

# Methoden moderner Röntgenphysik: Streuung und Abbildung

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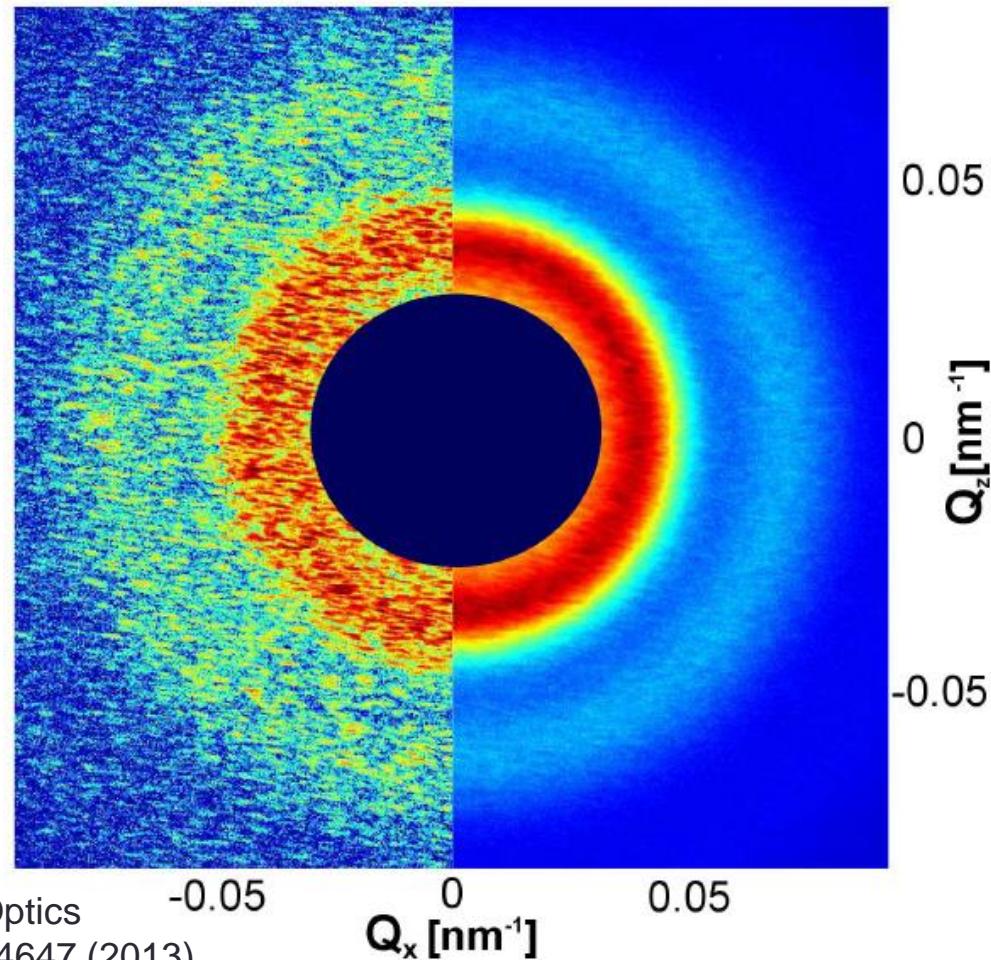
Lecture 19	Vorlesung zum Haupt- oder Masterstudiengang Physik, SoSe 2021 G. Grübel, O. Seeck, <u>F. Lehmkühler</u> , A. Philippi-Kobs, V. Markmann, M. Martins
Location	online
Date	Tuesday                    12:30 - 14:00 (starting 6.4.) Thursday                    8:30 - 10:00 (until 8.7.)

# Soft Matter – Timeline

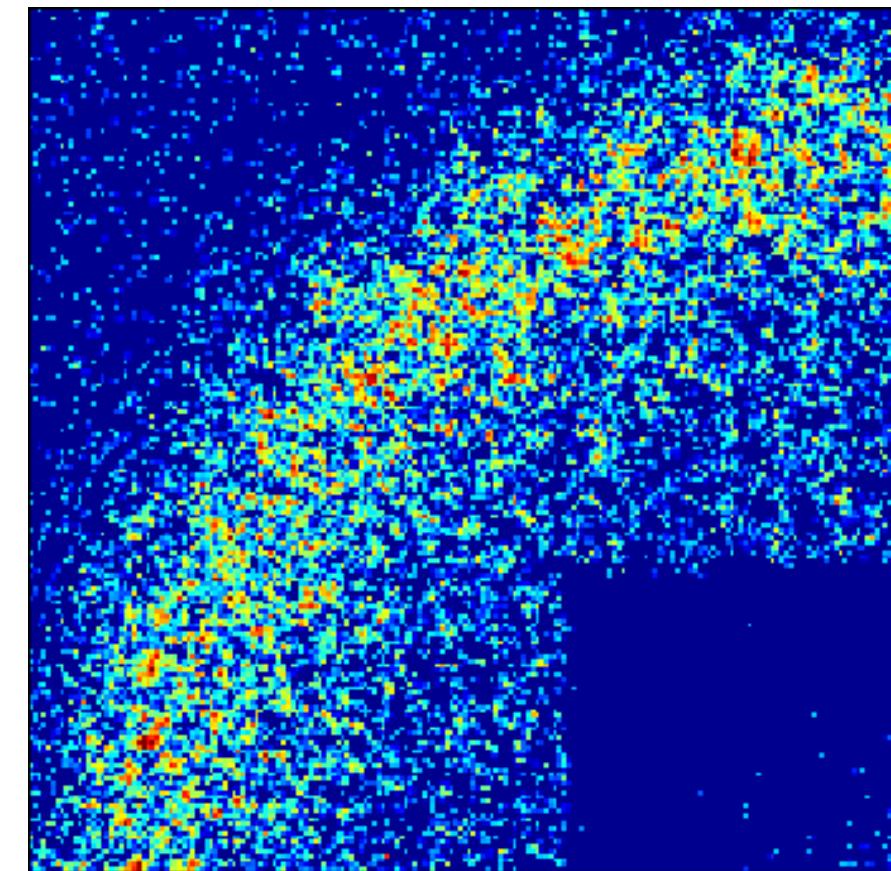
- Do 27.05.2021 Soft Matter studies I: Methods & experiments  
*Definitions, complex liquids, colloids, storage ring and FEL experiments, setups, liquid jets, ...*
- Di 01.06.2021 Soft Matter studies II: Structure  
*SAXS & WAXS applications, X-ray cross correlations, ...*
- Do 03.06.2021 Soft Matter studies III: Dynamics  
*XPCS applications, diffusion, dynamical heterogeneities, ...*
- Di 08.06.2021 XPCS & XCCA simulations and modelling
- Do 10.06.2021 Case study I: Glass transition  
*Supercooled liquids, glasses vs. crystals, glass transition concepts, structure-dynamics relations, ...*
- Di 15.05.2021 Case study II: Water  
*Phase diagram, anomalies, crystalline and glassy forms, FEL studies, ...*
- Do 17.06.2021 Outlook: Opportunities at new facilities

## Probing structure and dynamics with coherent X-rays: XPCS & XCCA

X-ray scattering from disordered samples: speckles → structure decoded



S. Lee et al. Optics  
Express 21, 24647 (2013)



## What is a glass?

- Disordered materials
  - Lack of periodicity (long-range order) as crystals
  - But: short-range order may exist
  - Behave mechanically like solids
- Glass is known since ancient times (e.g. silicate glass)
- Examples of glasses
  - Fused Silica ( $\text{SiO}_2$ )
  - Network glasses (phosphate glasses, borate glasses, ...)
  - Obsidian
  - Glass fibres
  - Metallic glasses
  - Polymer glasses (plastics)
  - Colloidal glasses
  - Glassy water ( $\rightarrow$  lecture 20)



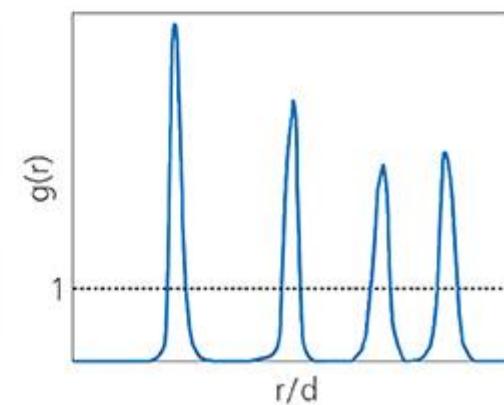
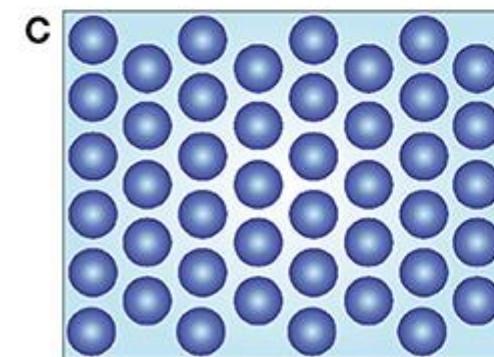
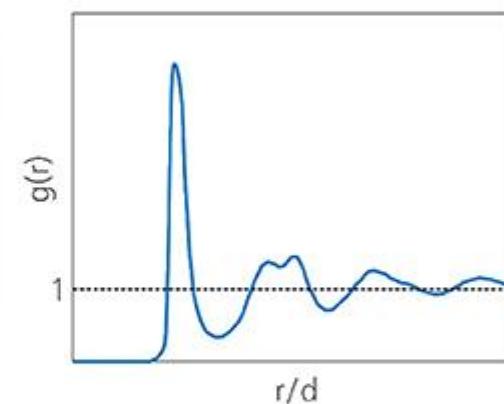
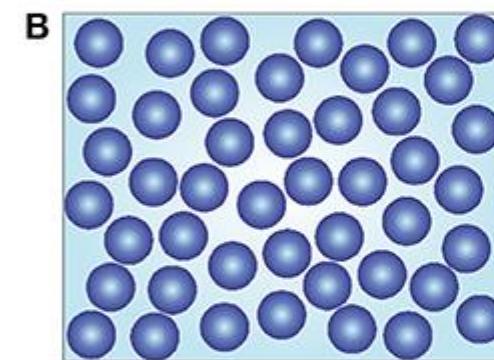
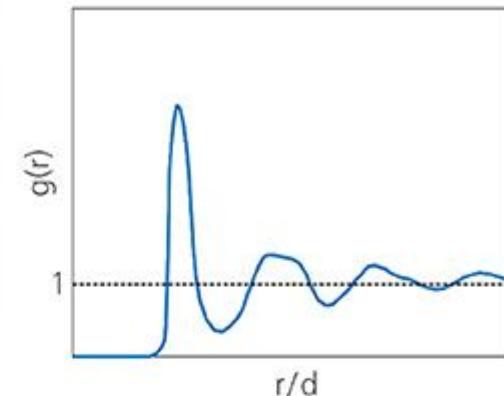
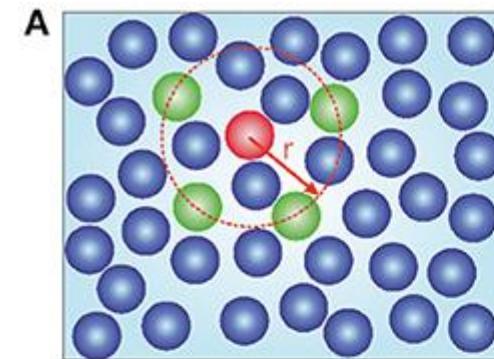
Roman glasses

Pictures: wikipedia

## What is a glass?

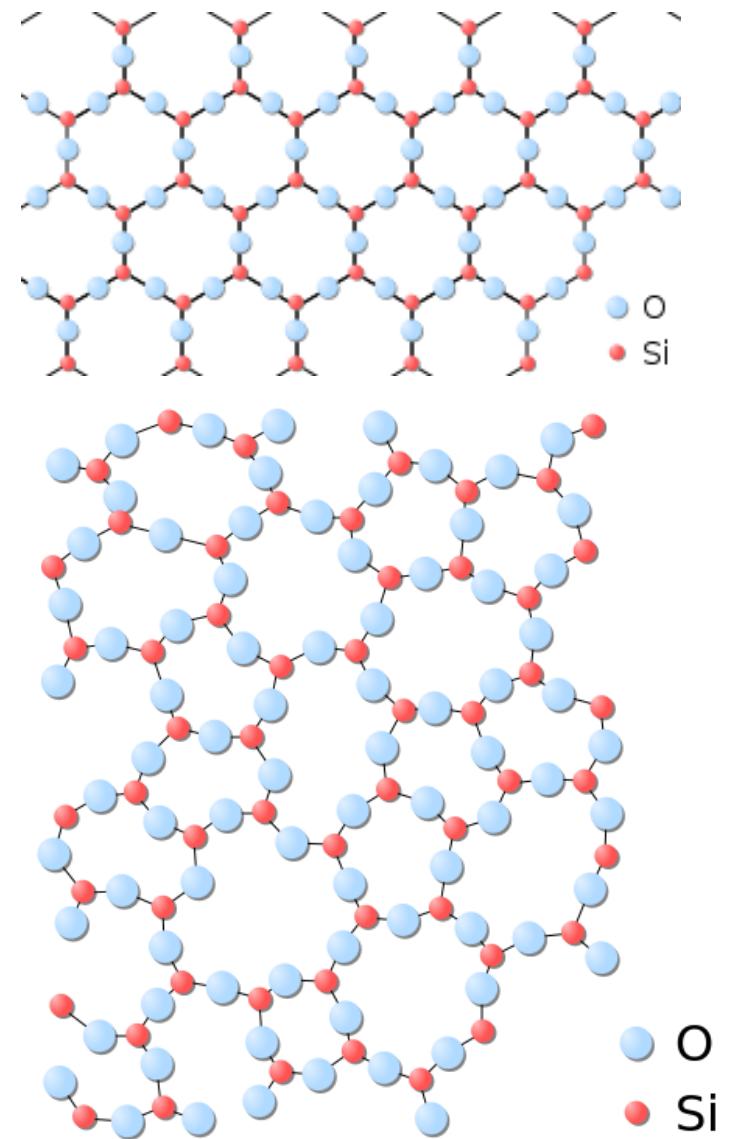
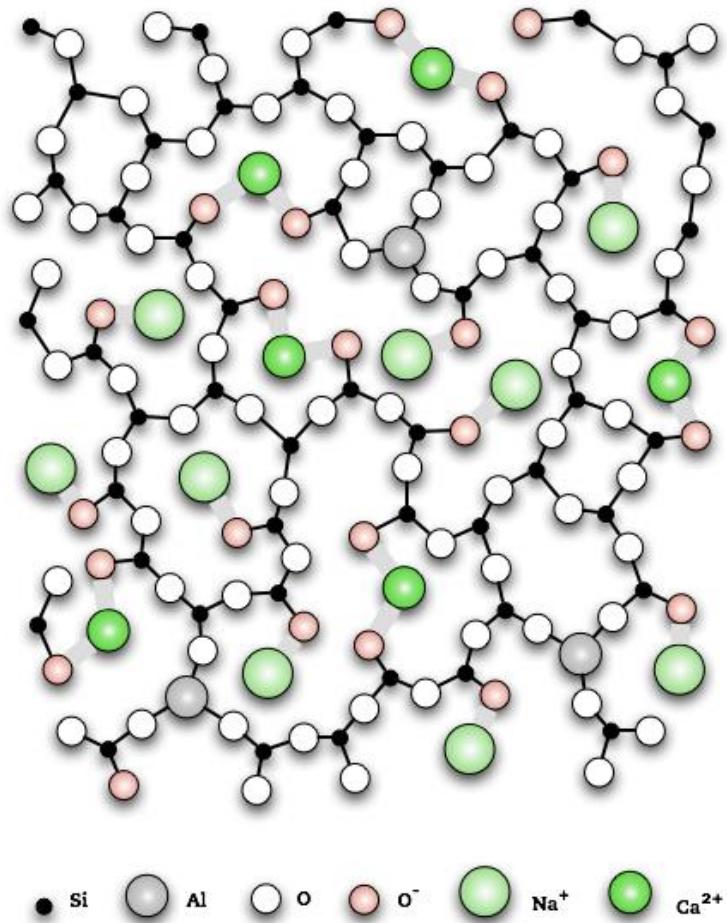
Structure of

- (A) a normal liquid
- (B) an amorphous solid, i.e., a glass
- (C) a crystalline solid



Frontiers in Physics 6, 97 (2018)  
arXiv:1806.01369 (2018)

## What is a glass?



## What is a glass?

Schematic dynamics of  
**(A)** a normal liquid  
**(B)** a glass

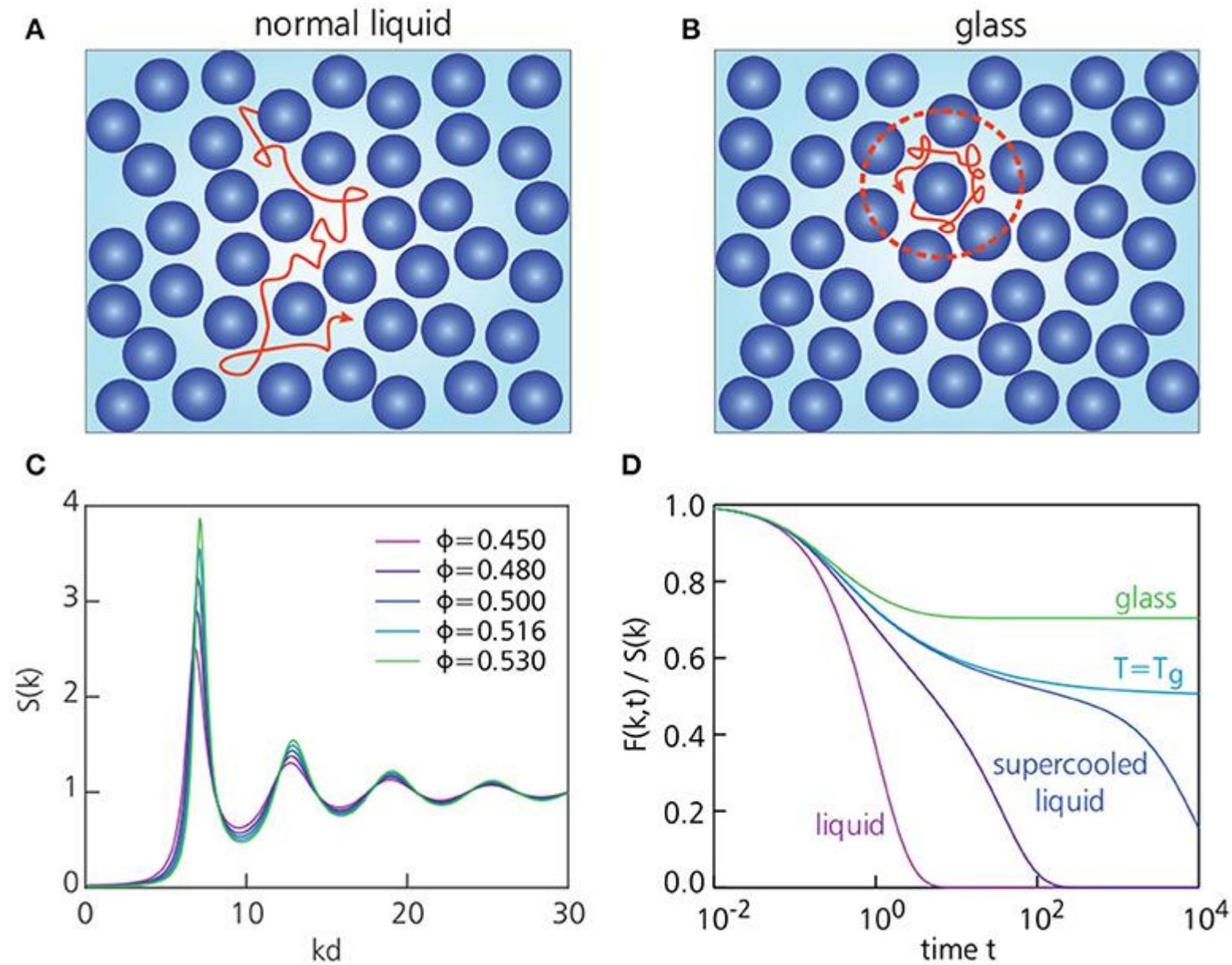
→[Video](#)

Hard spheres

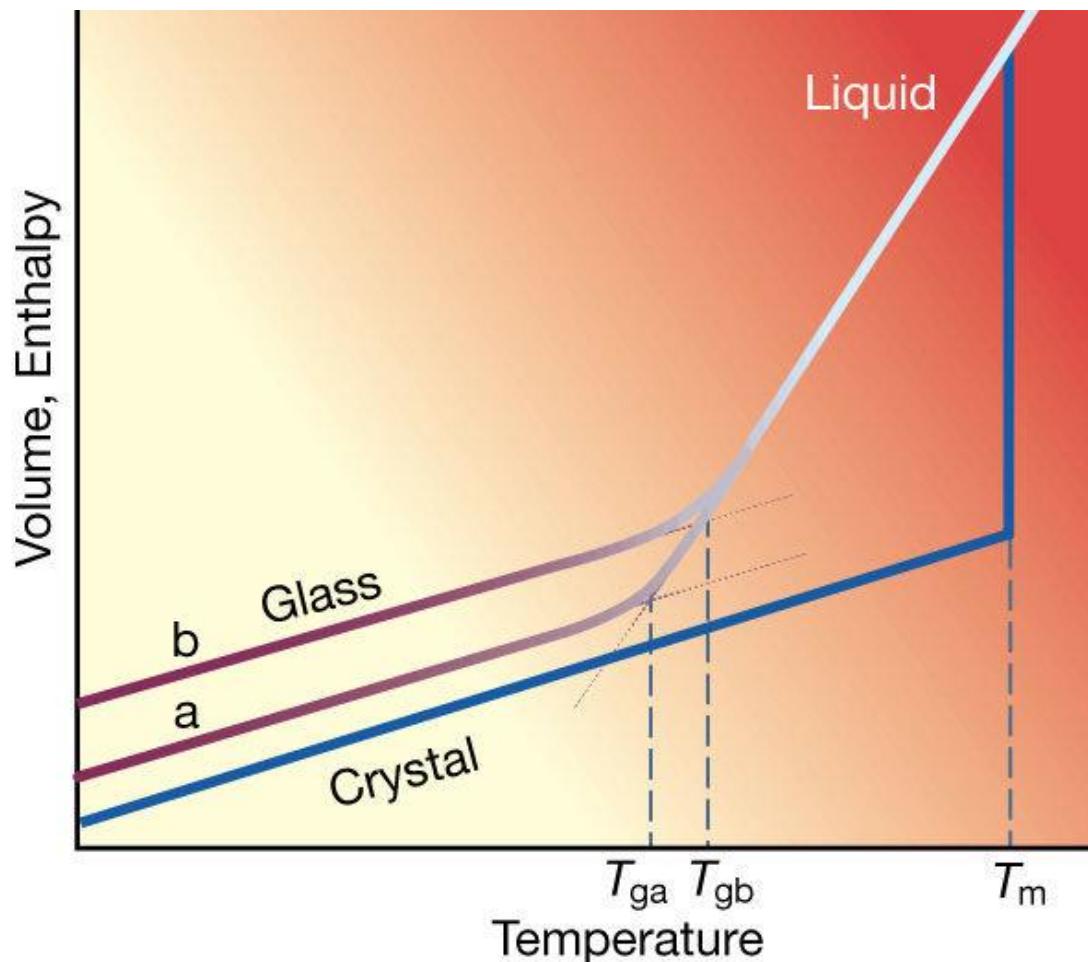
**(C)** Structure factor of hard spheres.

**(D)** Corresponding Intermediate scattering functions

Frontiers in Physics 6, 97 (2018)  
arXiv:1806.01369 (2018)



## Supercooled liquids and glass transition



Nature 410, 259 (2001)

Cooling a liquid below its freezing point  $T_m$   
→ slow down of molecular motion

If cooled sufficiently fast  
→ Crystallisation avoided  
→ Molecules rearrange too slowly  
→ Out-of-equilibrium  
→ Liquid is "frozen" on experimental timescales  
→ No phase transition!

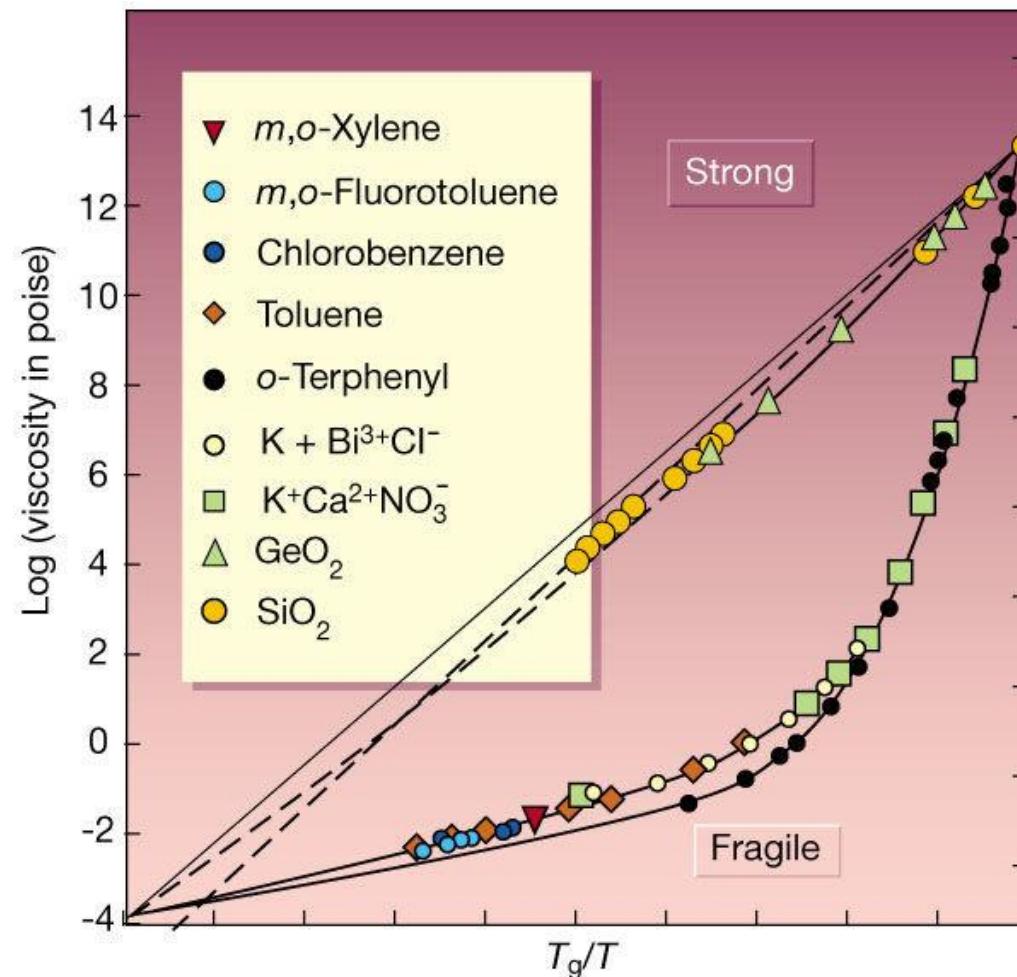
Glass transition temperature: convention!

- Depends on cooling rate
- Molecular relaxation  $\sim 100$  s
- Viscosity of  $10^{12}$  Pa s
- Change of heat capacity, thermal expansion, ...

## Glass transition temperatures

Material	Glass transition $T_g$ (°C)	Crystallisation $T_m$ (°C)
Silica $\text{SiO}_2$	~1200	1713
Borosilicate glass	~500	
$\text{GeO}_2$	~700	~1000
Polystyrene	95	~240-270
Teflon	115	327
PMMA (Plexiglas)	105	
Glycerol	~ -70	18
$\text{Zr}_{65}\text{Al}_{7.5}\text{Ni}_{10}\text{Cu}_{17.5}$	360	

## Fragility



Science 267, 1927 (1995)

Nature 410, 259 (2001)

### Viscosity towards $T_g$

- Arrhenius behaviour  $\eta = A \exp\left(\frac{E}{k_B T}\right)$   
 → "strong" glass former  
 → Broad range of  $T_g$
- Fragility: deviation from Arrhenius behaviour  
 → More pronounced viscous slow-down  
 → Described empirically by Vogel-Fulcher-Tamann law  

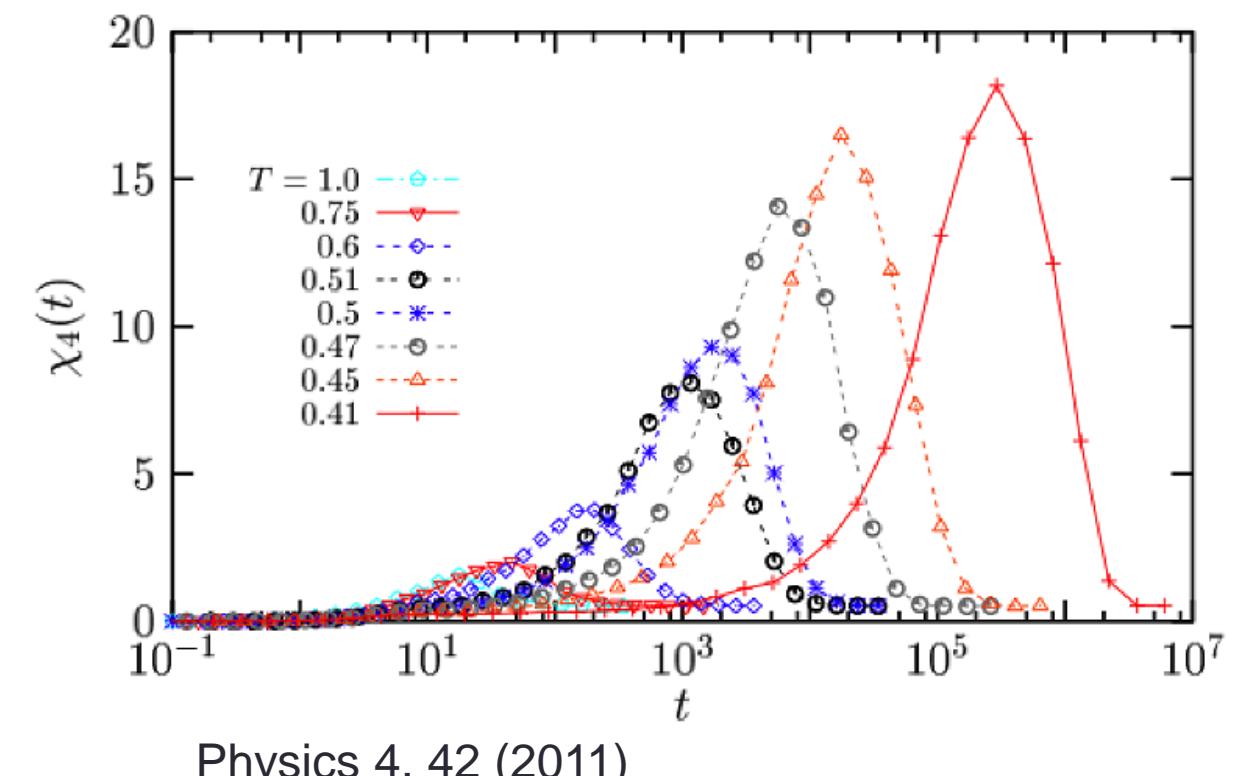
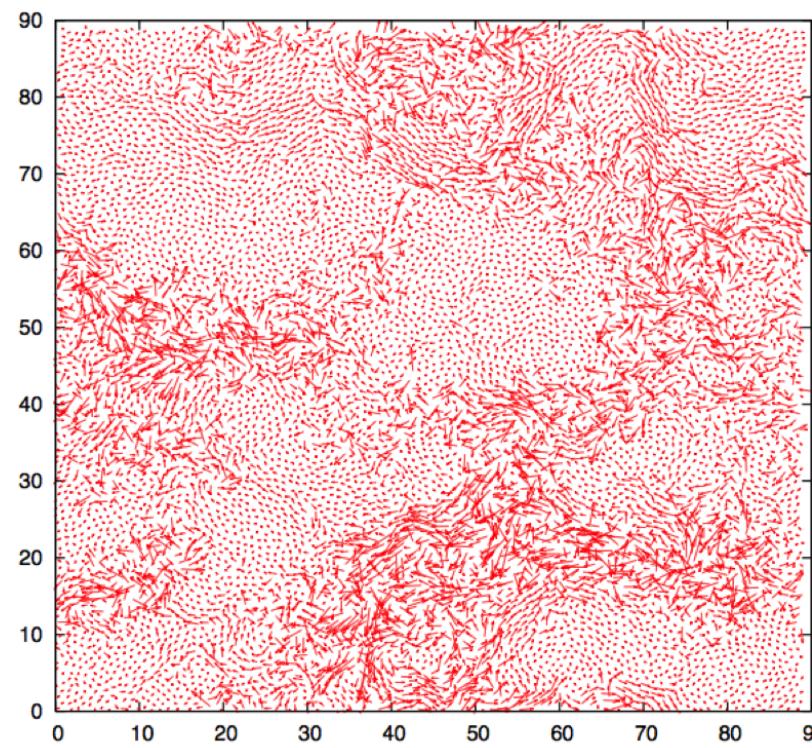
$$\eta = A \exp\left(\frac{B}{T-T_0}\right)$$
- Fragility index:  $m := \left(\frac{\partial \log_{10} \eta}{\partial \left(\frac{T_g}{T}\right)}\right)_{T=T_g}$

## Non-exponential relaxation

- Close to  $T_g$ , temporal behaviour of response functions becomes non-exponential
  - E.g. stress response on deformation, polarization response on applied electric field, ...
  - Likewise: particle dynamics
- Described by Kohlrausch-Williams-Watts function  $F(t) = \exp\left(-\left(\frac{t}{\tau}\right)^\gamma\right)$  with  $\gamma < 1$  (cf. Lecture 18) → XPCS experiments
- Contrasts to liquids → exponential response
- Spatial & dynamic heterogeneity: growth of domains with distinct relaxation

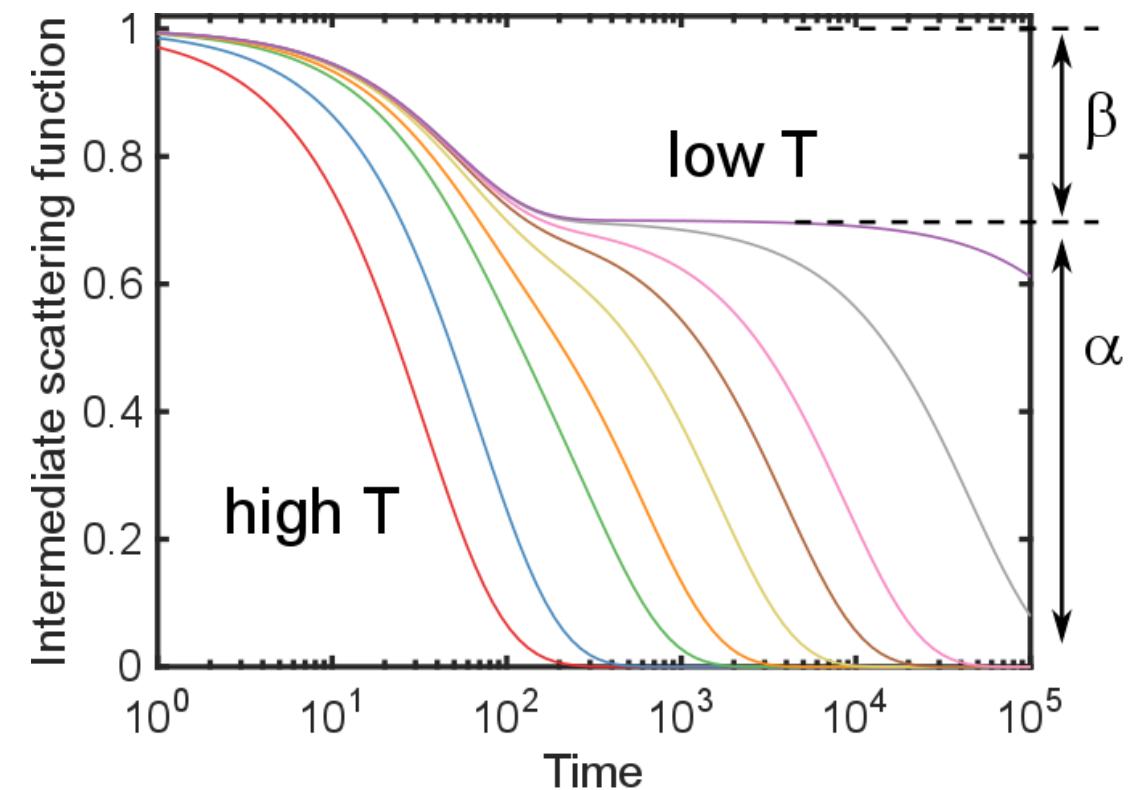
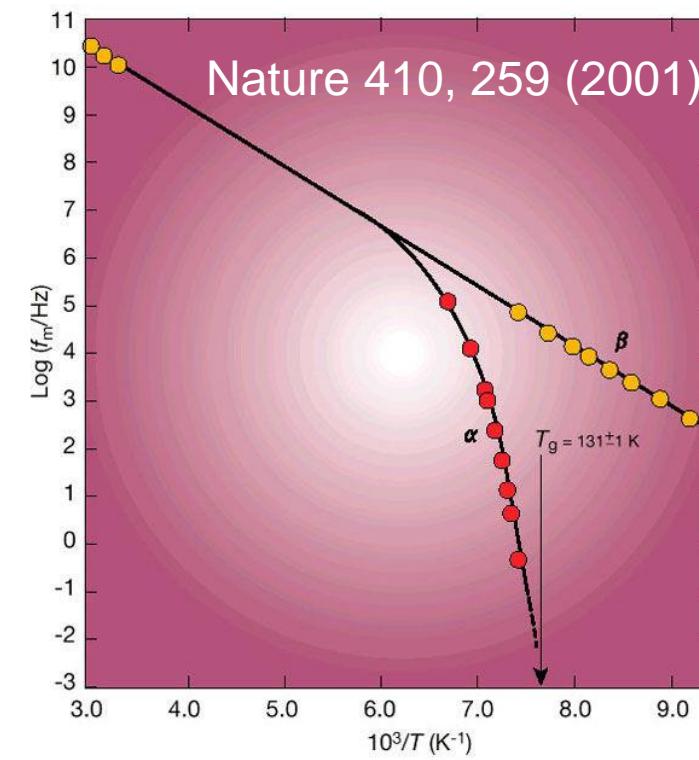
## Dynamic Heterogeneities

- Different dynamics at different regions of supercooled liquids
- Quantify via  $\chi_4(t) = N[\langle C(t)^2 \rangle - \langle C(t) \rangle^2]$ , with "total mobility"  $C(t)$
- Dynamic susceptibility  $\chi_4(t) \sim$  volume of correlated clusters
- Need higher-order correlations to be determined



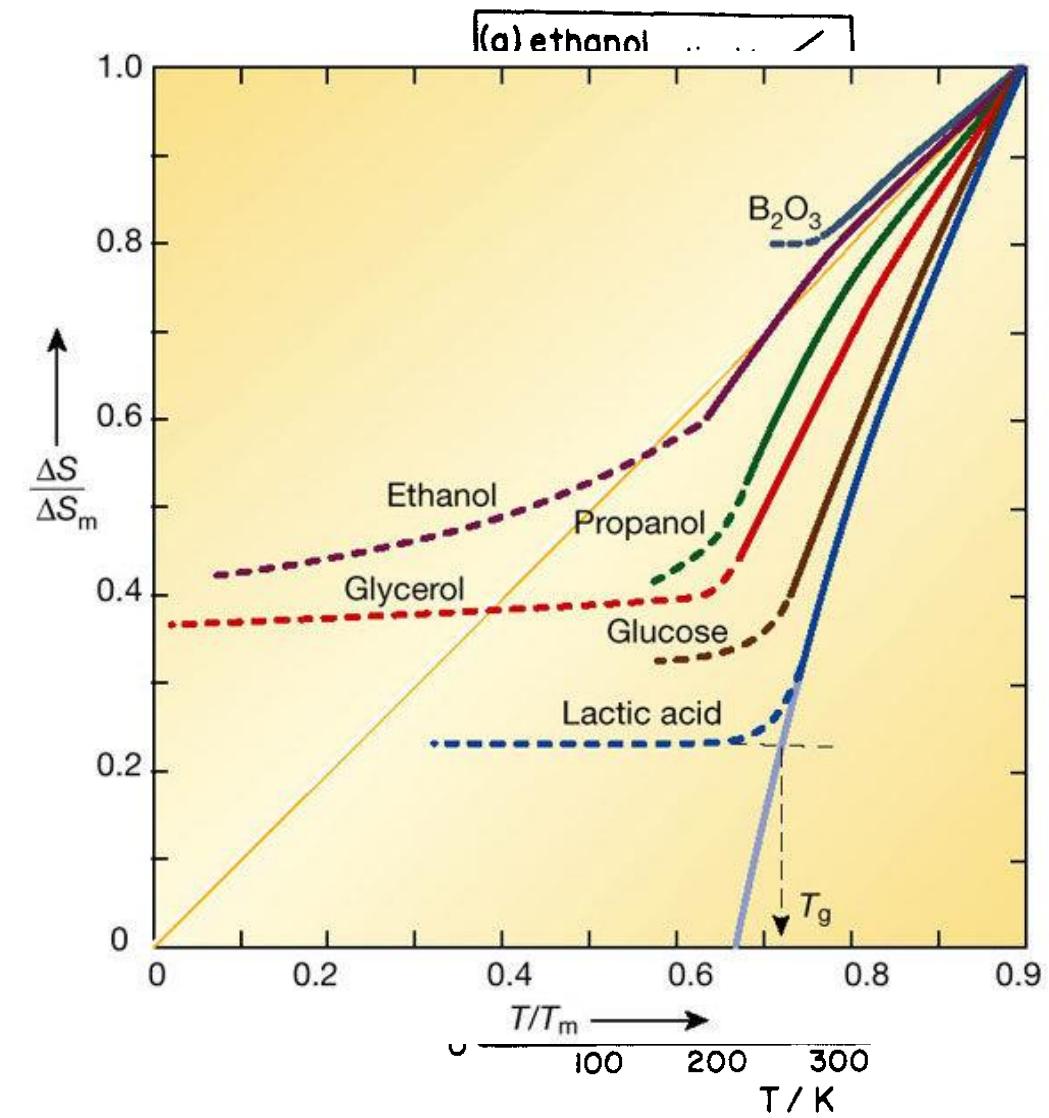
## Alpha- & beta-relaxation

- Decoupling of dynamics near  $T_g$ :  
 $\alpha$ - and  $\beta$ -relaxations



## Thermodynamics: Adam-Gibbs model

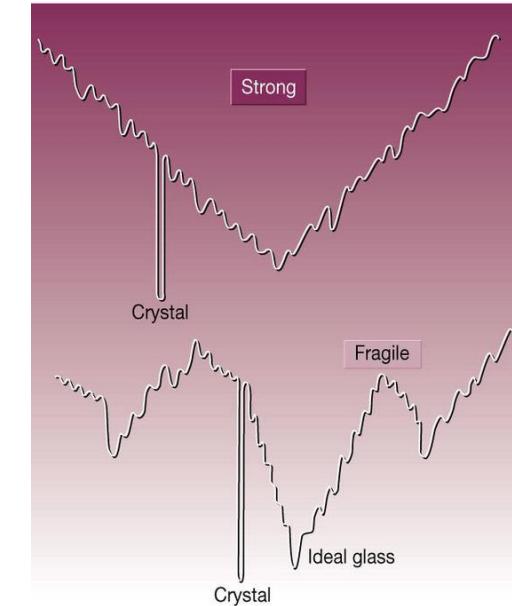
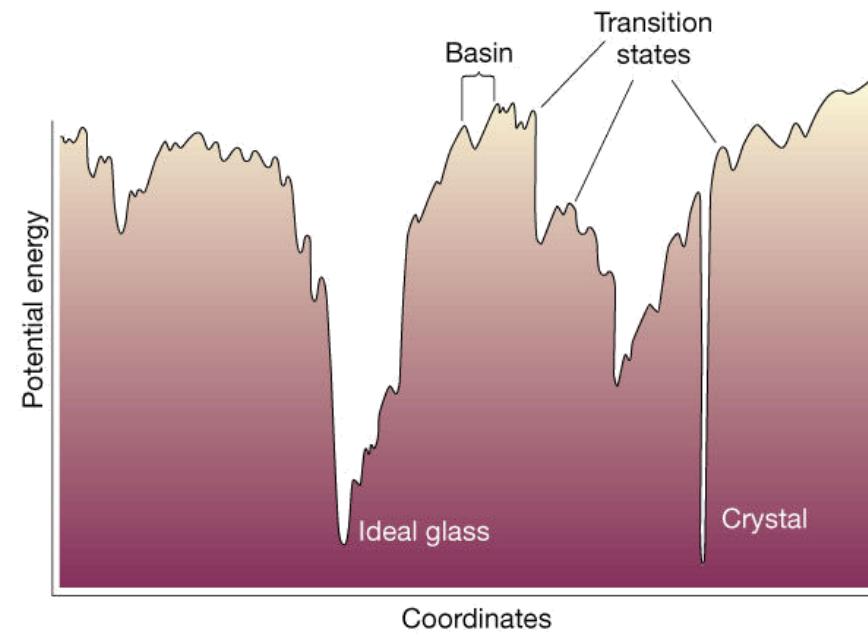
- Entropy difference liquid and crystal: glass transition before  $\Delta S = 0$
- Entropy crisis:  $S_{cryst}(T \rightarrow 0) \rightarrow 0 \rightarrow$  third law of thermodynamics
- Kauzmann temperature  $T_K$ :  $S_{liquid} = S_{crystal} \rightarrow$  ideal glass
- Glass transition: Kinetics vs. Thermodynamics?
- Adam-Gibbs model:  $\tau = A \exp\left(\frac{B}{T s_c}\right)$ 
  - $s_c$  configurational entropy
  - Slow down  $\rightarrow$  decreasing number of configurations
  - Energy landscapes
  - Cooperatively rearranging regions



Pure & Appl. Chem. 63, 1387 (1991).

## Energy landscapes

- Configurational entropy  $s_c \sim$  number of minima in potential energy surface



- At Kauzman temperature: non-crystalline state of lowest energy (ideal glass)
- Strong vs. Fragile – heterogeneous landscapes of fragile glass-formers → broad range of relaxation times  
→ dynamical heterogeneity
- $\alpha$ -relaxations correspond to configurational sampling of neighbouring "megabasins", whereas  $\beta$ -processes are thought to correspond to elementary relaxations between contiguous basins

## Mode coupling theory (MCT)

- Understanding dynamics of supercooled liquids
  - Time evolution (dynamics) of intermediate scattering function (as density-density correlation function) from time-independent structural properties, such as  $S(q)$
- Dynamics of ISF: four-point correlation function
- MCT: factorization of such four-point correlations to products of ISF's
- Mode coupling equations whose solutions provide the full time dependence of the ISF

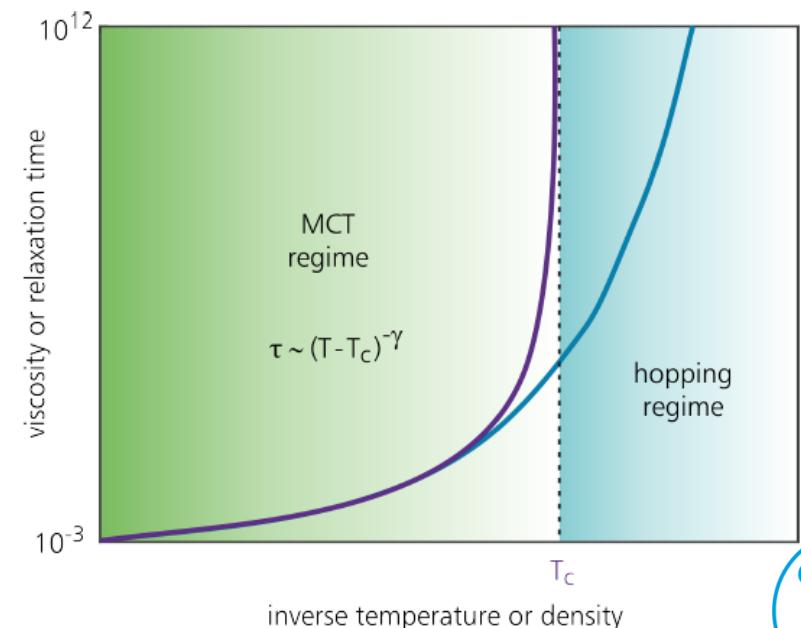
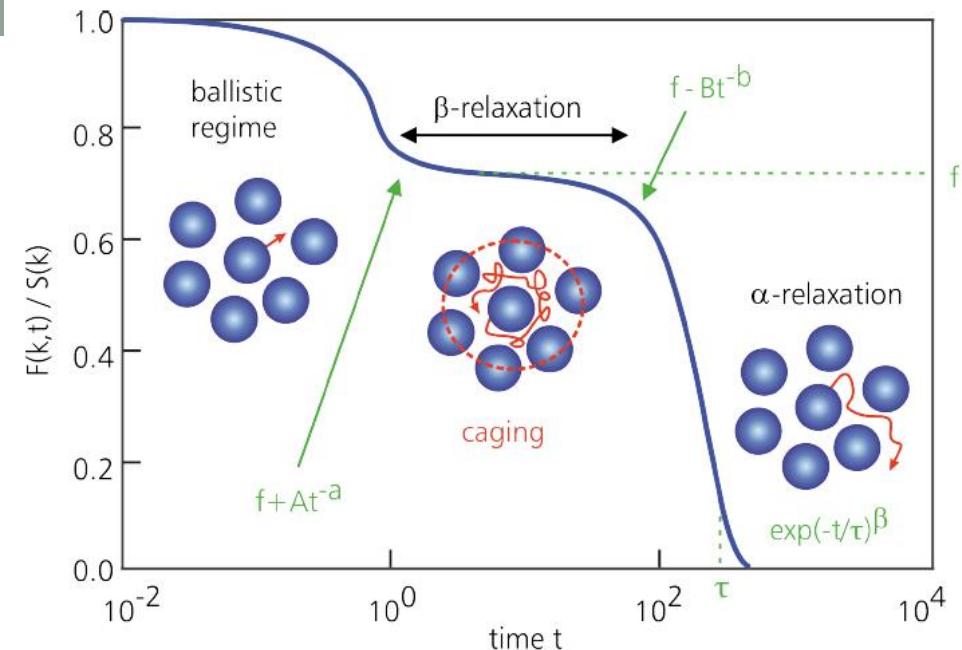
## Predictions from MCT

- Critical temperature  $T_c$  at which relaxation rate vanishes with a power law  $\Gamma = \frac{1}{\tau_0} \propto (T - T_c)^\delta$  with  $\delta > 1.5$
- Plateau regime of dynamics & scaling of  $\alpha, \beta$ -relaxation
- Slow relaxation at longer times showing KWW type stretched exponential

### Drawbacks:

- Ideal glass transition at relatively high temperature / low densities
- Fails to explain strong and fragile behaviour
- Currently approaches to overcome these shortcomings

Frontiers in Physics 6, 97 (2018)  
arXiv:1806.01369 (2018)

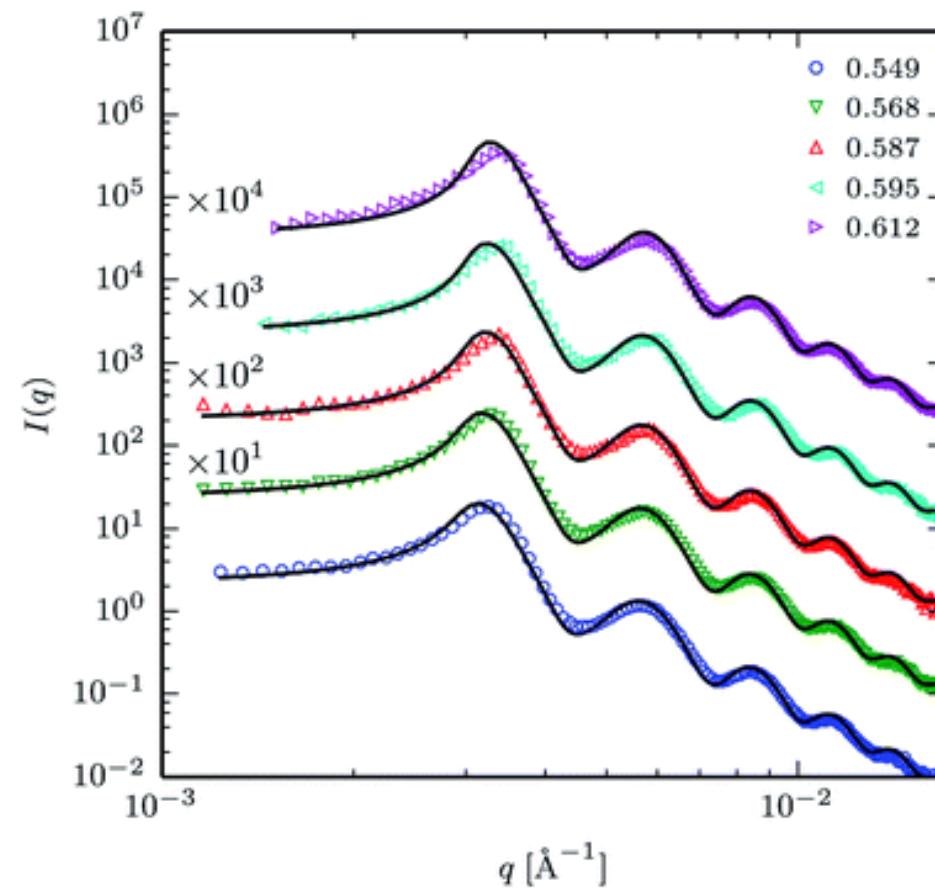


## X-ray scattering studies of glass transition

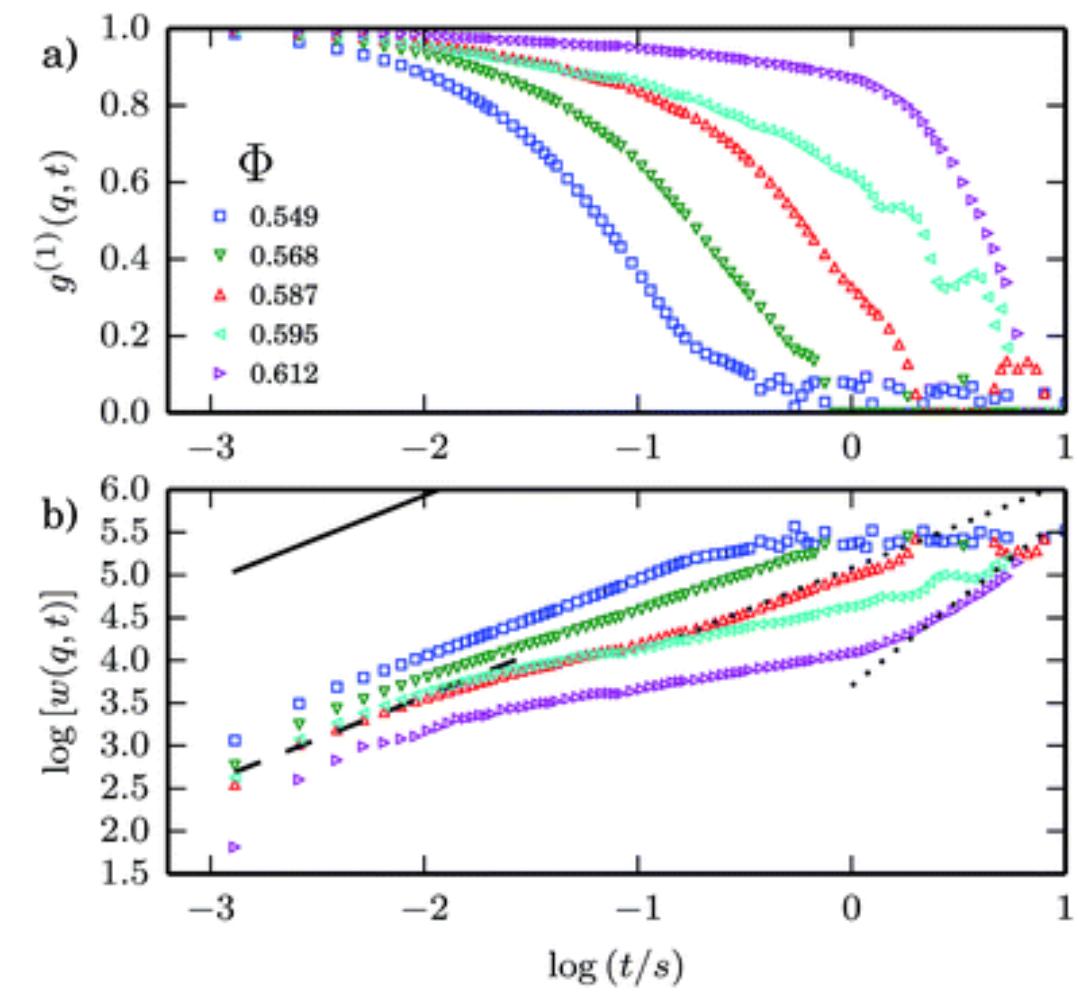
- Structure: Is there any difference to liquid state in  $S(q)$  and  $g(r)$ ?  
→ X-ray diffraction (SAXS / WAXS)
- Is there any orientational/bond order?  
→ higher order structural correlations (e.g. XCCA)
- Intermediate scattering function: Dynamics  
→ XPCS

## Example 1: Colloidal hard sphere glasses

Hard sphere colloidal glasses

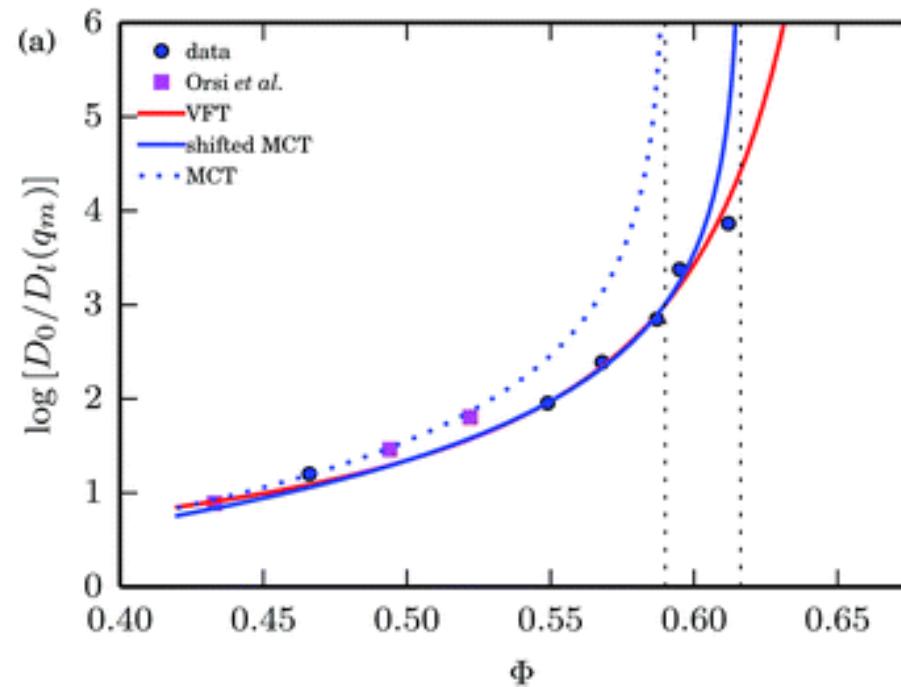


Soft Matter 10, 8698 (2014)



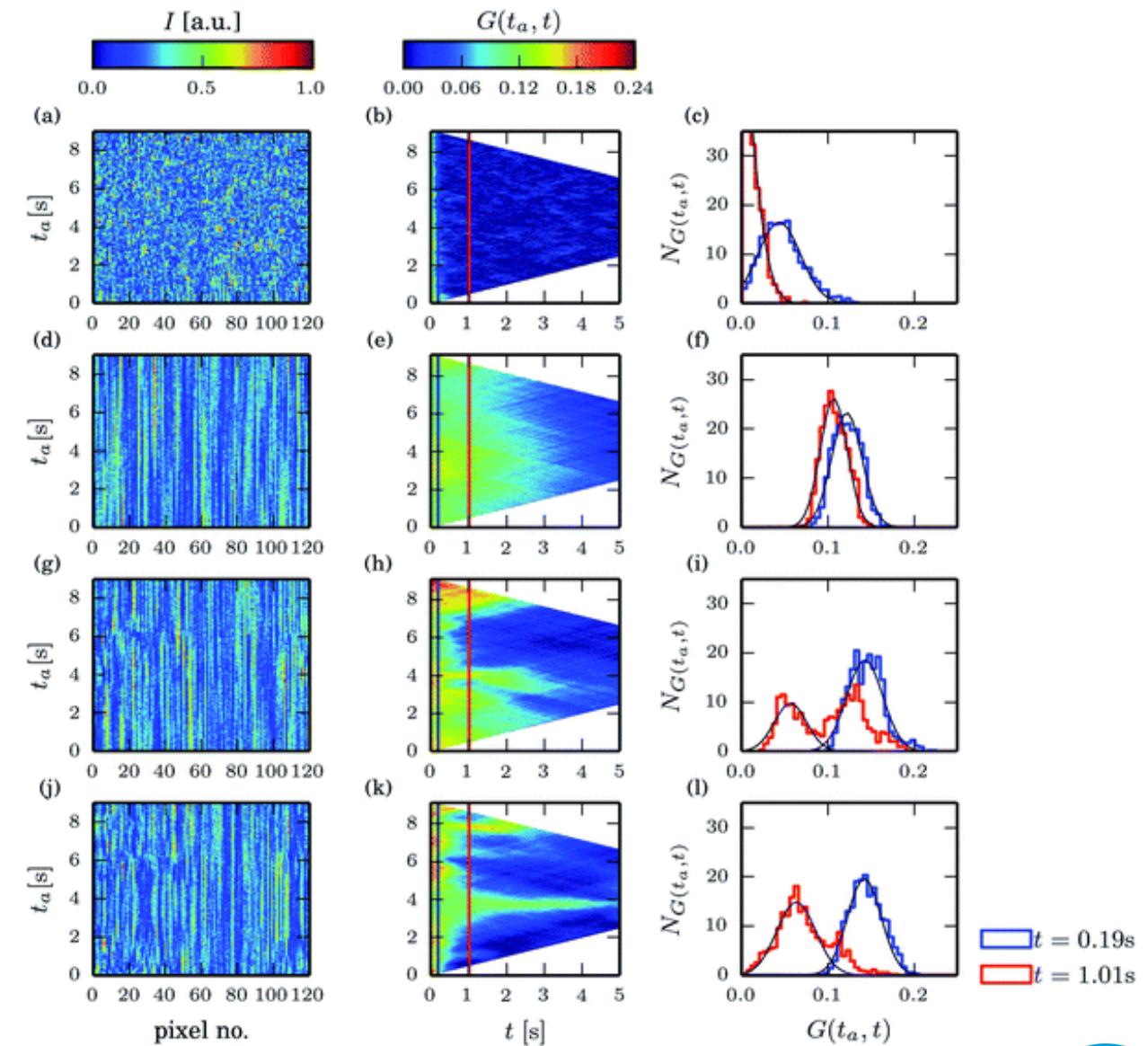
Intermediate scattering function  $g^{(1)}(q, t)$   
 $w(q, t) = -\ln(g^{(1)}(q, t))/q^2$  analog to MSD

## Example 1: Colloidal hard sphere glasses



Long time diffusion → structural relaxation

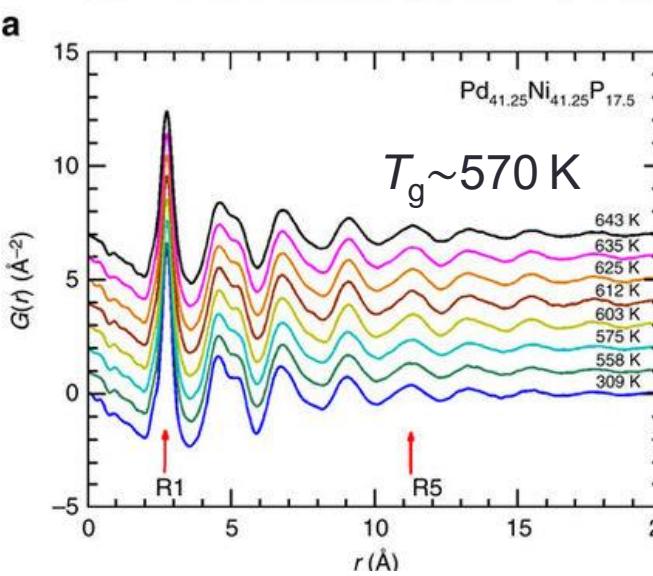
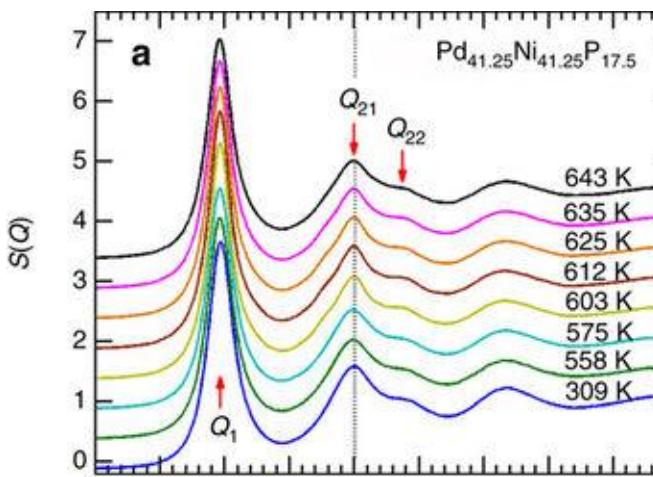
MCT & VFT modelling → 3% shift necessary



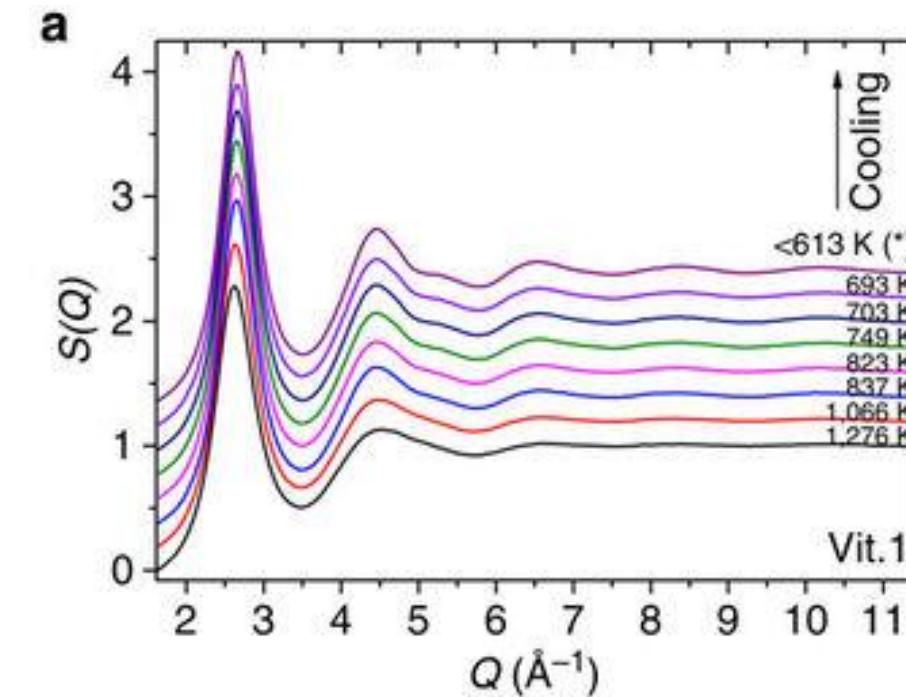
Soft Matter 10, 8698 (2014)

## Example 2: Structure factors

Metallic glasses



Nat. Commun. 8, 14679 (2017)



$Zr_{41.2}Ti_{13.8}Cu_{12.5}Ni_{10}Be_{22.5}$   
Nat. Commun. 4, 2083 (2013)

- Similar results for other glass formers
- Pair-correlations do not change significantly crossing the glass transition
- Is there any structure-dynamics relation?
- Higher-order correlations?

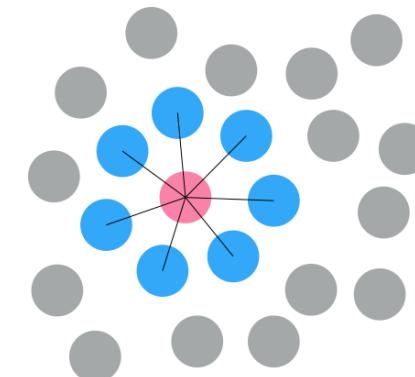
## Higher-order structure in simulation and microscopy

- Steinhardt parameters: local bond order of  $l$ -fold symmetry with distance  $\mathbf{r}$  and number of bonds  $N$

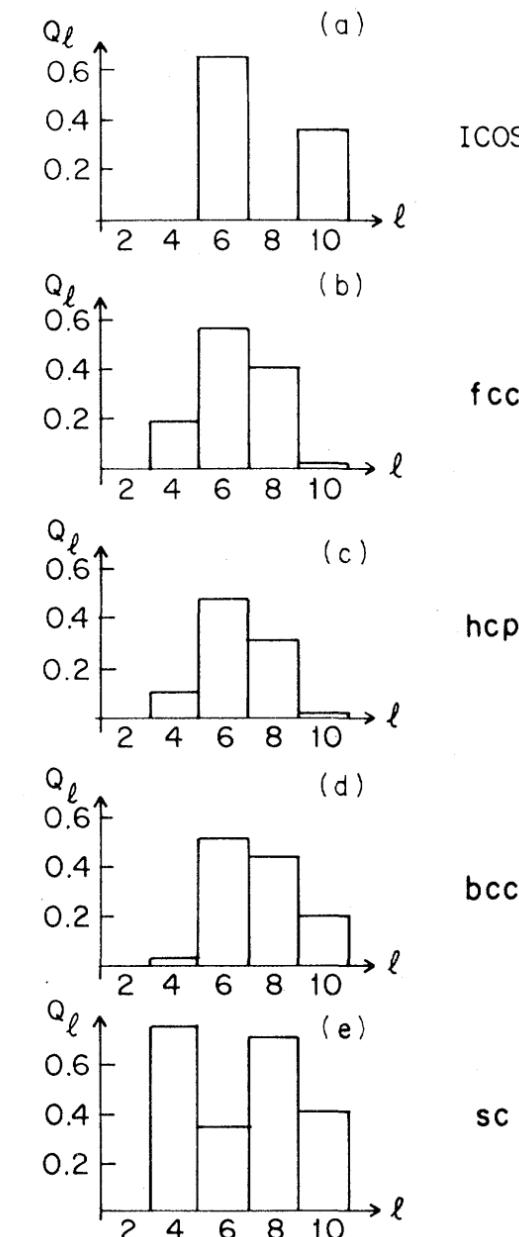
$$Q_{lm} = \frac{1}{N} \sum_0^N Y_{lm}(\theta(\mathbf{r}), \phi(\mathbf{r}))$$

- Rotationally invariant coordinate system

$$Q_l = \left( \frac{4\pi}{2l+1} \sum_{m=-l}^l |Q_{lm}|^2 \right)^{\frac{1}{2}}$$

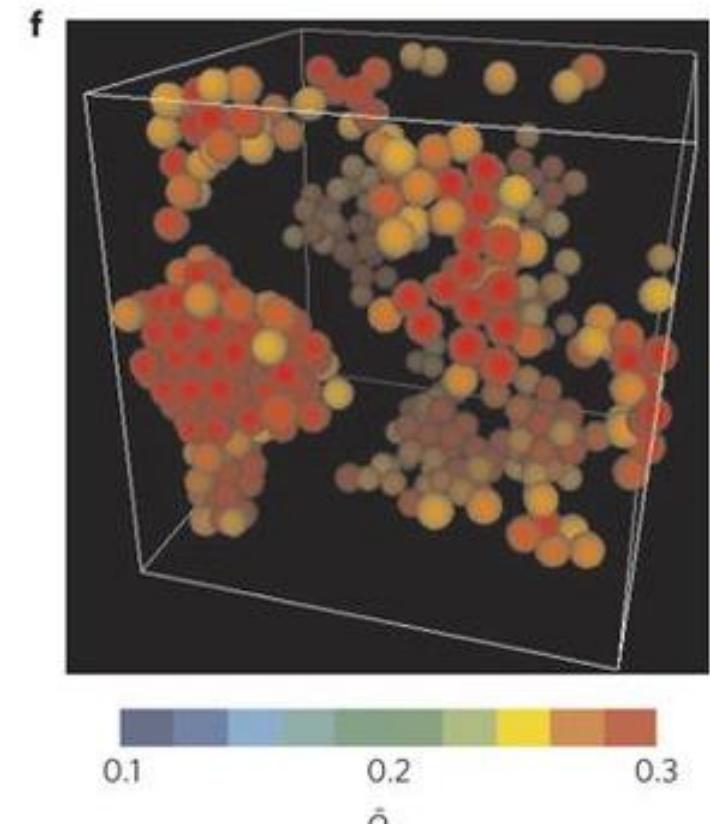
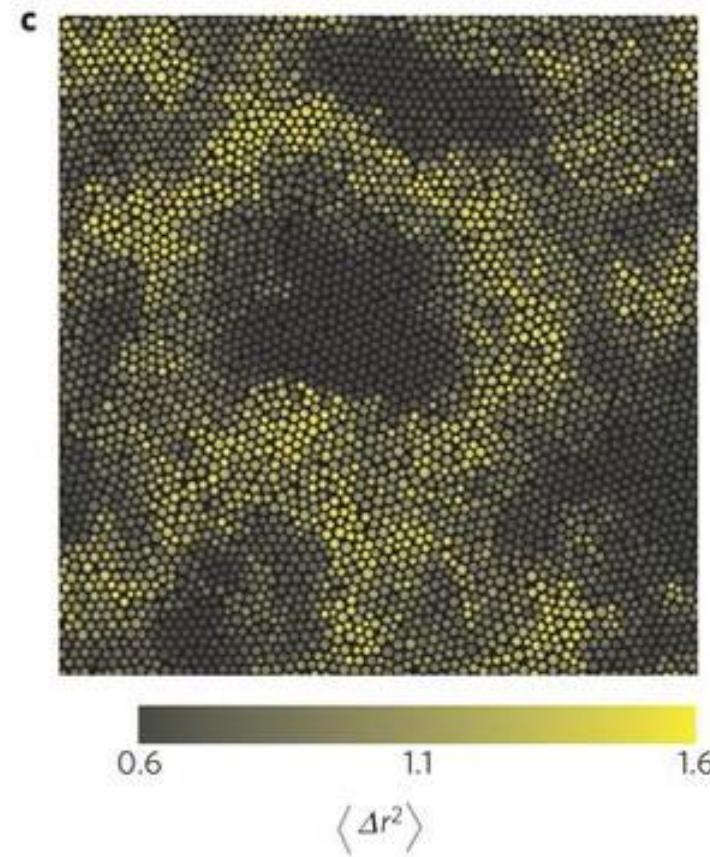
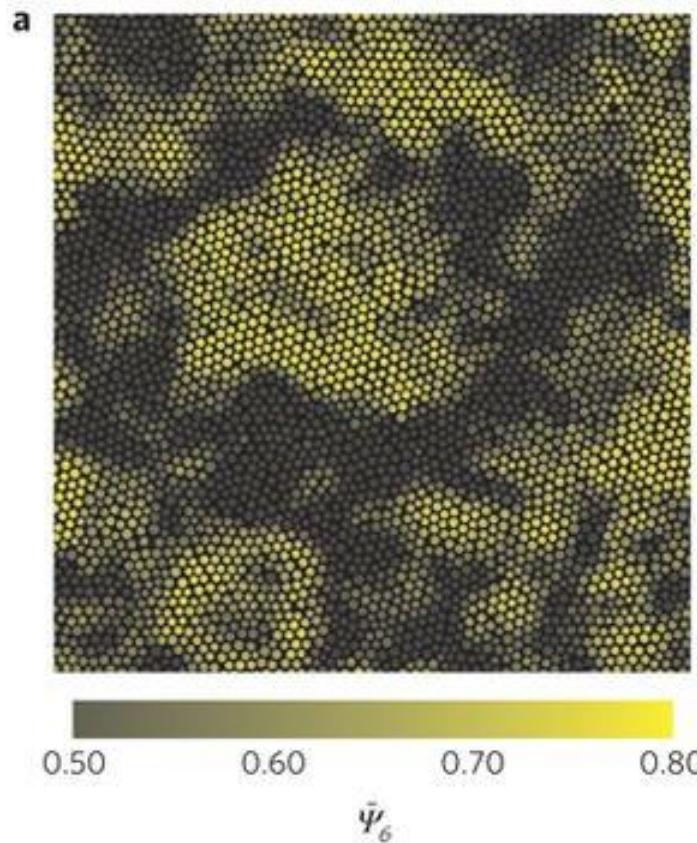


- Fingerprint for different local environments



Phys. Rev. B 28,  
784 (1983)

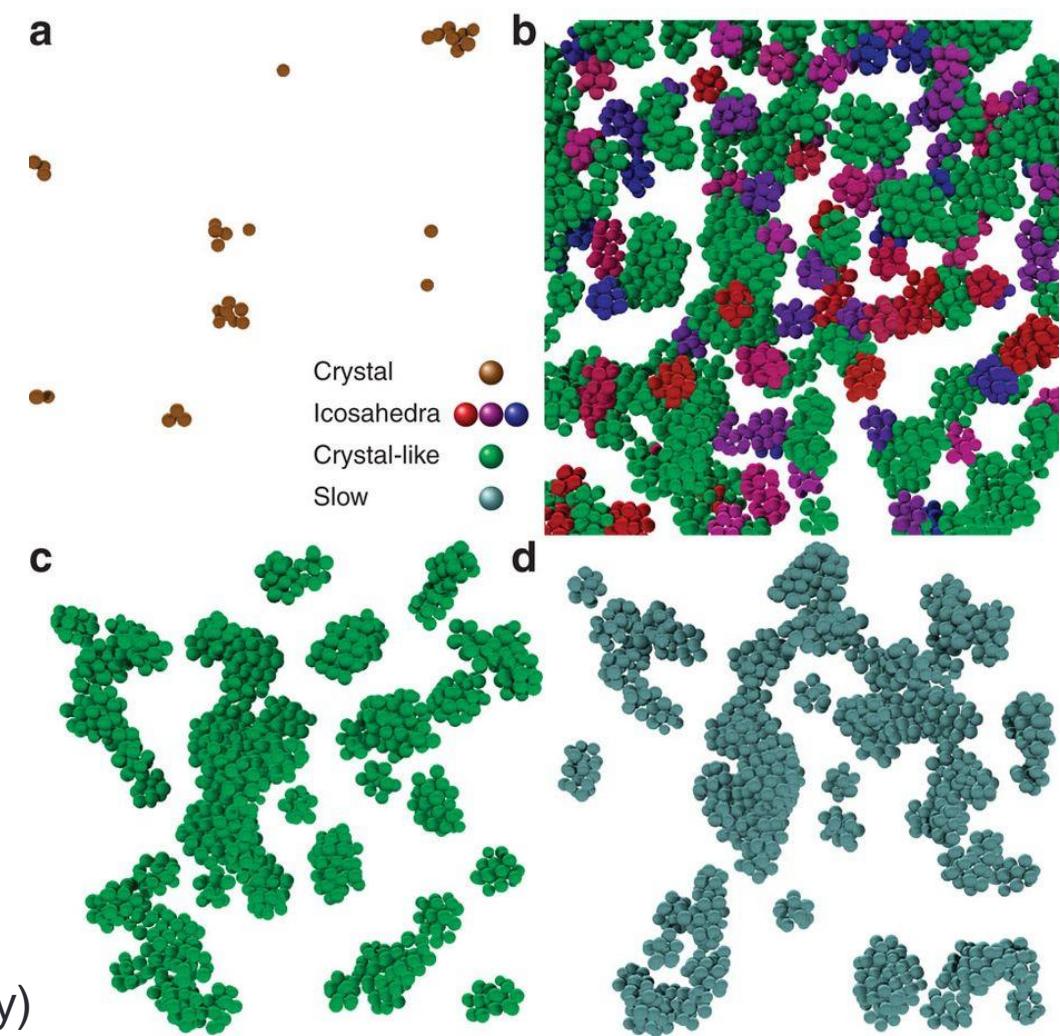
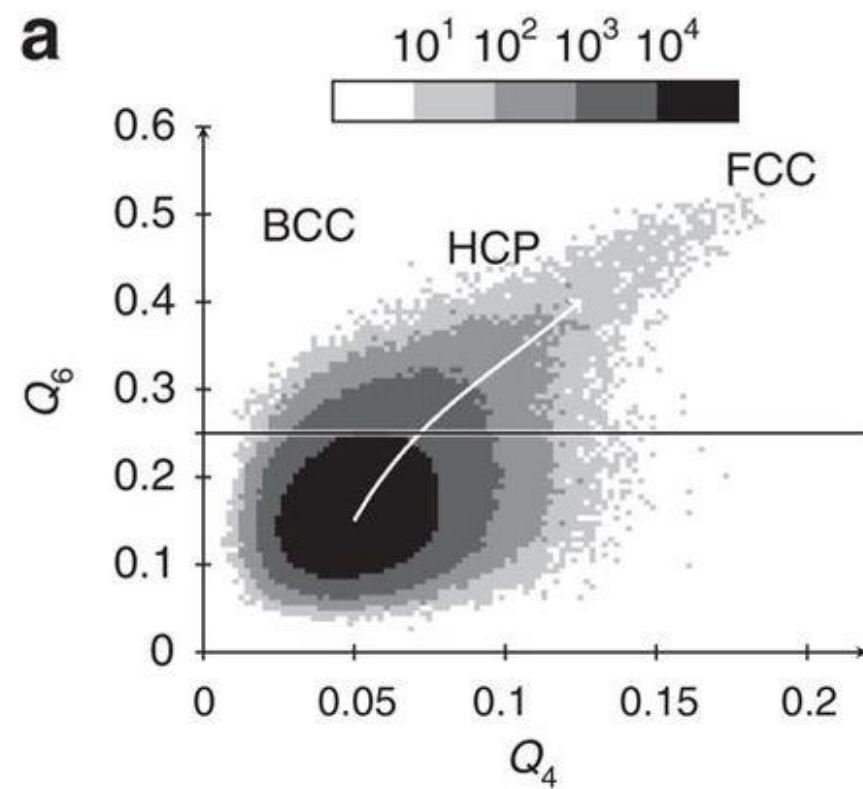
## Higher-order structure in simulation and microscopy



Structural and dynamical heterogeneities (simulation)

Nat. Mat. 9, 324 (2010)

## Higher-order structure in simulation and microscopy

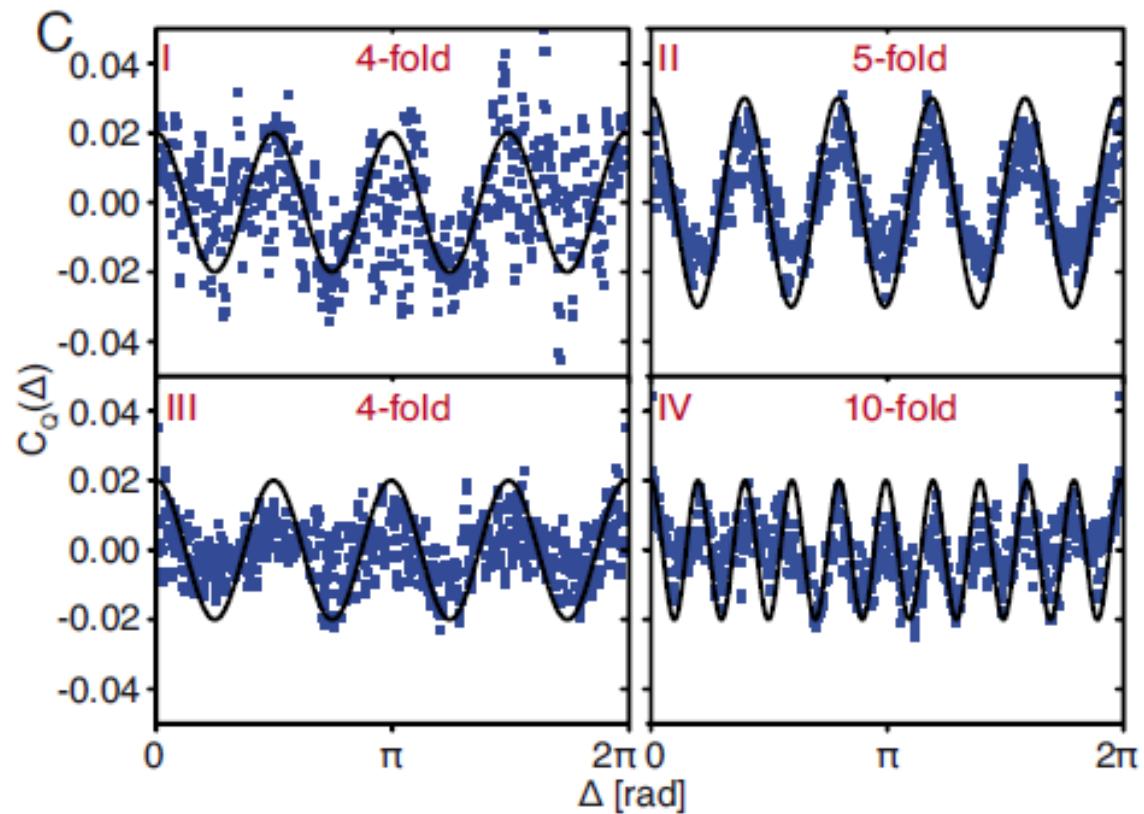
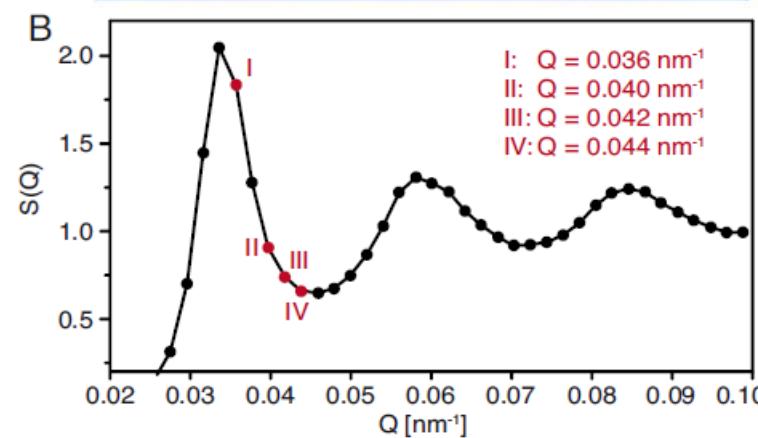
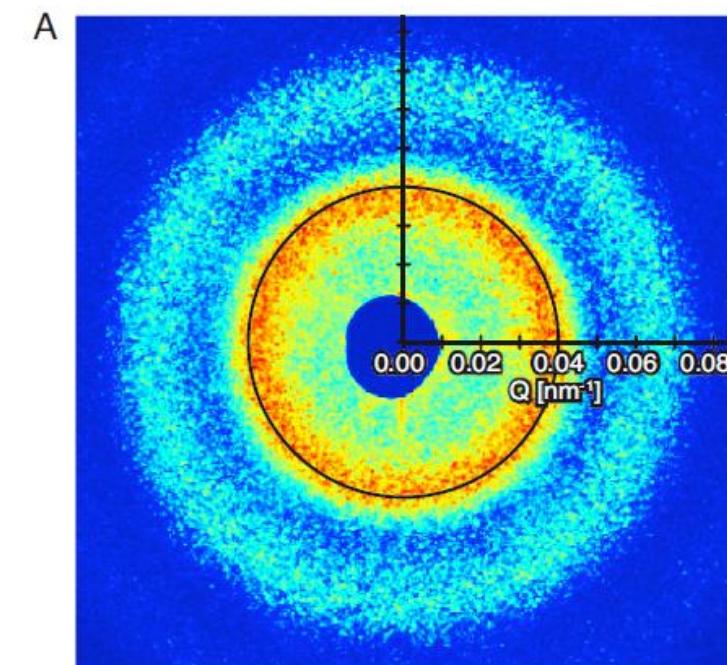


Crystal-like and icosahedral order in hard sphere fluids (microscopy)

Nat. Comm. 3, 974 (2012)

## Example 3: XCCA approaches

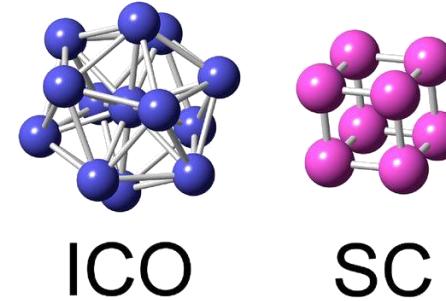
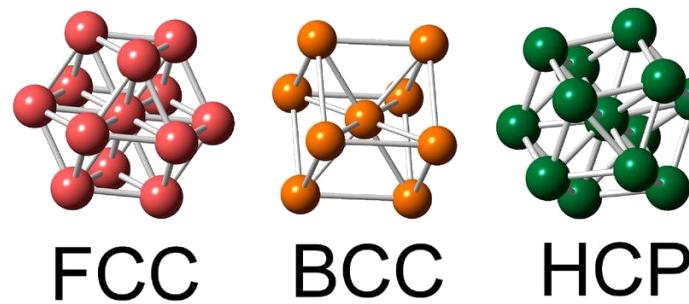
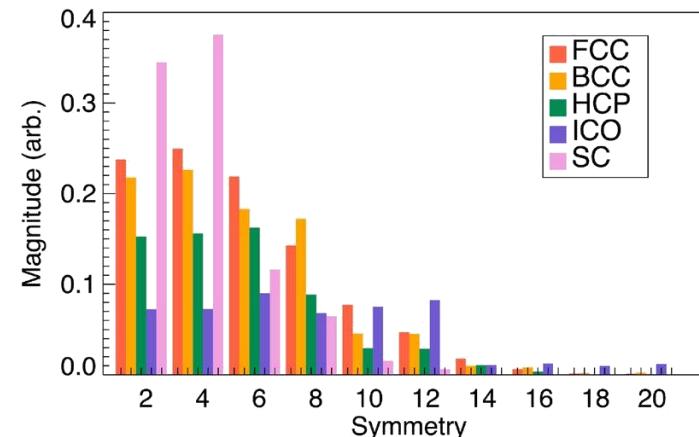
Hard-sphere glass



- Hidden symmetries
- Structural information beyond SAXS

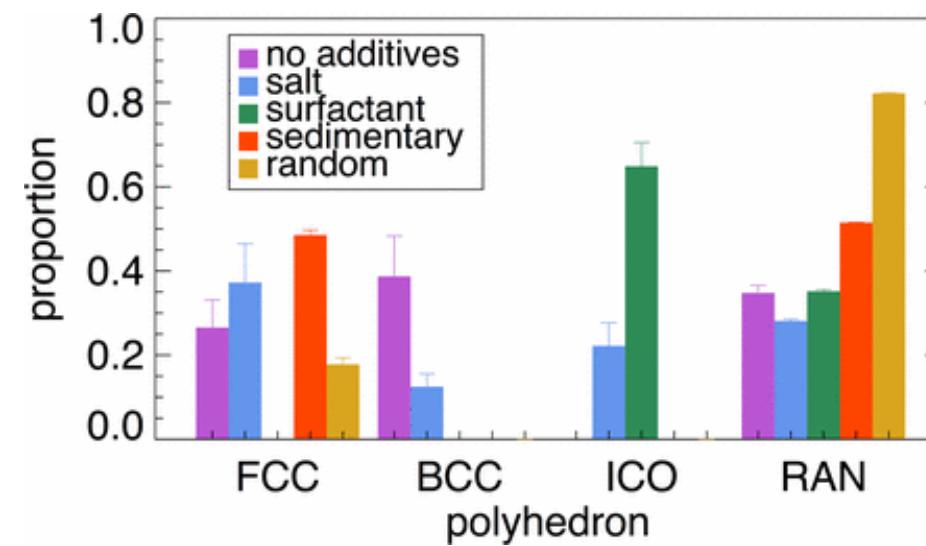
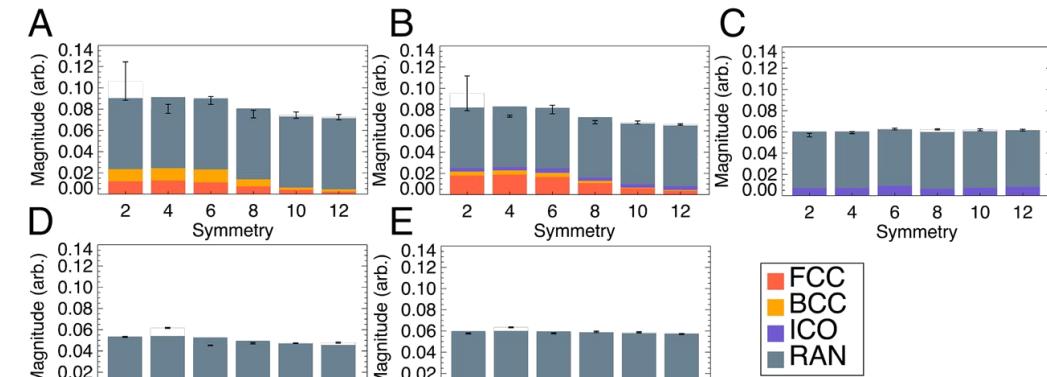
PNAS 109, 11511 (2009)

### Example 3: XCCA approaches

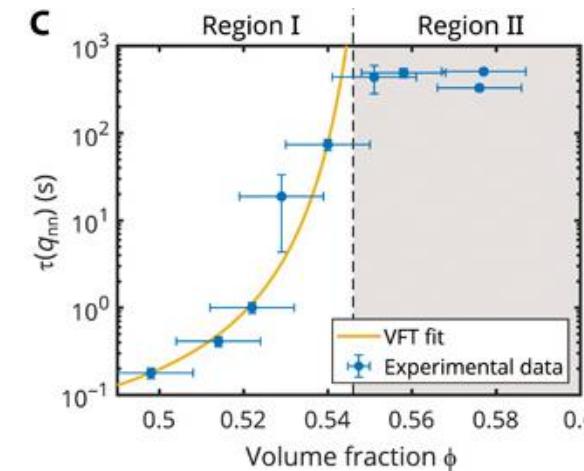
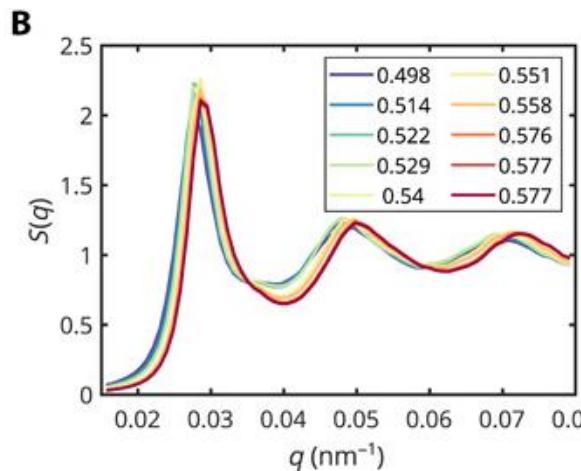


PNAS 114, 10344 (2017)

Colloidal glasses:  $\text{SiO}_2$  particles with additives  
Salt: screening  
Surfactant: short-range attractive compound



## Example 3: XCCA approaches

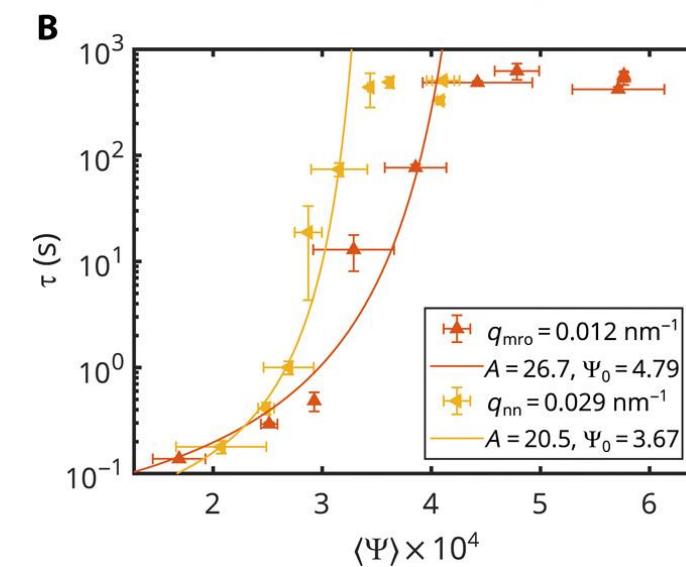
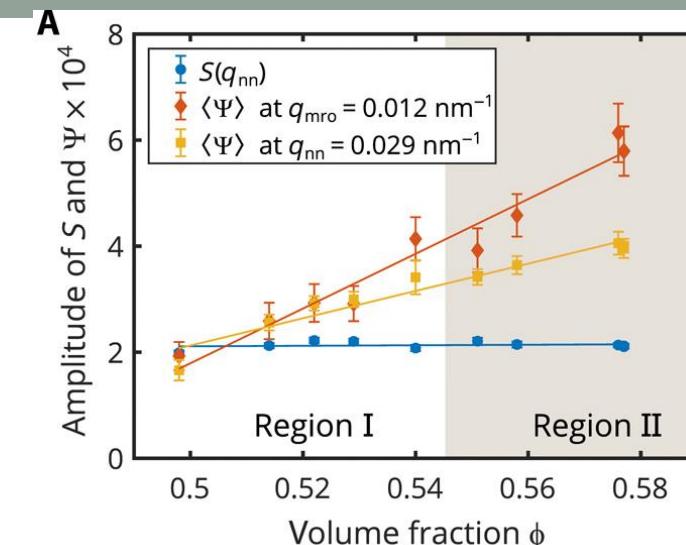
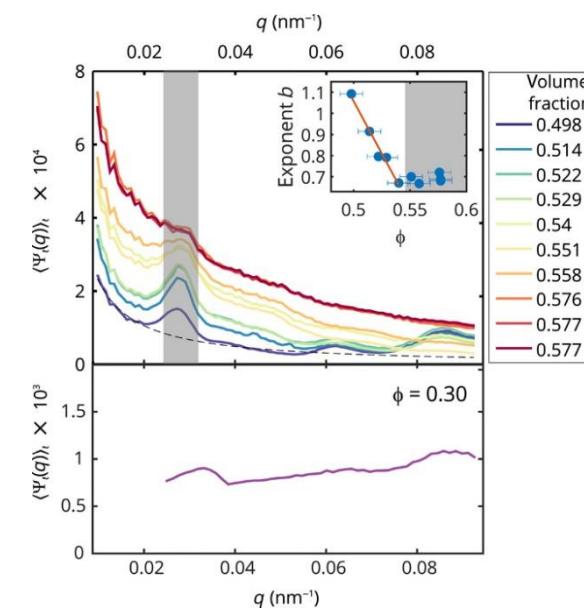


## Colloidal hard spheres

Around glass transition:

- $S(q)$  constant
- Relaxation time increases
- $\Psi_\ell \propto C_\ell$  from XCCA as measure of local order

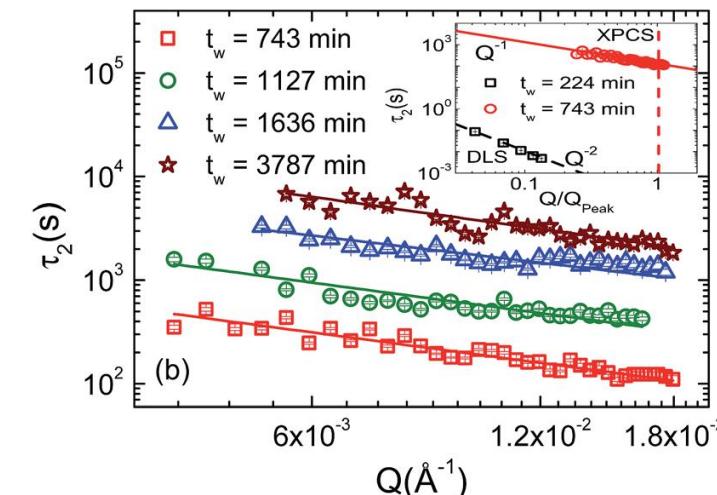
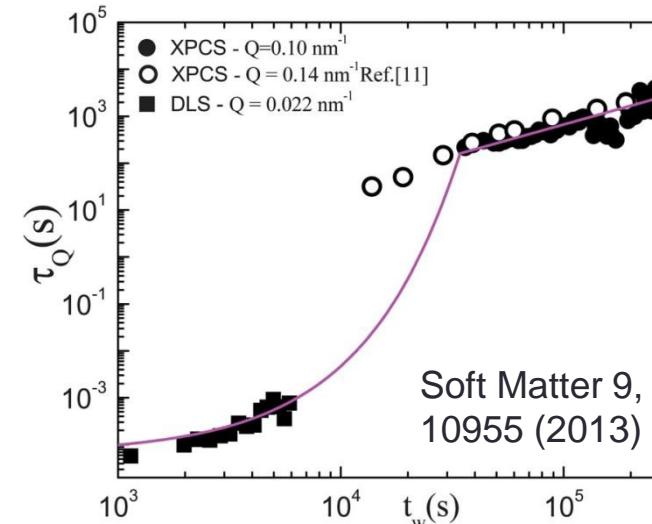
Sci. Adv. 6, eabc5916 (2020)



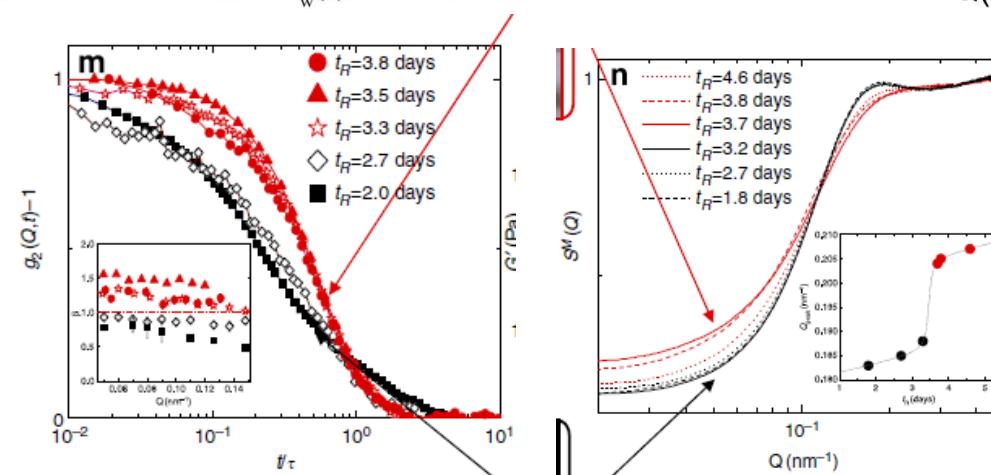
Correlation of dynamics and local order

## Example 4: Aging in colloidal glasses

Laponite glass (clay) – dynamics change with waiting (after rejuvenation)



Soft Matter 11, 466  
(2015)



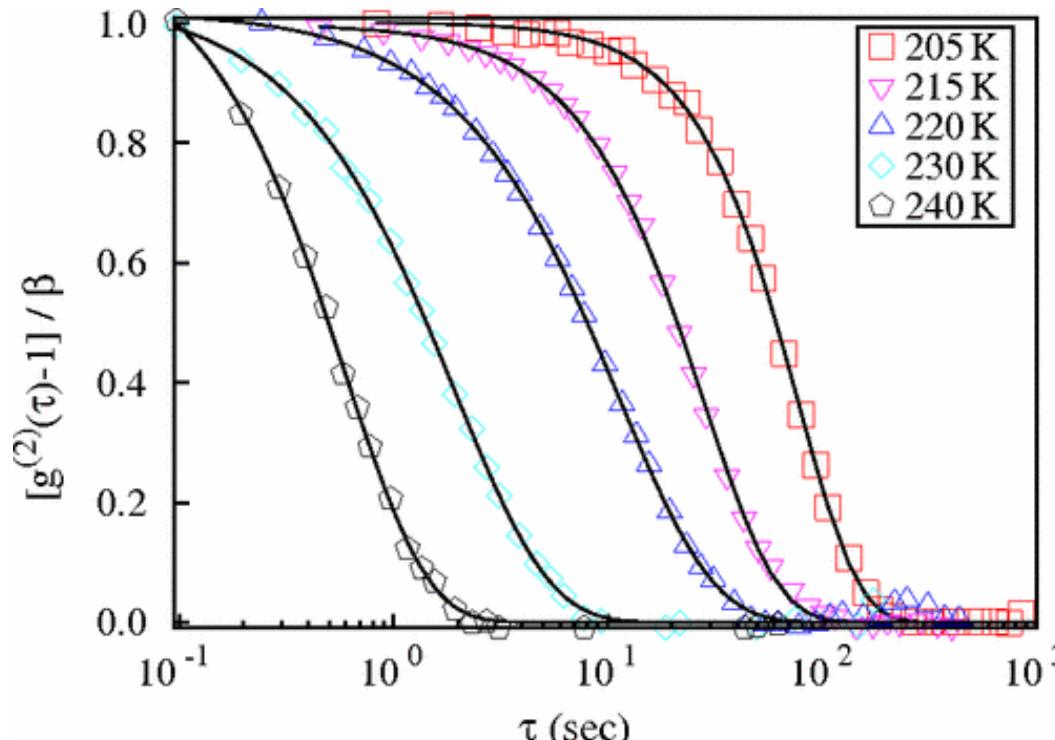
Glass-glass transition  
during aging

Nature Comm. 5, 4049 (2014)

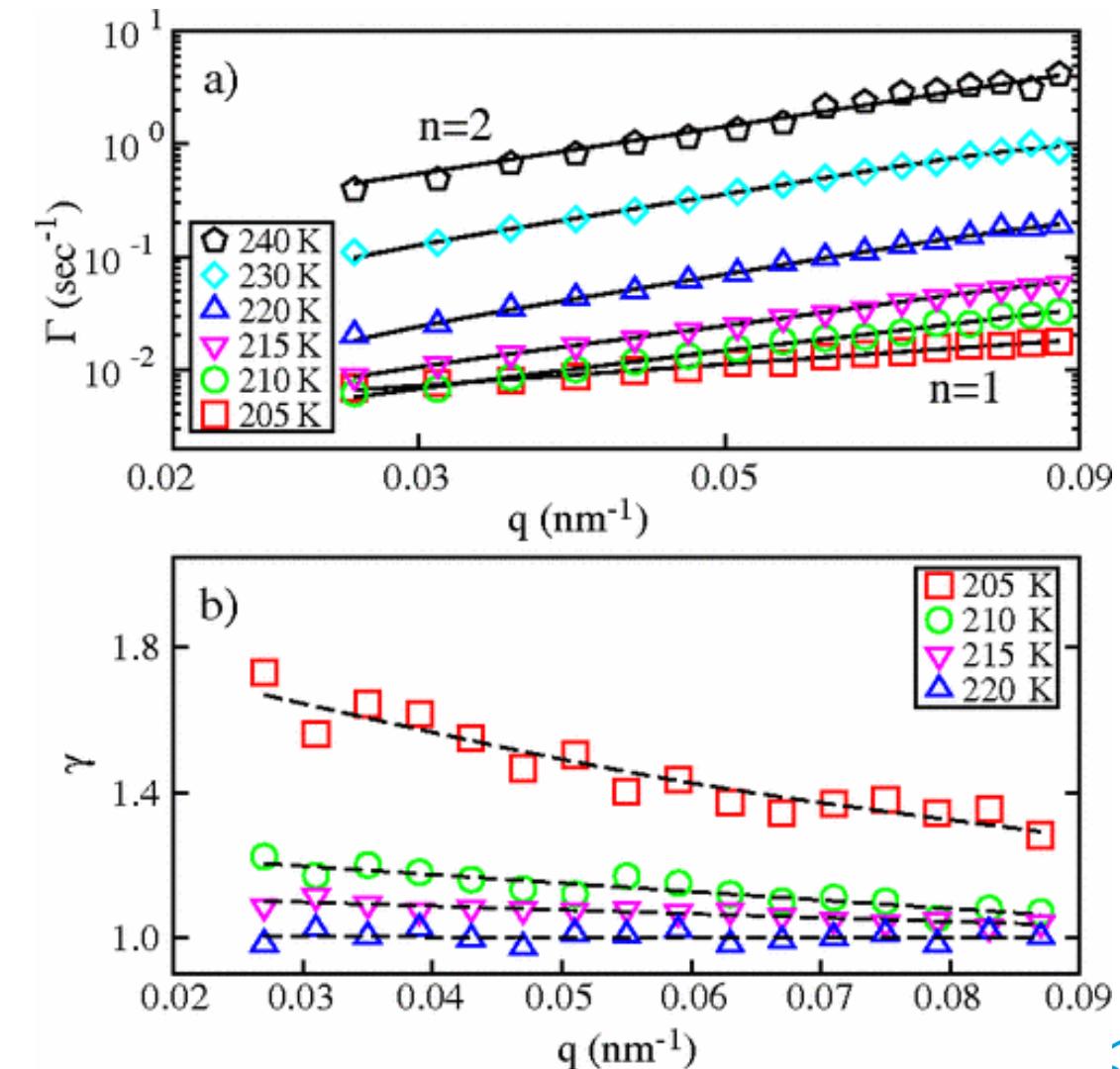
## Example 5: Microrheological glass transition studies on soft matter

Propanediol:  $T_m \approx 245$  K,  $T_g \approx 170$  K

Silica particles as tracer particles



Phys. Rev. Lett. 100, 055702 (2008)



## Example 5: Microrheological glass transition studies on soft matter

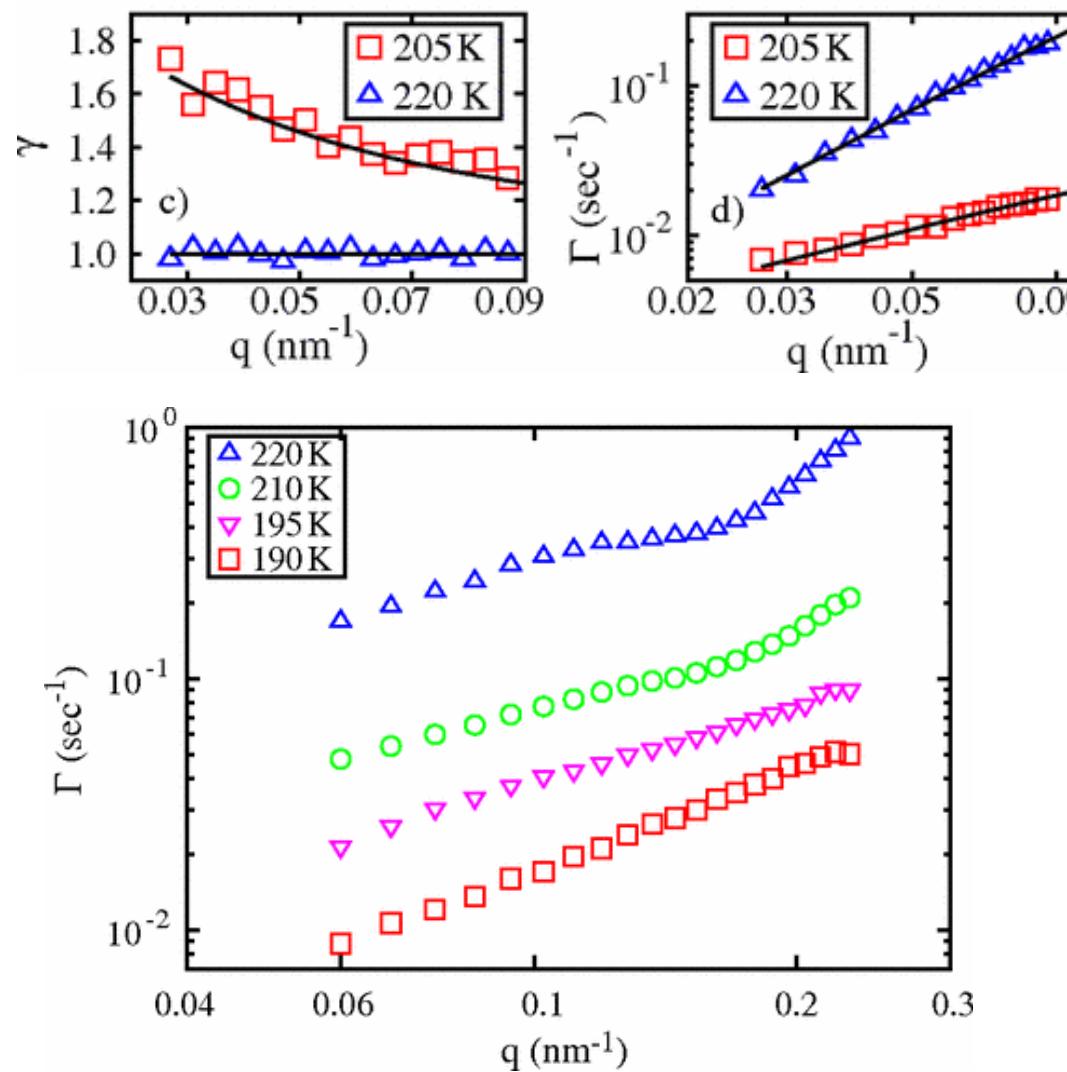
KWW function  $f(q, \tau) \propto \exp(-(q^n t)^\gamma)$

Model with continuous time random walk model: displacement of particle in time interval  $t$  consists of  $N$  discrete steps → ISF is determined by number of steps  $N$  and degree of decorrelation  $h(q, N)$  between steps

$$f(q, \tau) = \sum_N P_t(N) h(q, N)$$

- $P_t(N)$  probability of  $N$  events occurring during time interval  $t$  → Poisson distribution  $P_t(N) = \exp(-\Gamma_0 t)(\Gamma_0 t)^N / N!$ , with  $1/\Gamma_0$  the mean time between events
- $h(q, N) \simeq \exp[-(q N^\alpha \delta)^2]$  Gaussian distribution, with  $\alpha$  defining (non-)diffusive motion ( $\alpha = 0.5$  for diffusion) and  $\delta$  average lengths of single jumps

## Example 5: Microrheological glass transition studies on soft matter



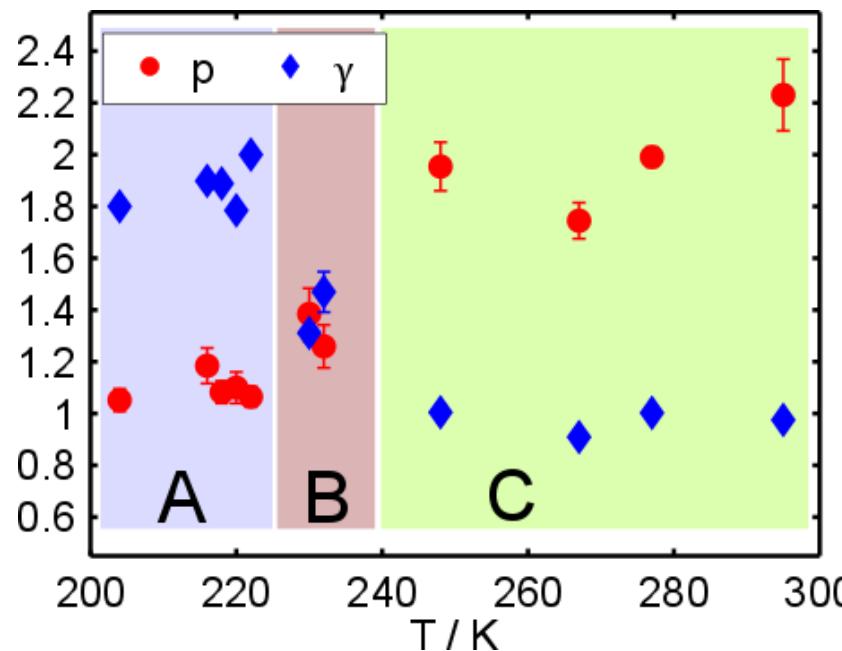
$(\delta, \Gamma_0, \alpha) = (5.4 \text{ nm}, 0.09 \text{ Hz}, 1)$  205 K  
 $(6.2 \text{ nm}, 2.0 \text{ Hz}, 0.5)$  for 220 K

→ From diffusive to ballistic motion!

- Increasing particle concentration:  
deGennes narrowing
- Disappears at low temperatures:  
cooperative behaviour close to  $T_g$

Phys. Rev. Lett. 100, 055702 (2008)

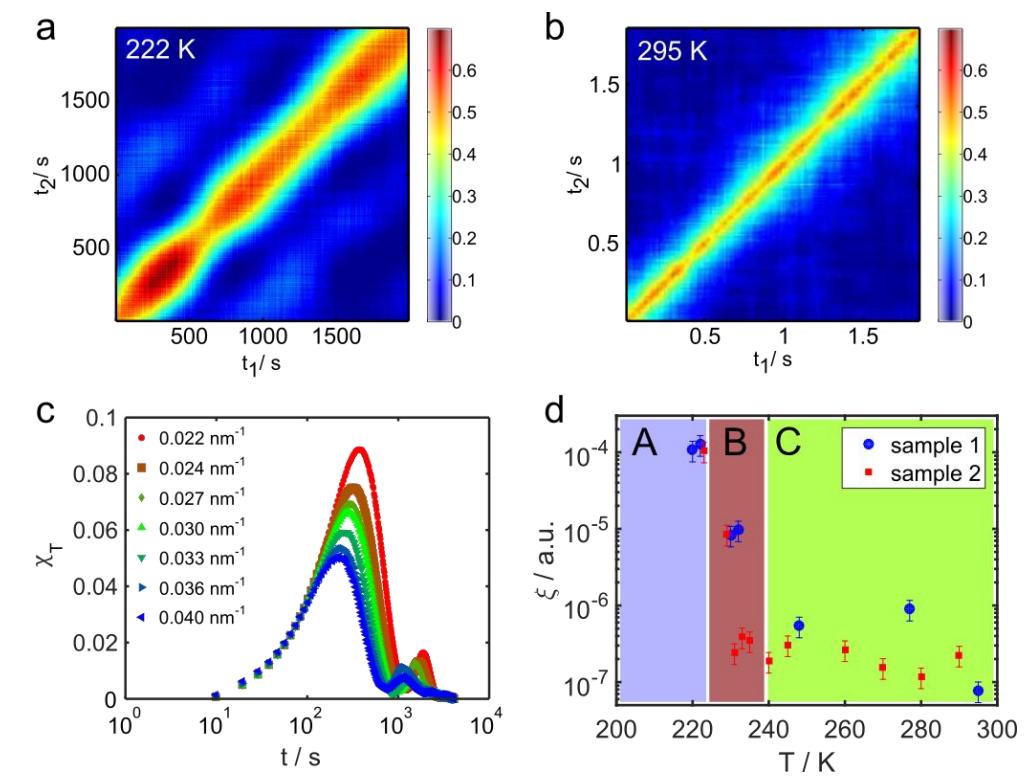
## Example 5: Microrheological glass transition studies on soft matter



Silica in PPG ( $T_g \approx 205$  K)

Exponents as function of temperature

- C: Brownian regime
- B: intermediate regime ( $T \approx 1.12 T_g$ )
- A: correlated motion



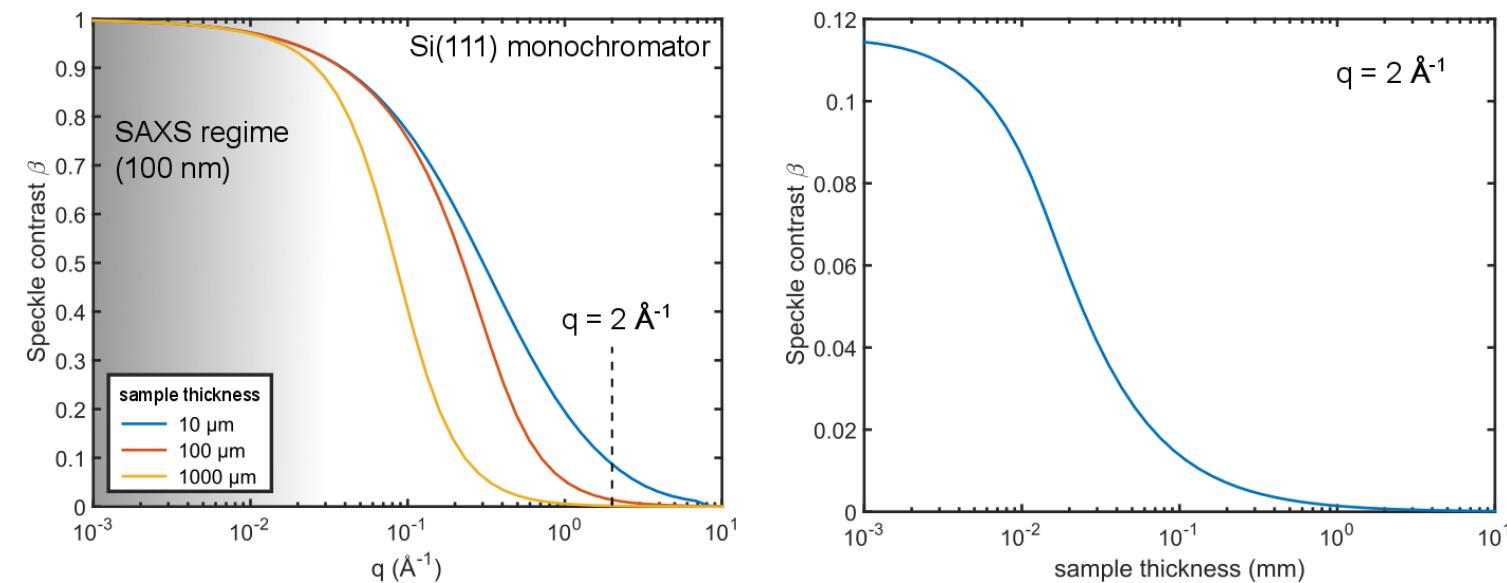
Dynamical heterogeneity

→ Correlated & heterogeneous dynamics close to  $T_g$

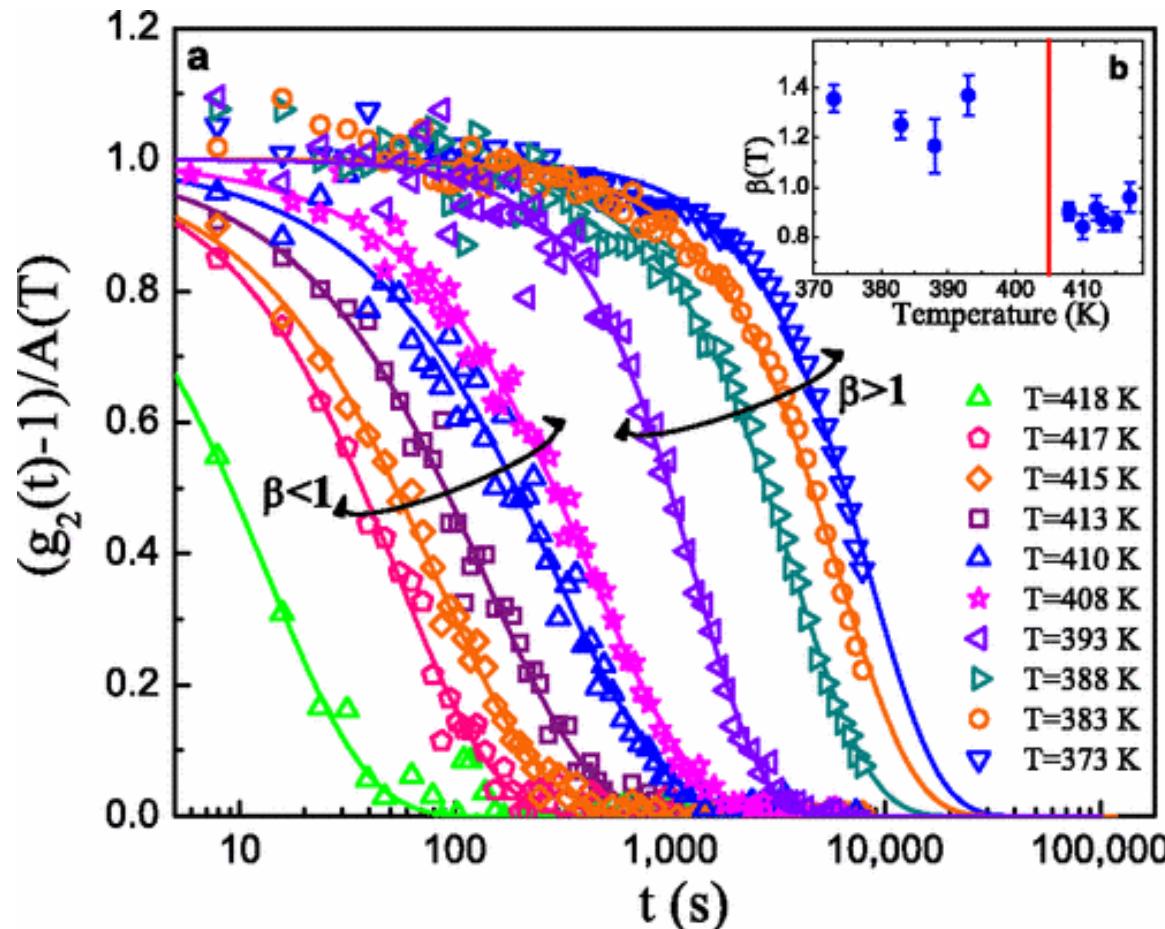
Phys. Rev. E 91, 042309 (2015)

## Example 6: dynamics of metallic & network glasses

- Molecular dynamics: large  $q$
- In general coherence factor (speckle contrast) as integral over coherence lengths  $\rightarrow$  lower value in large  $q$  XPCS  $\rightarrow$  Lecture 17
- $\beta = \beta_t \beta_l$  with correction factor (for beams with a Gaussian spectrum)
- $$\beta_l = \frac{2}{b^2 d^2} \int_0^b \int_0^d (b-x)(d-y) \exp\left[-\frac{x^2}{\xi_h^2}\right] [\exp(-2|Ax+By|) + \exp(-2|Ax-By|)], \text{ with } A = \frac{\Delta\lambda}{\lambda} q \sqrt{1 - \frac{q^2}{4k_0^2}}, B = -\frac{\Delta\lambda}{2\lambda} \cdot \frac{q^2}{k_0}, k_0 = \frac{2\pi}{\lambda}, \text{ width } b, \text{ depth } d$$

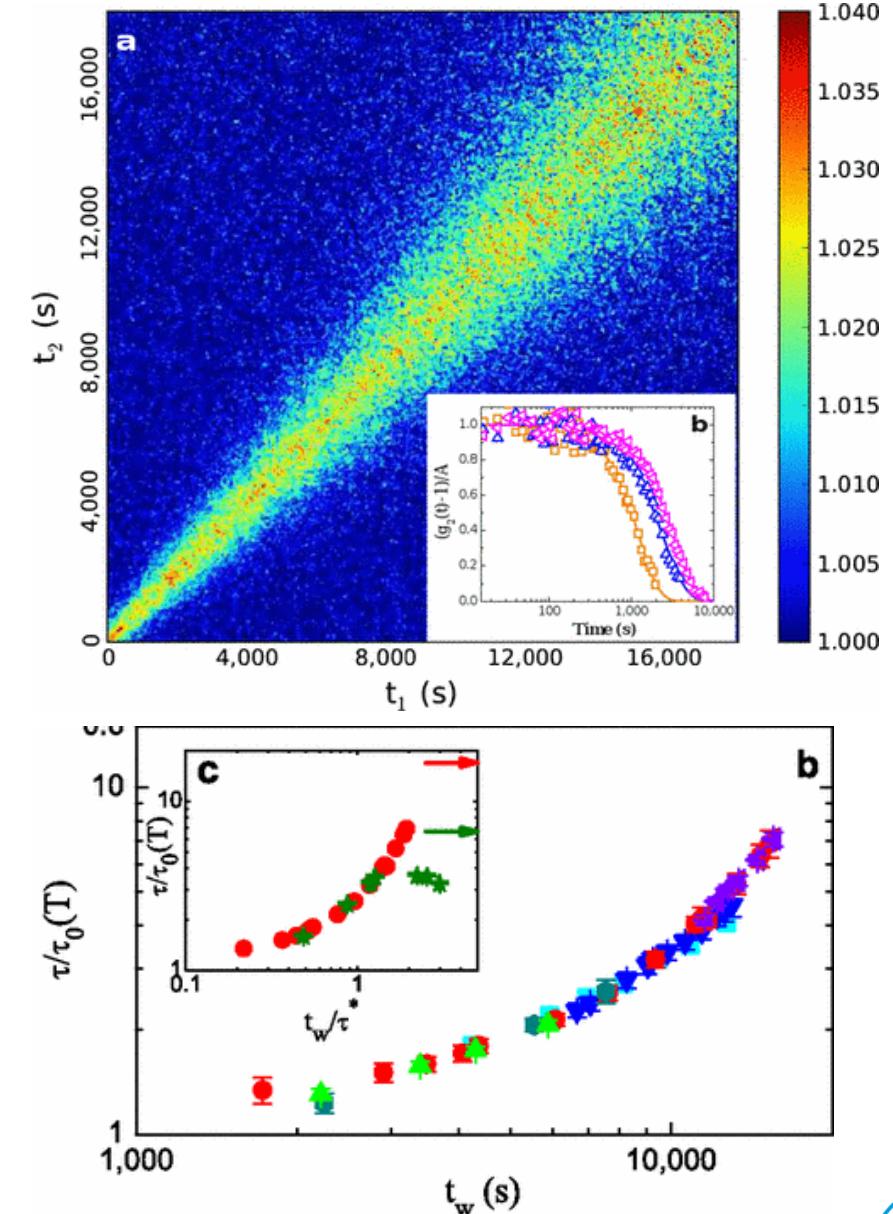


## Example 6: dynamics of metallic glasses

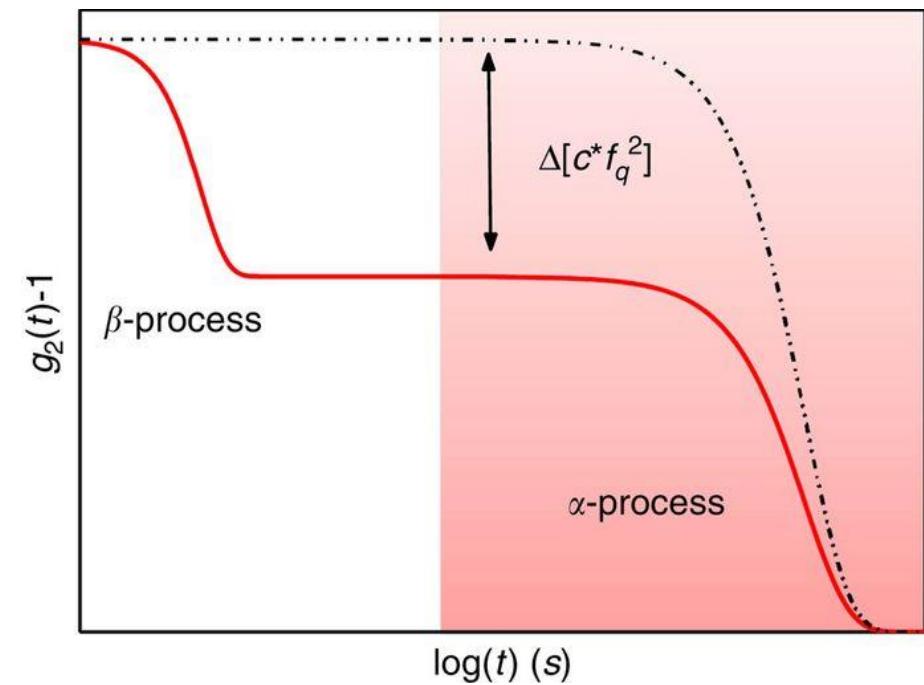
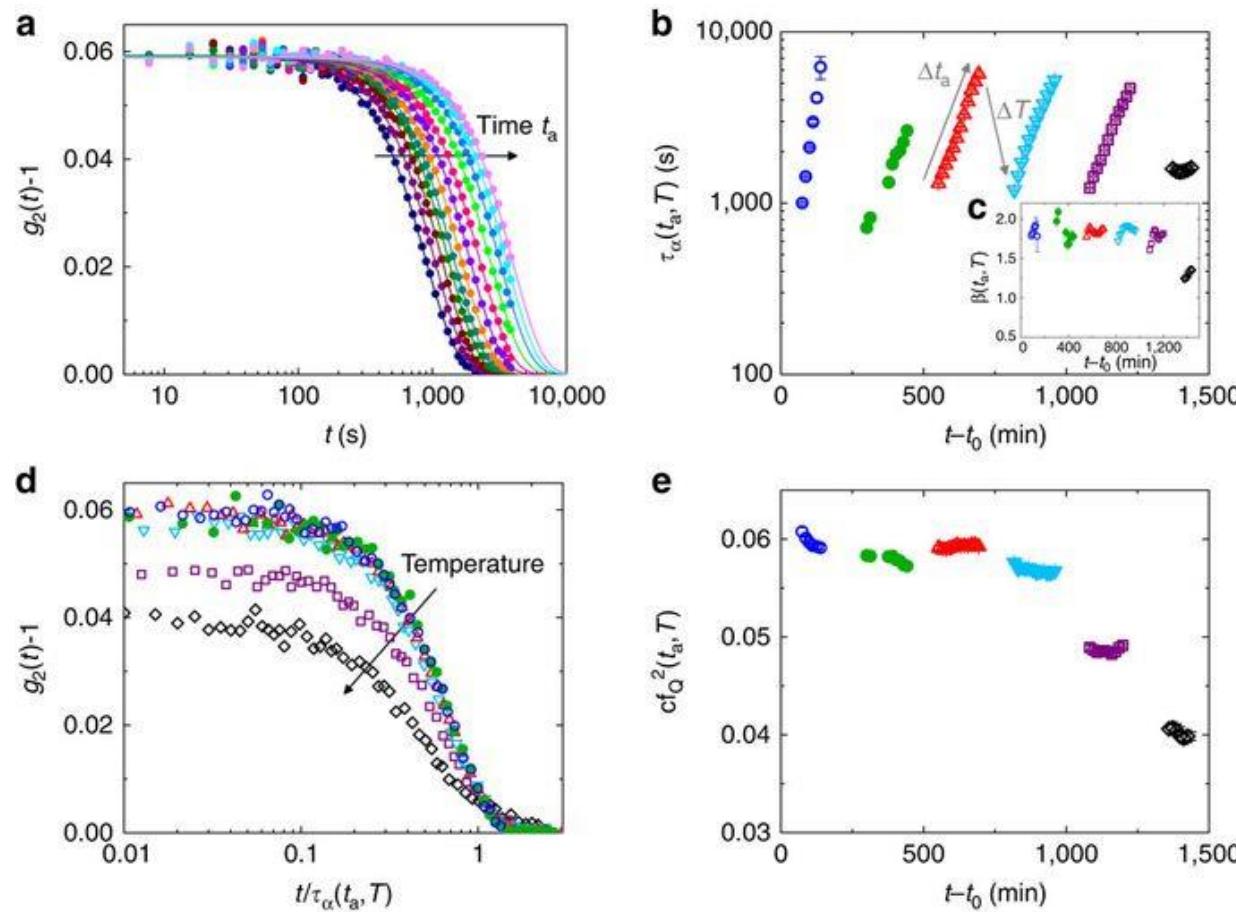


- Dynamics transition: stress relaxation below  $T_g$
- Aging

PRL 109, 165701 (2012)



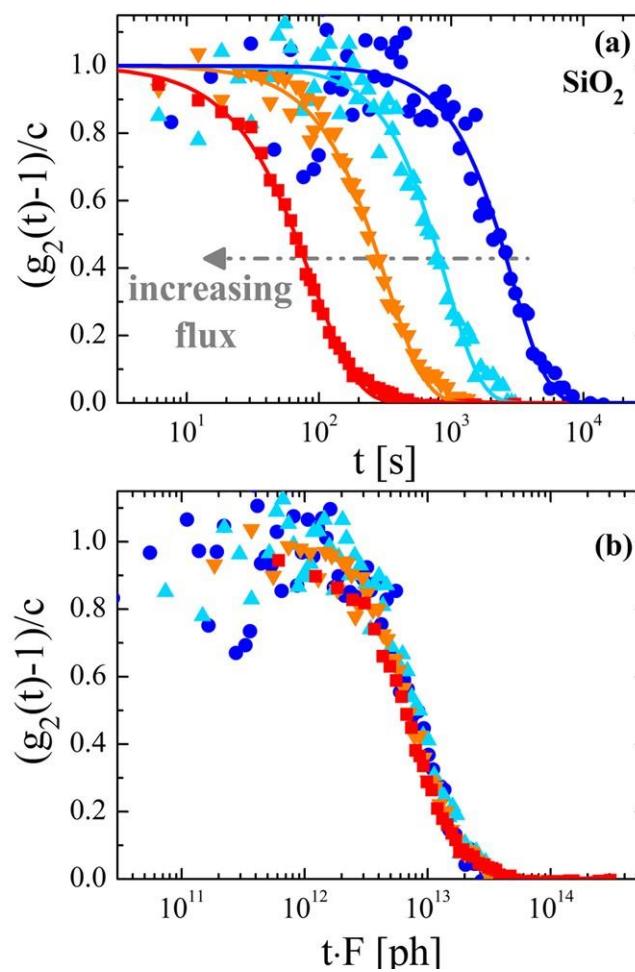
## Example 6: dynamics of metallic glasses



- Missing contrast: faster, non-accessible dynamics
- Aging

Nat. Commun. 7, 10344 (2016)

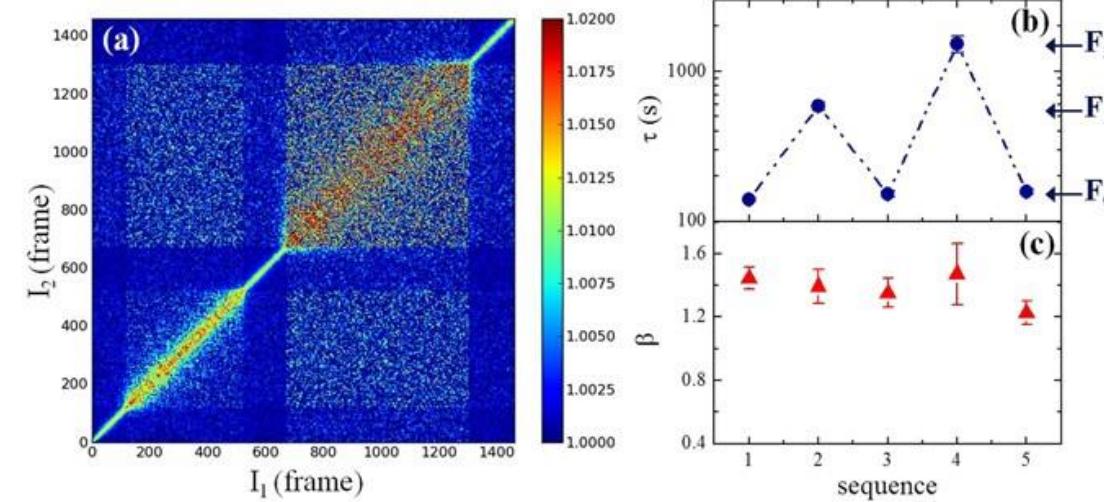
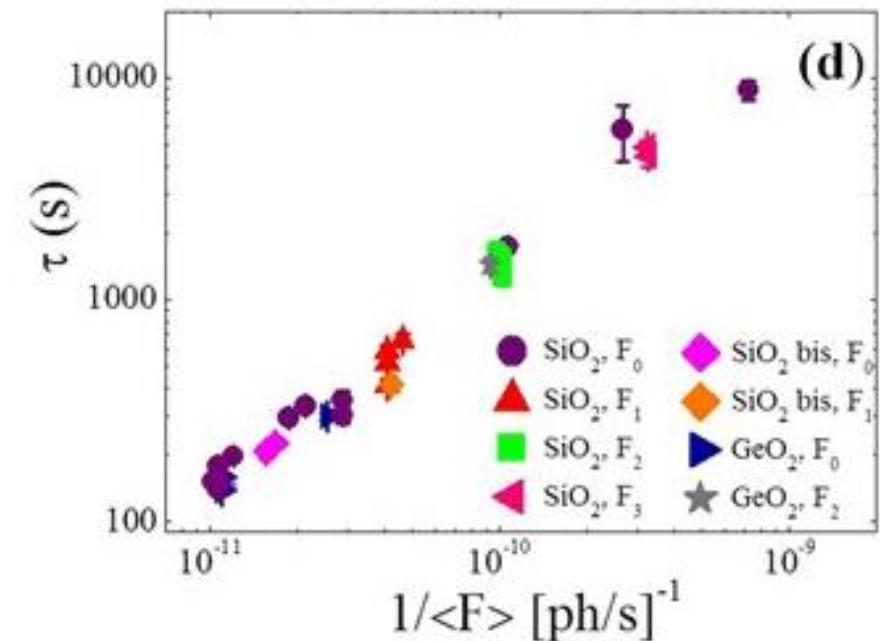
## Example 7: oxide glasses



Silica glass: dynamics scale with x-ray flux

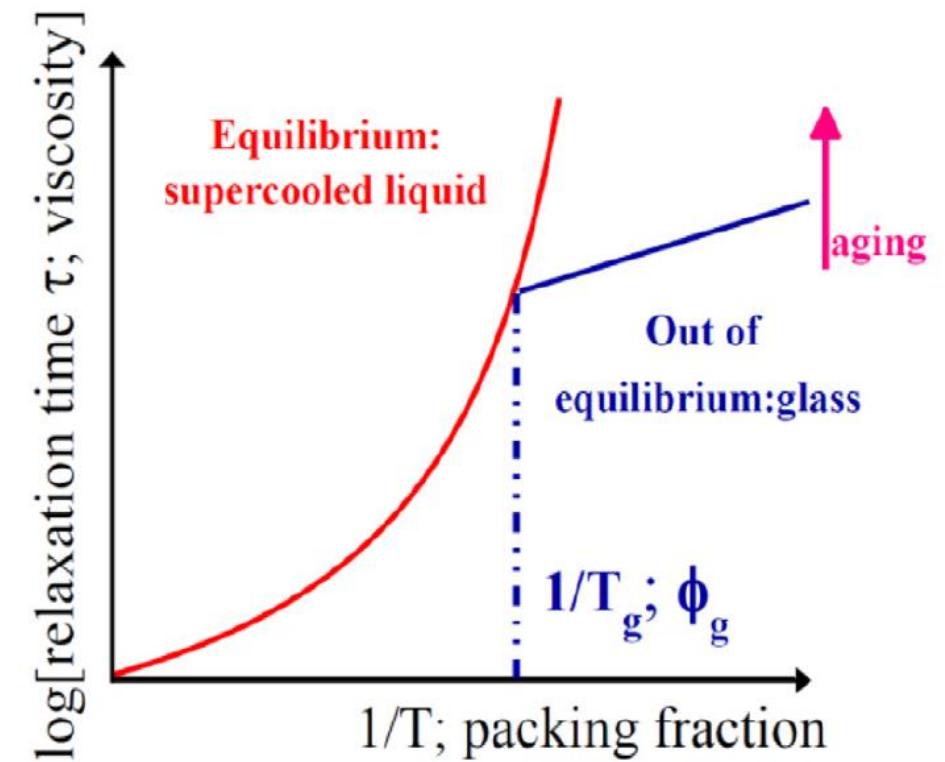
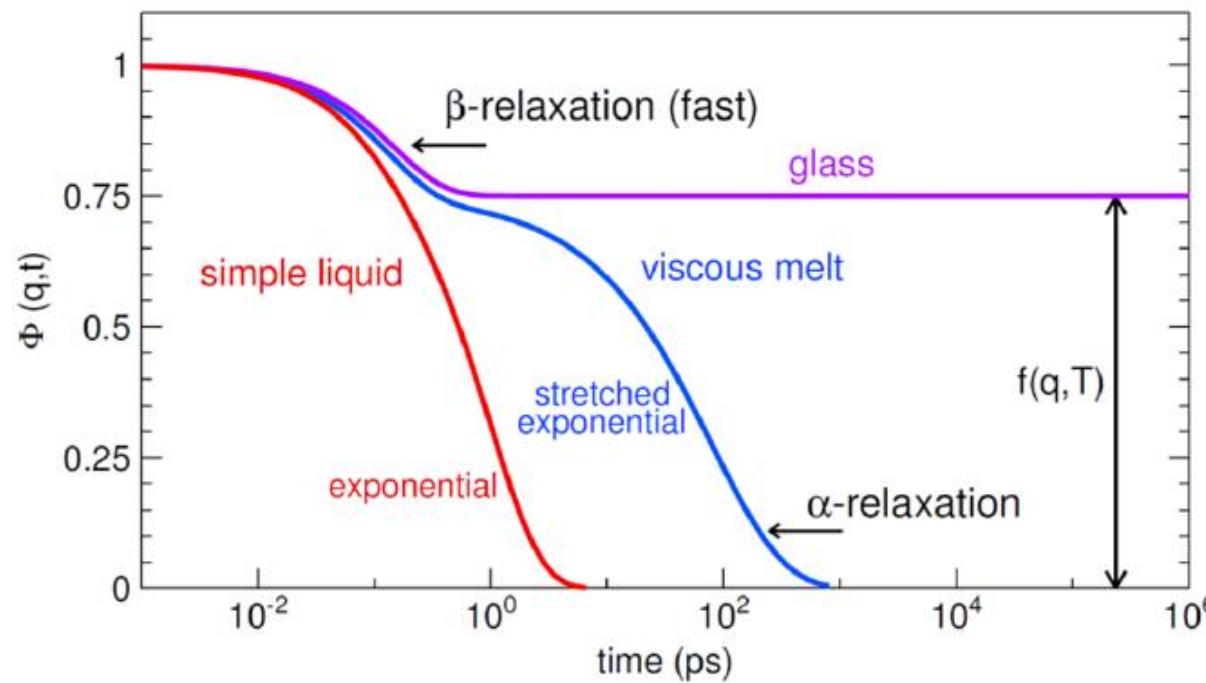
Fully reversible

Underlying process unclear



B. Ruta et al. Sci. Rep. 7, 3962 (2017)

## Dynamics towards glass transition



JPCD 29, 503002 (2017)