**Example**: Capillary waves (thermally induced surface fluctuations of liquid surfaces)

Even liquids do not have a perfectly smooth surface!
Theoretical description by the equilibrium of forces at the surface.

Break of symmetry at the surface =>

Resulting force directed to the bulk =>

**surface tension**

Equilibrium in the liquid: balance of forces => Navier Stokes Eq.

\[
\rho \frac{\partial}{\partial t} \mathbf{v} = -\nabla p + \eta \Delta \mathbf{v}
\]

with boundary conditions:

\[
\sigma_{zz} = \gamma \frac{\partial^2}{\partial z^2} u_z + g \rho \% u_z
\]

- \( \rho \): density, \( \mathbf{v} \): velocity
- \( \rho \): pressure, \( \eta \): viscosity
- \( \sigma_{zz} \): from stress tensor
- \( \gamma \): surface tension
- \( g \): gravitational constant
- \( u_z \): displacement at surface
With this knowledge the **dynamical susceptibility** $\chi_{zz}(q, \omega) = u_z / p_z$ can be calculated. 

$\chi_{zz}$ describes the response of the surface on external forces and depends on the frequency $\omega$ and the wave vector $q$.

The **equipartition theorem** states that each thermally excited surface wave (mode) has in average the energy $k_B T$. The PSD of the modes can be calculated by:

$$\tilde{C}(q, \omega) = 2k_B T \frac{\Im \{\chi_{zz}(q, \omega)\}}{\omega}$$

In the static case for a bulk liquid (average in time)

$$\tilde{C}(q) = \frac{k_B T}{4\pi^2 \gamma} \left[ q^2 + \frac{g \rho}{\gamma} \right]^{-1}$$
Surface Sensitive X-ray Scattering

After 2-dim Fourier-backtransformation leads to a logarithmic auto-correlation function $C(r)$

$$\tilde{C}(q) = \frac{k_B T}{4 \pi^2 \gamma} \left[ q^2 + \frac{g \rho}{\gamma} \right]^{-1}$$

$$C(r) = -\frac{k_B T}{2 \pi^2 \gamma} \left[ \ln\left( \frac{g \rho}{\gamma} r \right) + 0.5772 \right]$$

With logarithmic auto-correlation function $C(r)$ the diffuse scattering in Born approximation is for $q^2 > g \rho / \gamma$ in good approximation given by

$$I_{\text{diff, liquid}}^{BA} (q) \sim q \frac{k_B T}{2 \pi \gamma} q_x^2 - 1$$

$q_z$-dependent power low: slope contains the surface tension
Surface Sensitive X-ray Scattering

Yoneda peak


X-ray diffuse scattering at a liquid ethanol surface

From the data extracted parameters:
Roughness = 0.69nm
surface tension = 0.022N/m (exactly the known value)
Summary

- Rough surfaces and interfaces can be described via their power spectral density (PSD) or their auto-correlation function.

- The PSD is a measure of the number of modes in a wave vector interval.

- Rough interfaces will cause diffuse x-ray scattering.

- Diffuse x-ray scattering can be calculated in Born approximation and in DWBA (better). The formula contains the PSD of the interface.

- The PSD of liquids contain macroscopic material parameters such as temperature, surface tension and density. These parameters show up in the diffuse scattering.