \mathbf{e}_f) \\ &= \mathbf{e}_f^t \cdot [\hat{f}_1(\omega) - \hat{f}_2(\omega)] \cdot \mathbf{e}_i \\ &\doteq \mathbf{e}_f^t \cdot \hat{F}(300) \cdot \mathbf{e}_i, \end{aligned}$$

$$\hat{F}(300) = \hat{f}_1 - \hat{f}_2 = \begin{pmatrix} f_{\perp} - f_{\parallel} & 0 & 0 \\ 0 & f_{\parallel} - f_{\perp} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$$I = I_0 \cdot \left| \mathbf{e}_f \cdot (U \hat{F}(300) U^t) \cdot \mathbf{e}_i \right|^2, \quad \text{where}$$

$$U \hat{F}(300) U^t = \begin{pmatrix} 0 & f_{\parallel} - f_{\perp} & 0 \\ f_{\parallel} - f_{\perp} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

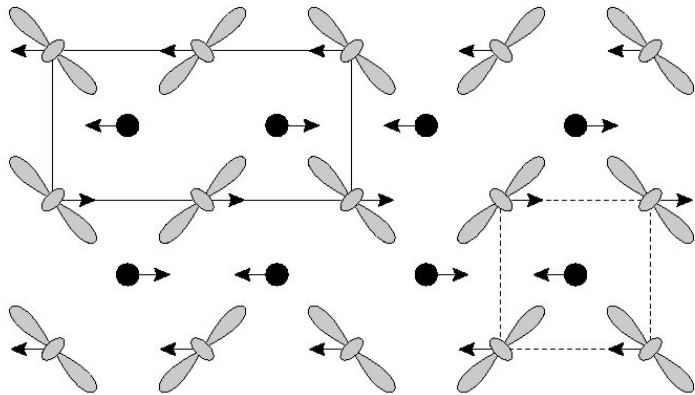
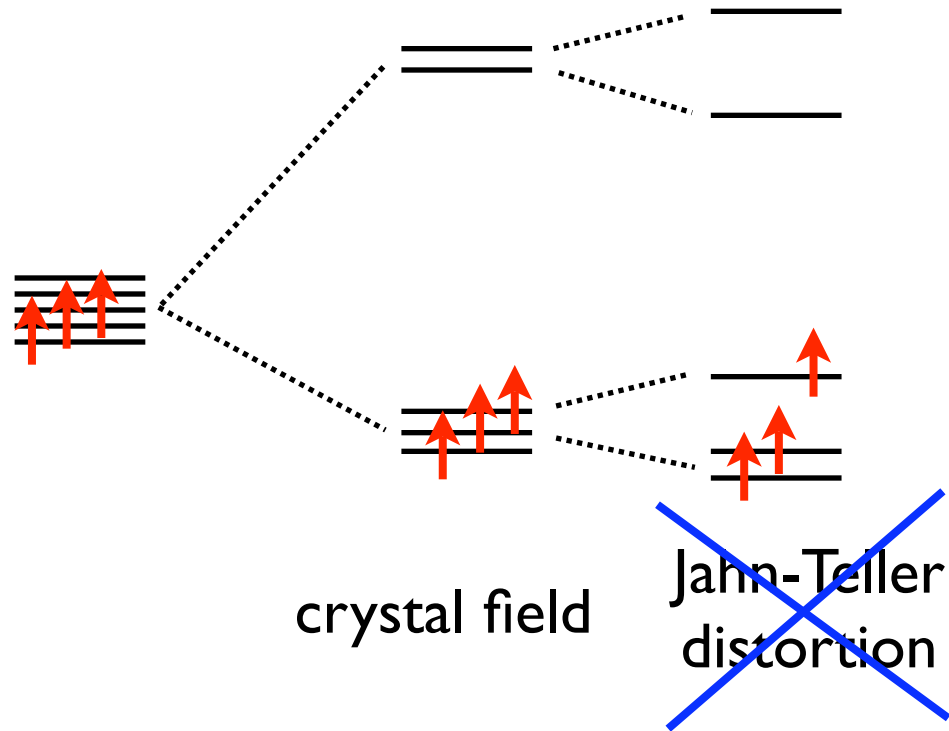
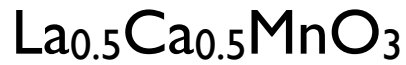
azimuthal dependence



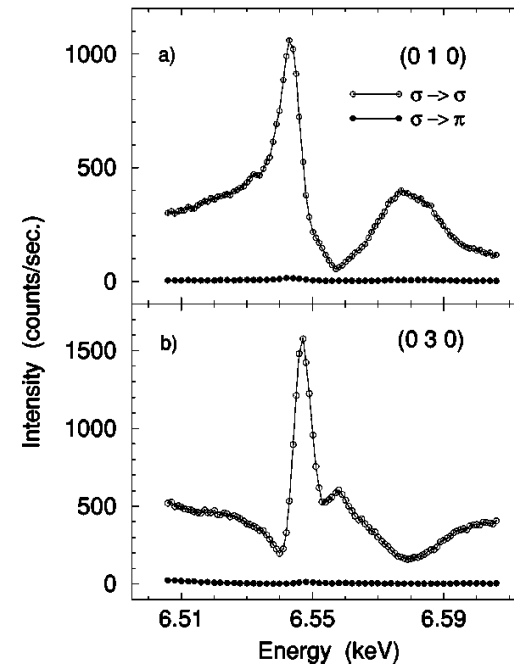
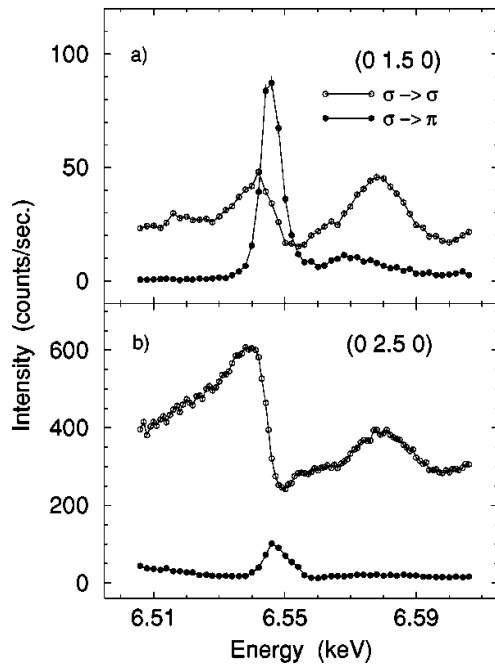
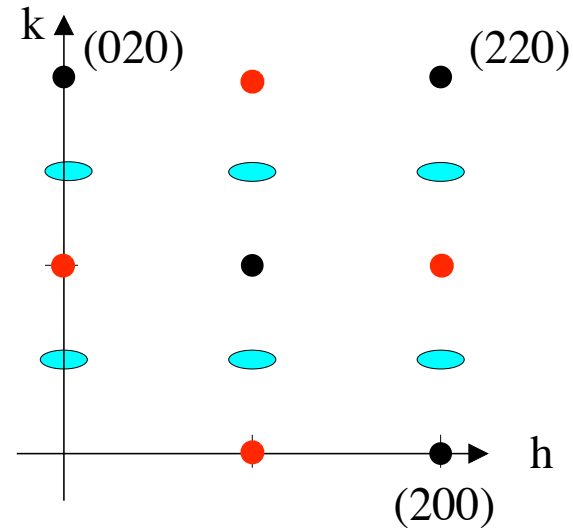
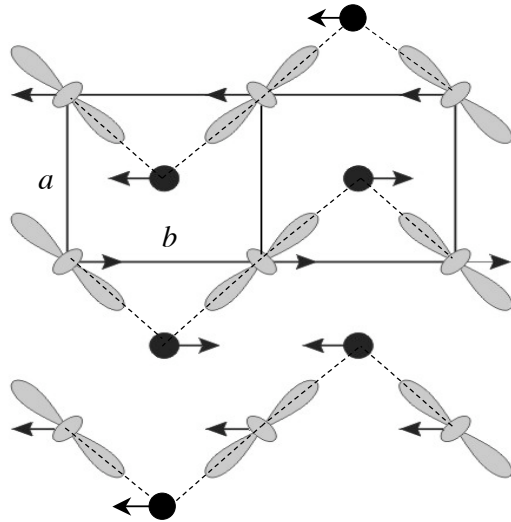
resonant scattering at transition metal L-edges

- direct sensitivity for d-electrons, thus orbital order is probed directly, not the Jahn-Teller distortion as for k-edge
- large resonant enhancement for magnetic order
- small momentum transfers achievable, (100) of LaMnO_3 out of reach
- surface sensitive
- ultra high vacuum conditions necessary

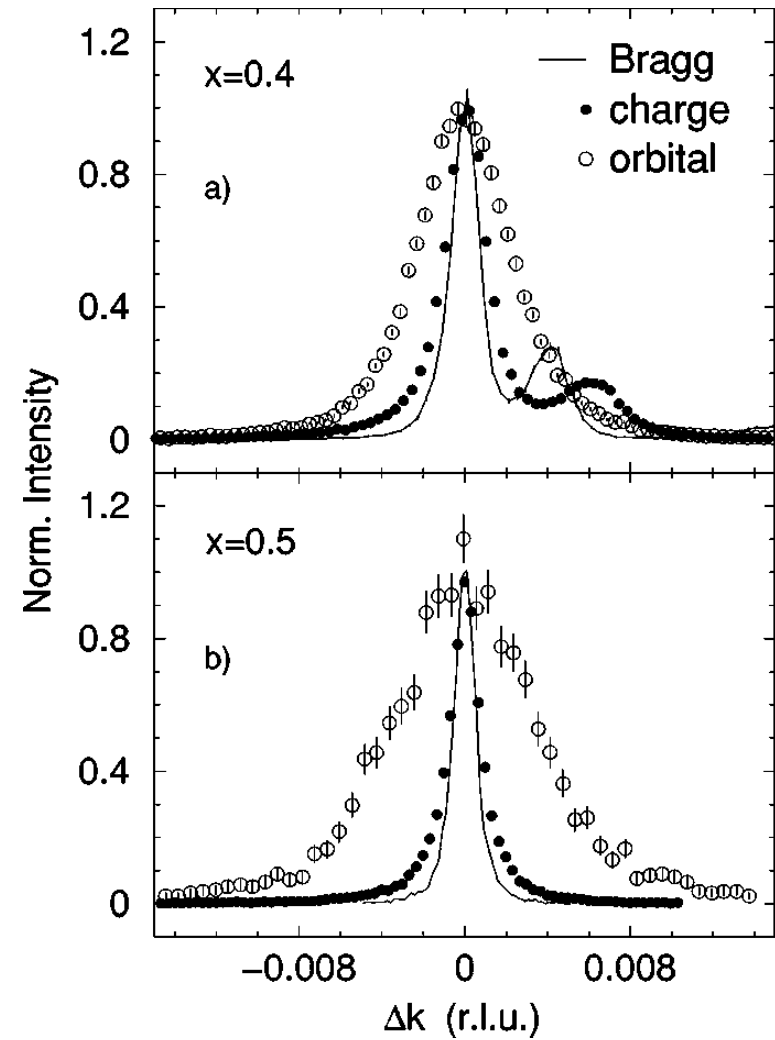
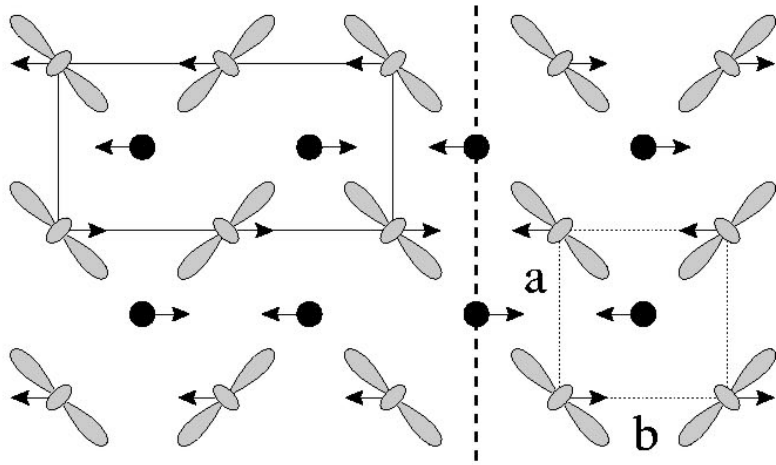
doping - charge order



charge/orbital order resonant diffraction



domains - correlation length



summary

- solids state phase transitions
 - order parameter
 - power laws with critical exponents
 - correlation length
 - superlattice reflection

- transition metal oxides
 - symmetry of d-electrons in cubic crystal field
 - Jahn-Teller effect
 - resonant x-ray scattering

literature

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W. Gebhard and U.Krey, *Phasenübergänge und kritische Phänomene*, Friedr. Vieweg & Sohn, Braunschweig/Wiesbaden 1980

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S.W. Lovesey and S.P. Collins, *X-ray Scattering and Absorption by Magnetic Materials*, Oxford Series on Synchrotron Radiation (Clarendon Press-Oxford, 1996)