

Methoden moderner Röntgenphysik: Streuung und Abbildung

Lecture 9	Vorlesung zum Haupt- oder Masterstudiengang Physik, SoSe 2021 G. Grübel, O. Seeck, V. Markmann, F. Lehmkuhler, Andre Philippi-Kobs, M. Martins
Location	online
Date	Tuesdays 12:30 - 14:00 (starting 6.4.) Thursdays 8:30 - 10:00 (until 8.7.)

Methoden moderner Röntgenphysik: Online Info

Tuesday Zoom-Meeting

<https://desy.zoom.us/j/92674682486>

Meeting ID: 926 7468 2486

Passcode: 144456

Thursday Zoom-Meeting

<https://desy.zoom.us/j/99738625981>

Meeting ID: 997 3862 5981

Passcode: 841881

Tutorial Zoom-Meeting

<https://desy.zoom.us/j/95288979489>

Meeting ID: 952 8897 9489

Passcode: 832350

Literature

Basic concepts:

Moderne Röntgenbeugung
Röntgendiffraktometrie für
Materialwissenschaftler,
Physiker und Chemiker

Authors

[\(view affiliations\)](#)

Lothar Spieß

Robert Schwarzer

Herfried Behnken

Gerd Teichert

<https://link.springer.com/book/10.1007/978-3-663-10831-3>

Elements of Modern X-Ray Physics

J. A. Nielsen and D. McMorrow, J. Wiley&Sons (2001)

X-Ray Diffraction

B.E. Warren, DOVER Publications Inc., New York

Principles of Optics

M. Born and E. Wolf, Cambridge University Press, 7th ed.

Soft X-rays and Extreme Ultraviolet Radiation

D. Attwood, Cambridge University Press (2000)

<http://www.coe.berkeley.edu/AST/sxrev/>

Physik der Teilchenbeschleuniger und Synchrotronstrahlungsquellen

K. Wille, Teubner Studienbücher 1996

Lecture Notes

https://photon-science.desy.de/research/research_teams/coherent_x_ray_scattering/teaching/index_eng.html

Methoden moderner Röntgenphysik: Streuung und Abbildung

Small Angle Scattering, and Soft Matter

Introduction, Form Factor, Structure Factor, Applications, ...

Anomalous Diffraction

Introduction into Anomalous Scattering, ...

Introduction into Coherence

Concept, First Order Coherence, Spatial Coherence, Second Order Coherence, ...

Coherent Scattering

Imaging and Correlation Spectroscopy, ...



The Concept of Coherence: Classical Light

First Order Coherence

Coherence and Emission Spectrum

Spatial Coherence

Second Order Coherence

Chaotic Light

Basic concepts: **The quantum theory of light**

Rodney Loudon, Oxford University Press (1990)

Quantum optics

Marlan O. Scully, M. Suhail Zubairy,
Cambridge University Press (1997)

Coherence: Applications

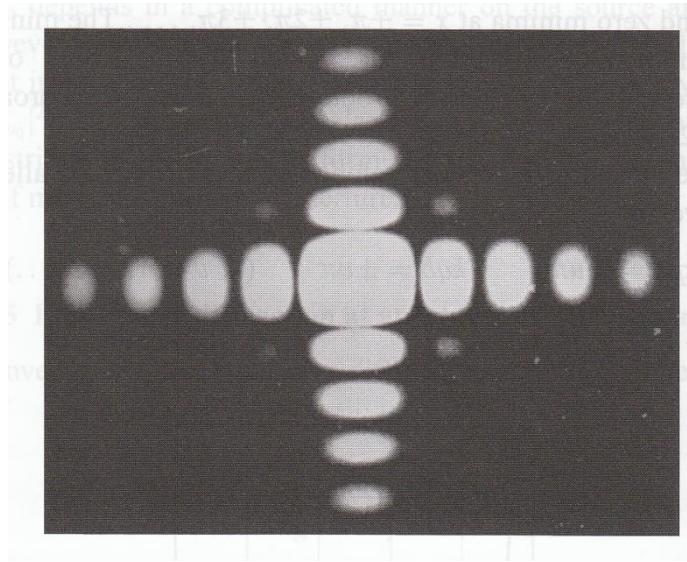
Interference Patterns

X-ray Speckle

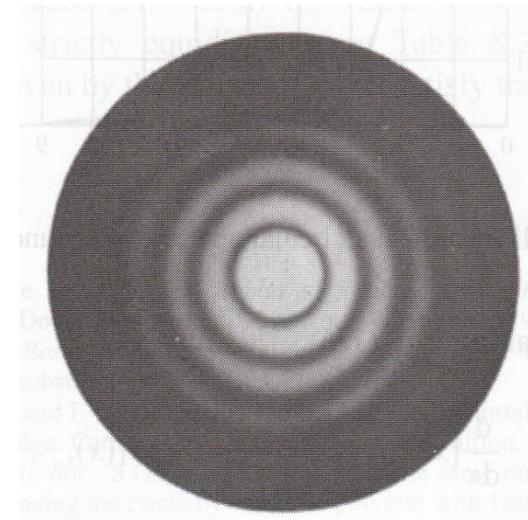
(Imaging)

X-Ray Photon Correlation Spectroscopy (XPCS)

Fraunhofer Diffraction

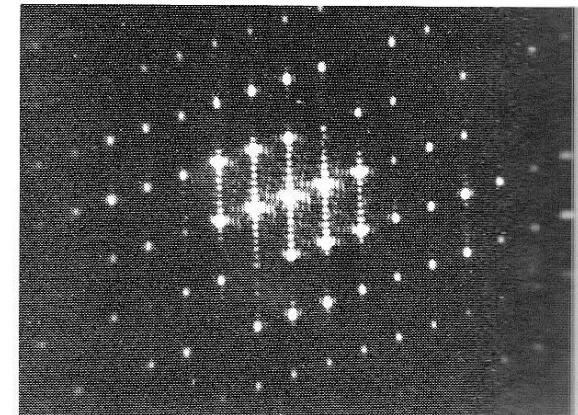
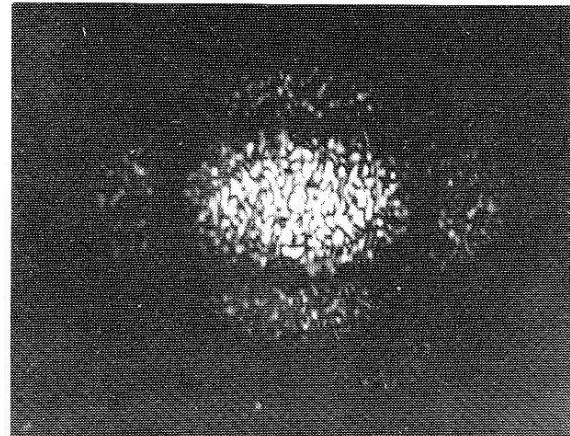


Fraunhofer diffraction of a rectangular aperture $8 \times 7 \text{ mm}^2$, taken with mercury light $\lambda=579\text{nm}$ (from Born&Wolf, chap. 8)



Fraunhofer diffraction of a circular aperture, taken with mercury light $\lambda=579\text{nm}$ (from Born&Wolf, chap. 8)

Speckle Pattern

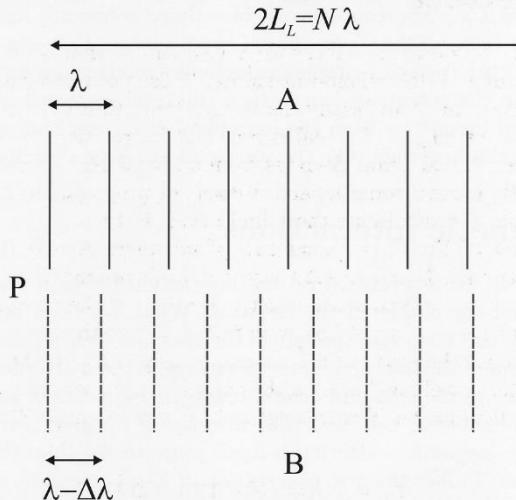


Random arrangement of
apertures: speckle

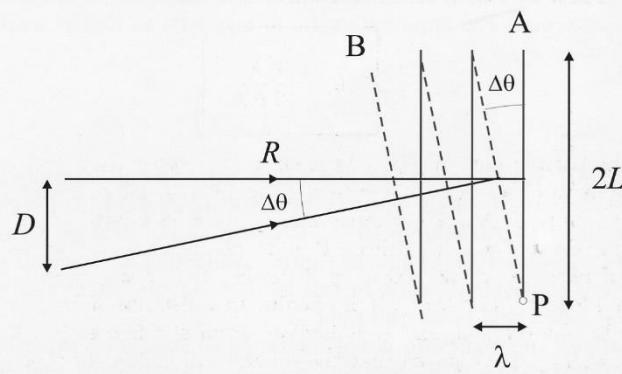
Regular arrangement
of apertures

Coherence Lengths (0.1 nm X-Rays)

(a) Longitudinal coherence length, L_L



(b) Transverse coherence length, L_T



Longitudinal coherence:

Two waves are in phase at point P. How far can one proceed until the two waves have a phase difference of π :

$$\xi_L = \left(\frac{\lambda}{2}\right) \left(\frac{\lambda}{\Delta\lambda}\right)$$

$$\lambda = 0.1\text{nm} \quad \frac{\Delta\lambda}{\lambda} = 10^{-4} \quad \Rightarrow \xi_L \approx 1 \mu\text{m}$$

Transverse coherence:

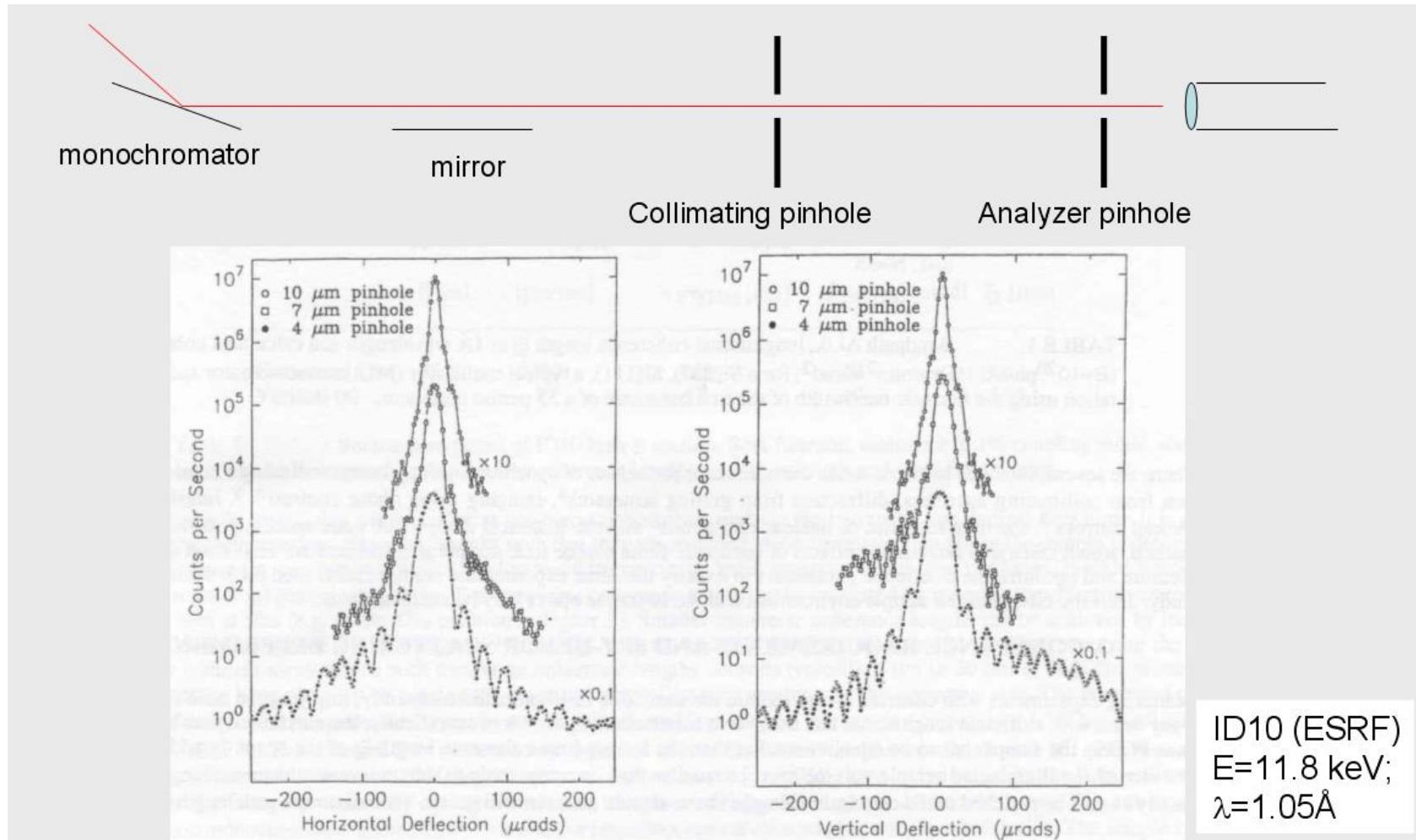
Two waves are in phase at P. How far does one have to proceed along A to produce a phase difference of π :

$$2\xi_t \Delta\theta = \lambda$$

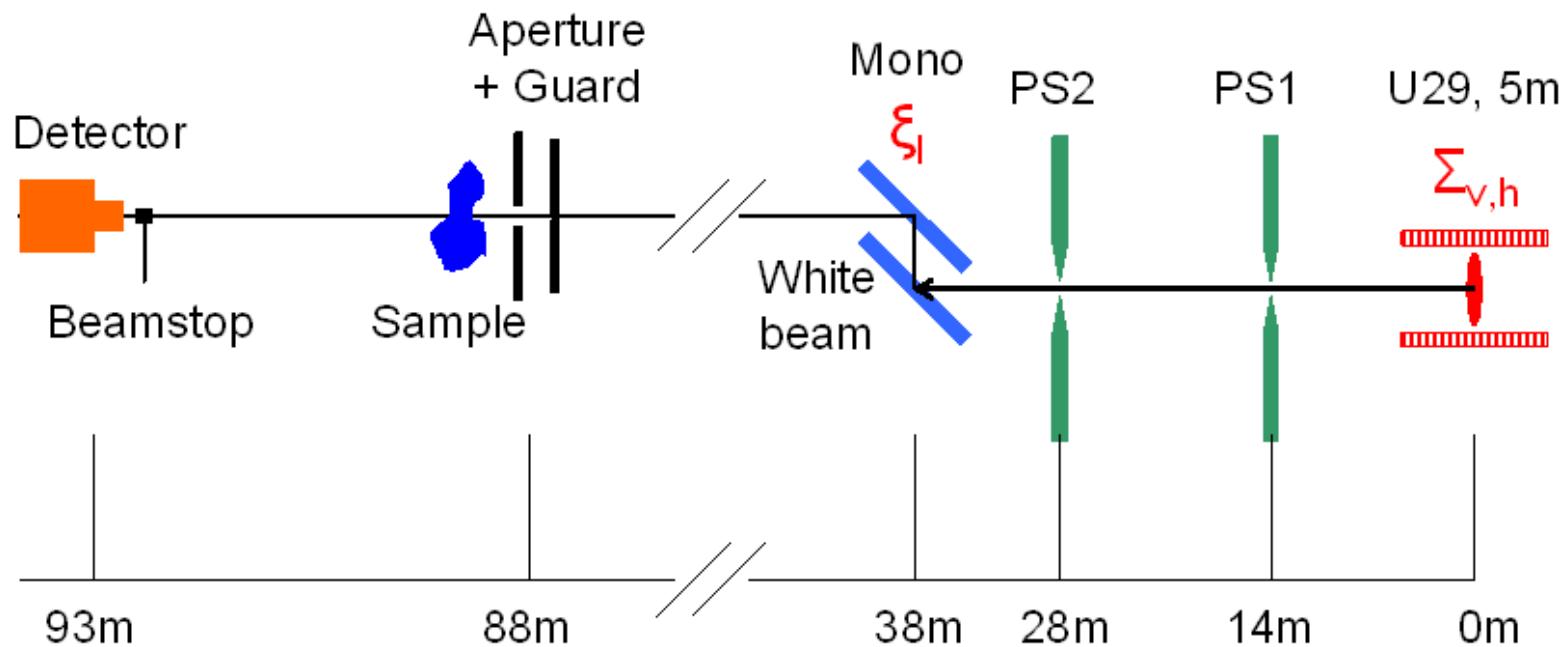
$$\xi_t = \left(\frac{\lambda}{2}\right) \left(\frac{R}{D}\right)$$

$$\lambda = 0.1\text{nm}, R = 100\text{ m}, D = 20 - 150 \mu\text{m} \\ \Rightarrow \xi_t \approx 100 \mu\text{m}$$

Fraunhofer Diffraction ($\lambda = 0.1\text{nm}$)



Coherence Lengths of a Storage Ring Beamline



$$\frac{\Delta\lambda}{\lambda} = 10^{-4}$$

$$\Sigma_v \approx 5 - 10 \mu\text{m}$$

$$\Sigma_h \approx 100 - 200 \mu\text{m}$$



Speckle

If coherent light is scattered from a disordered system it gives rise to a random (grainy) diffraction pattern, known as “**speckle**”. A speckle pattern is an interference pattern and related to the exact spatial arrangement of the scatterers in the disordered system.

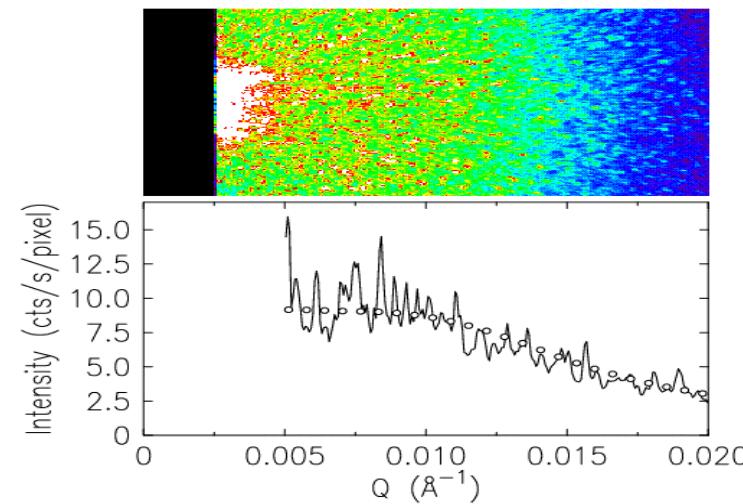
$$I(Q, t) \propto S_C(Q, t) \propto \left| \sum e^{iQ R_j(t)} \right|^2$$

j in coherence volume $V_c = \xi_t^2 \xi_l$

Incoherent Light:

$S(Q, t) = \langle S_c(Q, t) \rangle_{V \gg V_c}$ ensemble average

Aerogel
 $\lambda=1\text{\AA}$
 CCD (22 μm)



Abernathy, Grübel, et al.
 J. Synchrotron Rad. 5, 37,
 1998



Speckle Statistics

If the source is fully coherent and the scattering amplitudes and phases of the scattering are statistically independent and distributed over 2π one finds for the probability amplitude of the intensities:

$$P(I) = \left(\frac{1}{\langle I \rangle} \right) e^{\frac{-I}{\langle I \rangle}}$$

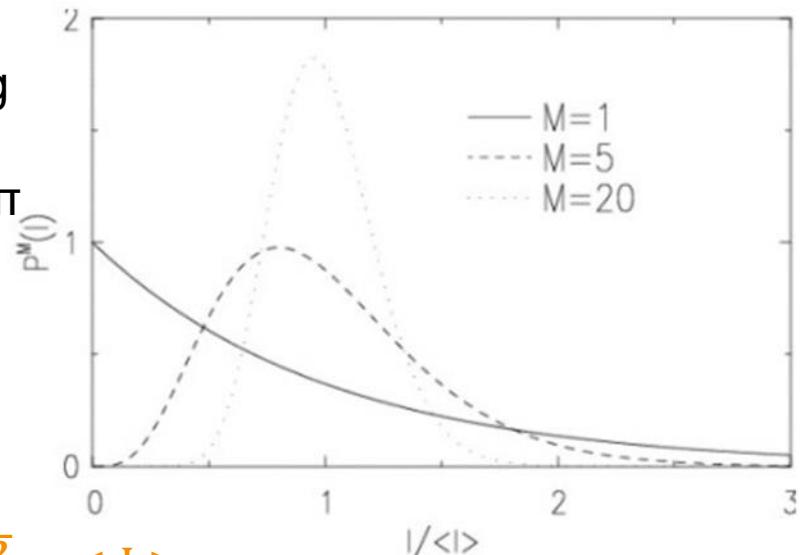
Mean: $\langle I \rangle$ **Std. Dev.** σ : $\sqrt{\langle I^2 \rangle - \langle I \rangle^2} = \langle I \rangle$

Contrast: $\beta = \sigma^2 / \langle I \rangle^2 = 1$

Partially coherent illumination: the speckle pattern is the sum of M independent speckle patterns

$$P_M(I) = M^M \cdot \frac{\left(\frac{1}{\langle I \rangle}\right)^{M-1}}{\Gamma(M)\langle I \rangle} \cdot e^{-\frac{MI}{\langle I \rangle}}$$

Mean: $\langle I \rangle$; $\sigma = \frac{\langle I \rangle}{\sqrt{M}}$ $\beta = 1/M$



Speckle

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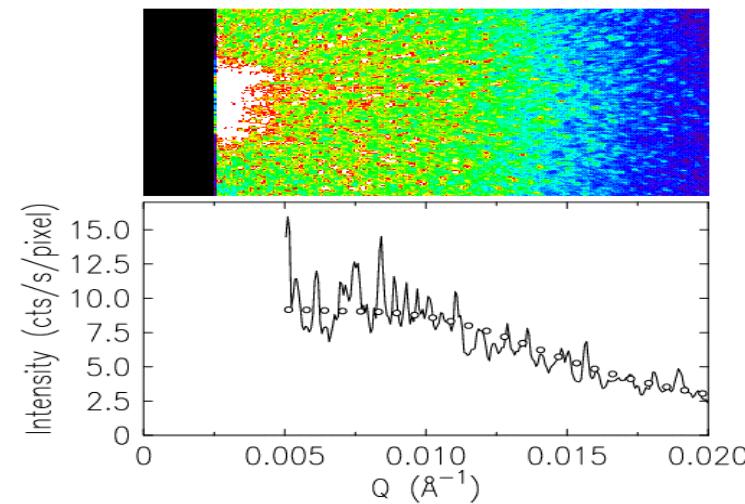
$$I(Q, t) \propto S_C(Q, t) \propto \left| \sum e^{iQR_j(t)} \right|^2$$

j in coherence volume $V_c = \xi_t^2 \xi_l$

Incoherent Light:

$S(Q, t) = \langle S_c(Q, t) \rangle_{V \gg V_c}$ ensemble average

Aerogel
 $\lambda=1\text{\AA}$
 CCD (22 μm)

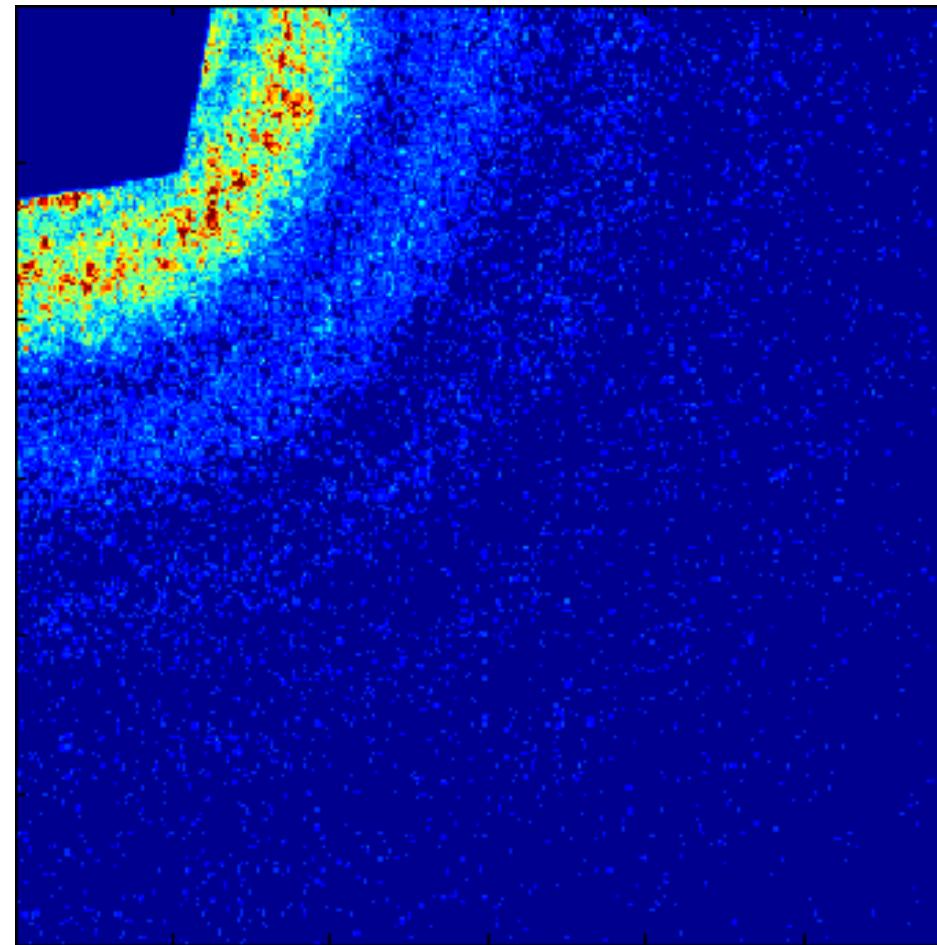


Abernathy, Grübel, et al.
 J. Synchrotron Rad. 5, 37,
 1998



Fluctuating Speckle Patterns

Silica: 2610 Å, $\frac{\Delta R}{R} = 0.03$, 10 vol% in glycerol, $T = -13.6 \text{ }^\circ\text{C}$, $\eta \approx 56000 \text{ cp}$



V. Trappe
& A. Robert

X-Ray Photon Correlation Spectroscopy (XPCS)

$$g_2(Q, t) = \frac{\langle I(Q, 0) \bullet I(Q, t) \rangle}{\langle I(Q) \rangle^2}$$

$$I(Q, t) = |E(Q, t)|^2 = \left| \sum b_n(Q) e^{iQ \bullet r_n(t)} \right|^2$$

Note: $E(Q, t) = \int dr' \rho(r') e^{iQ \bullet r'(t)}$ $\rho(r')$: charge density

If $E(Q, t)$ is a zero mean, complex Gaussian variable:

$$g_2(Q, t) = 1 + \beta(Q) \frac{\langle E(Q, 0) E^*(Q, t) \rangle^2}{\langle I(Q) \rangle^2} \quad \langle \rangle: \text{ensemble av.}; \quad \beta(Q): \text{contrast}$$

$$g_2(Q, t) = 1 + \beta(Q) |f(Q, t)|^2 \quad \text{with } f(Q, t) = S(Q, t)/S(Q, 0)$$

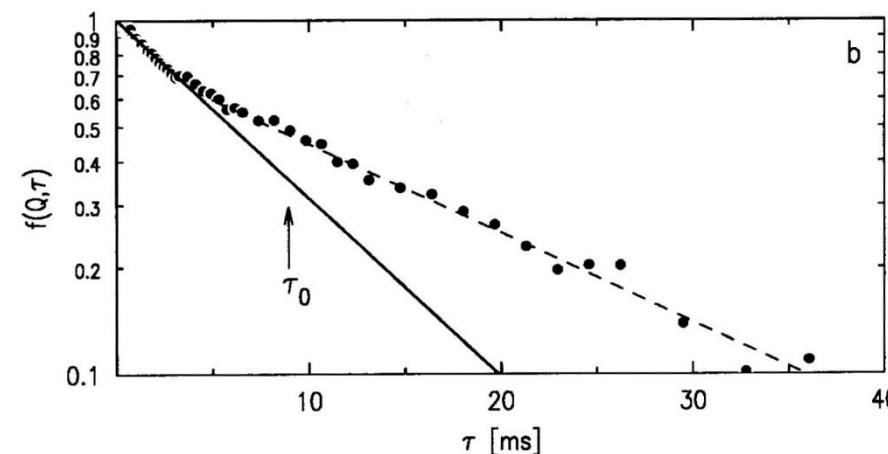
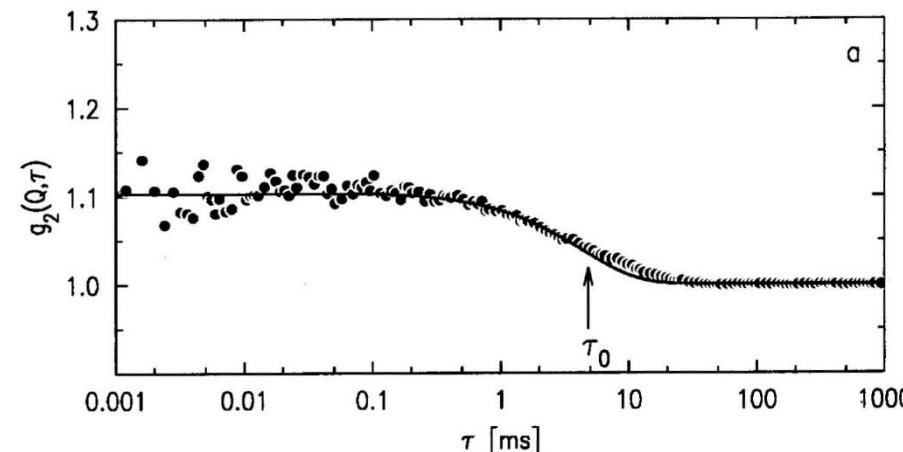
$S(Q, 0)$: static structure factor

N: number of scatterers

$$S(Q, t) = \frac{1}{N \{b^2(Q)\}} \sum_{m=1}^N \sum_{n=1}^N \langle b_n(Q) b_m(Q) e^{iQ[r_n(0) - r_m(t)]} \rangle$$

Time Correlation Function $g_2(Q,t)$

$$g_2(Q, t) = 1 + \beta(Q)|f(Q, t)|^2 \text{ and } f(Q, t) = e^{(-\Gamma t)} = e^{\left(\frac{-t}{\tau_0}\right)}$$



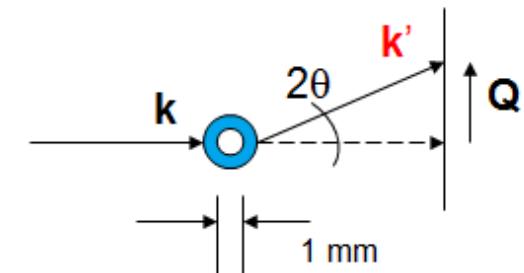
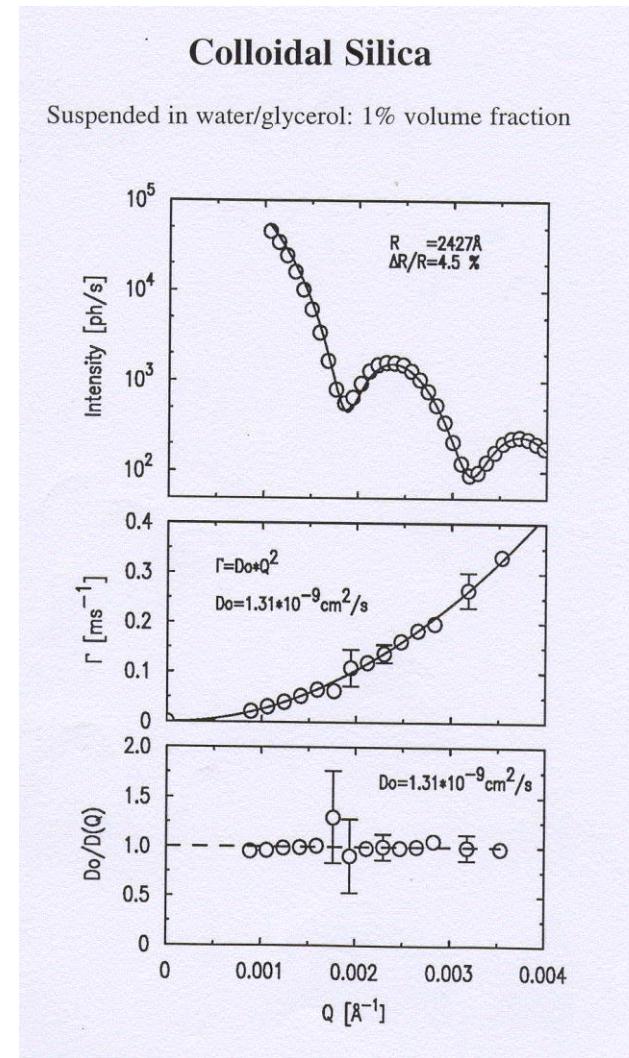
Dynamics in a Dilute, Non-interacting System

$$I \sim |F(Q)|^2 S(Q)$$

$$\sim \left[\frac{\sin QR - QR \cos QR}{(QR)^3} \right]^2$$

$$\Gamma = D_0 Q^2$$

$$D_0 = \frac{k_B T}{6\pi\eta R}$$



$$\mathbf{Q} = \mathbf{k}' - \mathbf{k}$$

$$Q = 2k \sin \theta$$

$$k = 2\pi/\lambda$$

G. Grübel, A. Robert, D. Abernathy
 8th Tohwa University International
 Symposium on "Slow Dynamics in
 Complex Systems", 1998, Fukuoka, Japan



Outlook

Imaging Holographic Imaging, Ptychography,....

Impact of FEL sources.....

XPCS Equilibrium, non-equilibrium dynamics delay line techniques at FEL sources.....