May 16, 2001

Formulas for synchrotron radiation from wigglers and undulators

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Radiated power of a wiggler or undulator with sinusoidal field:

$$P[W] = \frac{1265.382}{2} L[m] \cdot I[A] \cdot E[GeV]^2 \cdot B_{max}[T]^2 \;\; for\ electrons$$

$(Example: \; P = 1.461kW; \; L = 2.112m; \; I = 0.15A;$
$E = 4.5GeV; \; B_{max} = 0.60T)$
$(Example: \; P = 5.228kW; \; L = 4.000m; \; I = 0.1A;$
$E = 4.5GeV; \; B_{max} = 1.01T)$
$(Example: \; P = 6.614kW; \; L = 4.000m; \; I = 0.06A;$
$E = 12GeV; \; B_{max} = 0.55T)$

Central power density of a wiggler or undulator for $\epsilon = 0$:

$$P[W/mrad^2] = 10.85 \cdot N \cdot E[GeV]^4 \cdot B_{max}[T] \cdot I[A] \;\; for\ electrons$$

$(Example: \; P = 85.6kW/mrad^2; \; N = 127; \; E = 4.5GeV;$
$B_{max} = 1.01T; \; I = 0.15A)$
$(Example: \; P = 94.3kW/mrad^2; \; N = 127; \; E = 12GeV;$
$B_{max} = 0.55T; \; I = 0.06A)$

Fundamental wavelength and energy of an undulator:

$$\lambda_{ph} = \frac{\lambda}{2\gamma^2} \cdot (1 + \frac{k^2}{2} + \gamma^2 \cdot \theta^2); \quad \gamma = \frac{E}{E_0}$$

$\theta \; = \; observation\ angle\ with\ respect\ to\ the\ undulator\ beam$

$$E_{ph}[eV] = \frac{1239.842}{\lambda_{ph}[nm]}$$

$(Example: \; \lambda_{ph} = 0.4648nm; \; \lambda = 3.14cm; \; E = 4.5GeV;$
$k = 1.61; \; \theta = 0)$
$(Example: \; E_{ph} = 2667eV; \; \lambda_{ph} = 0.4648nm)$

$(Example: \ \lambda_{ph} = 0.06537nm; \ \lambda = 3.14cm; \ E = 12GeV;$
$k = 1.61; \ \theta = 0)$
$(Example: \ E_{ph} = 18967eV; \ \lambda_{ph} = 0.06537nm)$

Undulator beam divergence for the i^{th} harmonics:

$$\sigma'_{ph} = \sqrt{\frac{1 + \frac{k^2}{2}}{2i \cdot N \cdot \gamma^2}}$$

$(Example: \ \sigma'_{ph} = 10.8\mu rad; \ k = 1.61; \ i = 1; \ N = 128; \ \gamma = 8806)$
$(Example: \ \sigma'_{ph} = 2.33\mu rad; \ k = 1.61; \ i = 3; \ N = 128; \ \gamma = 23483)$

Undulator beam divergence (fundamental mode; zero electron emittance):

$$\sigma'_{ph} = \sqrt{\frac{\lambda_u \cdot (1 + \frac{k^2}{2})}{2\gamma^2 \cdot L}}$$

$$\sigma'_{ph} = \sqrt{\frac{\lambda_{ph}}{L}}; \quad \lambda_{ph} = \frac{\lambda_u}{2\gamma^2}(1 + \frac{k^2}{2})$$

$(Example: \ \sigma'_{ph} = 6.12\mu rad; \ \lambda_{ph} = 0.15nm; \ L = 4m)$

Undulator beam size for zero electron emittance:

$$\sigma_{ph} = \frac{1}{4\pi} \cdot \sqrt{\lambda_{ph} \cdot L}$$

$(Example: \ \sigma_{ph} = 31.5\mu m; \ \lambda_{ph} = 6.28nm; \ L = 25m)$

Wiggler beam size for nonzero electron emittance:

$$\sigma_{photon(x)} = \sqrt{\sigma^2_{electron(x)} + (\frac{L}{2} \cdot \frac{k}{\gamma})^2}$$

$(horizontal\ photon\ and\ electron\ beam\ divergence\ neglected)$

$$\sigma_{photon(z)} = \sqrt{\sigma^2_{electron(z)} + (\frac{L}{2} \cdot \sqrt{\sigma'^2_{photon(z)} + \sigma'^2_{electron(z)}})^2}$$

Critical energy of synchrotron radiation from a wiggler:

$$E_c = \frac{3}{2} \cdot \frac{\hbar \cdot c^2}{E_0^3} \cdot B \cdot E^2$$

$$E_c[eV] = 665.0255 \cdot B[T] \cdot E[GeV]^2$$

$(Example: \; E_c = 16160eV; \; B = 1.2T; \; E = 4.5GeV)$

Undulator line width for the N^{th} harmonics and for n periods:

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{n \cdot N}$$

$(Example: \; \Delta\lambda/\lambda = 0.26\%; \; n = 128; \; N = 3)$

Undulator line broadening because of the electron beam divergence:

$$\frac{\Delta\lambda}{\lambda} = \frac{(\gamma \cdot \sigma_r')^2}{1 + \frac{k^2}{2}}$$

$(Example: \; \Delta\lambda/\lambda = 8.8\%; \; E = 4.5GeV; \sigma_r' = 51\mu rad; \; k = 1.61)$

Undulator line broadening because of the acceptance angle θ:

$$\frac{\Delta\lambda}{\lambda} = \frac{(\gamma \cdot \theta)^2}{1 + \frac{k^2}{2}}$$

Undulator line broadening because of the electron energy spread:

$$\frac{\Delta\lambda}{\lambda} = 2\frac{\Delta E}{E}$$

$(Example: \; \Delta\lambda/\lambda = 0.22\%; \; \Delta E/E = 0.11\%)$

Spectral flux (wiggler):

$$I[phot./(sec\,mrad\,0.1\%\,bandw.)] = 2.458 \cdot 10^{10} \cdot 2N \cdot I_e[mA] \cdot E_e[GeV] \cdot \frac{E}{E_c} \cdot \int_{E/E_c}^{\infty} K_{5/3}(\eta) d\eta$$

$$\int_1^{\infty} K_{5/3}(\eta) d\eta = 0.6522$$

$(Example: \; I = 6.06 \cdot 10^{14} phot./(sec\,mrad\,0.1\%\,bandw.);$
$N = 28; \; I_e = 150mA; \; E_e = 4.5GeV; \; E = E_c)$

Spectral central brightness for $\epsilon = 0$ (wiggler):

$$I[phot./(sec\,mrad^2\,0.1\%\,bandw.)] = 1.325 \cdot 10^{10} \cdot 2N \cdot I_e[mA] \cdot E_e[GeV]^2 \cdot (\frac{E}{E_c})^2 \cdot K_{2/3}(\frac{E}{2E_c})$$

$$K_{2/3}(\frac{1}{2}) \approx 1.45$$

$$(Example:\ I \approx 3.3 \cdot 10^{15} phot./(sec\, mrad^2\ 0.1\%\ bandw.);$$
$$N = 28;\ I_e = 150mA;\ E_e = 4.5GeV;\ E = E_c)$$