

Methoden moderner Röntgenphysik II: Streuung und Abbildung

Lecture 8

Vorlesung zum Haupt/Masterstudiengang Physik

SS 2013

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Location: Hörs AP, Physik, Jungiusstrasse

Tuesdays 12.45 – 14.15

Thursdays 8:30 – 10.00

■ Methoden moderner Röntgenphysik II: Streuung und Abbildung

Introduction

Overview, Introduction to X-ray scattering

X-ray Scattering Primer

Elements of X-ray scattering

Sources of X-rays, Synchrotron Radiation

Laboratory sources, accelerator based sources

Reflection and Refraction

Snell's law, Fresnel equations,

Kinematical Diffraction (I)

Diffraction from an atom, molecule, liquids, glasses,...

Kinematical Diffraction (II)

Diffraction from a crystal, reciprocal lattice, structure factor,...

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Small Angle Scattering, and Soft Matter

Introduction, form factor, structure factor, applications, ..

Anomalous Diffraction

Introduction into anomalous scattering,..

Introduction into Coherence

Concept, First order coherence, ..

Coherent Scattering

Spatial coherence, second order coherence,..

Applications of coherent Scattering

Imaging and Correlation spectroscopy,..

Resonant Scattering (phasing, magnetism,...)

Scattering length of an atom: $-r_0 f^0(\mathbf{Q})$

$f^0(\mathbf{Q})$ atomic form factor (fourier transform of charge distribution)

r_0 thomson scattering length of single electron

in order to include absorption effects (f'') atoms a more elaborate model than the free electron gas is needed.

→ Electrons are bound to atoms

→ Forced oscillator model with resonant frequency ω_s and damping constant Γ

include dispersion corrections (f' , f''):

[note: $f'' = (k/4\pi r_0) \sigma_a$]

$$f(\mathbf{Q}, \omega) = f^0(\mathbf{Q}) + f'(\omega) + i f''(\omega)$$

[in units of r_0]

Resonant Scattering

classical model of
an electron bound
in an atom in E field

$$\mathbf{E}(\mathbf{r},t) = \hat{\mathbf{x}} E_0 \exp\{-i\omega t\} \longrightarrow$$

equation of motion
of the electron

$$\ddot{x} + \Gamma \dot{x} + \omega_s^2 x = - \left(\frac{e E_0}{m} \right) \exp\{-i\omega t\}$$

Γ = damping
 ω_s resonant
frequency

Solution: $x(t) = x_0 \exp\{-i\omega t\} \longrightarrow x_0 = - \left(\frac{e E_0}{m} \right) \frac{1}{(\omega_s^2 - \omega^2 - i\omega\Gamma)}$ (A)

radiated field strength at
distance R and time t

$$E_{\text{rad}}(R,t) = \left(\frac{e}{4 \epsilon_0 R c^2} \right) \ddot{x}(t - R/c) \quad (B)$$

↑
acceleration at “earlier” time (t-R/c)

Resonant scattering

inserting $\ddot{x}(t - R/c) = \omega^2 x_0 \exp\{-i\omega t\} \exp\{i(\omega/c)R\}$ using (A) into (B):

$$E_{\text{rad}}(R,t) = \frac{\omega^2}{(\omega_s^2 - \omega^2 - i\omega\Gamma)} \left(\frac{e^2}{4 \epsilon_0 m c^2} \right) E_0 \exp\{-i\omega t\} \left(\frac{\exp\{ikR\}}{R} \right)$$

or

$$\frac{E_{\text{rad}}(R,t)}{E_{\text{in}}} = \underbrace{-r_0 \frac{\omega^2}{(\omega_s^2 - \omega^2 + i\omega\Gamma)}}_{\text{atomic scattering length } f_s} \left(\frac{\exp\{ikR\}}{R} \right)$$

atomic scattering length f_s (in units of $-r_0$) for bound electron (C)
 note: $f_s \rightarrow 1$ ($\omega \gg \omega_s$)

total cross-section: $\sigma_T = 8\pi/3) r_0^2$ (free electron)

$$\sigma_T = \left(\frac{8\pi}{3} \right) \frac{\omega^4}{(\omega^2 - \omega_s^2)^2 + (\omega\Gamma)^2} r_0^2$$

for $\Gamma = 0$ and $\omega \ll \omega_s$: $\sigma_T = (8\pi/3)r_0^2 (\omega / \omega_s)^4$: “Rayleigh Scattering”

Resonant scattering

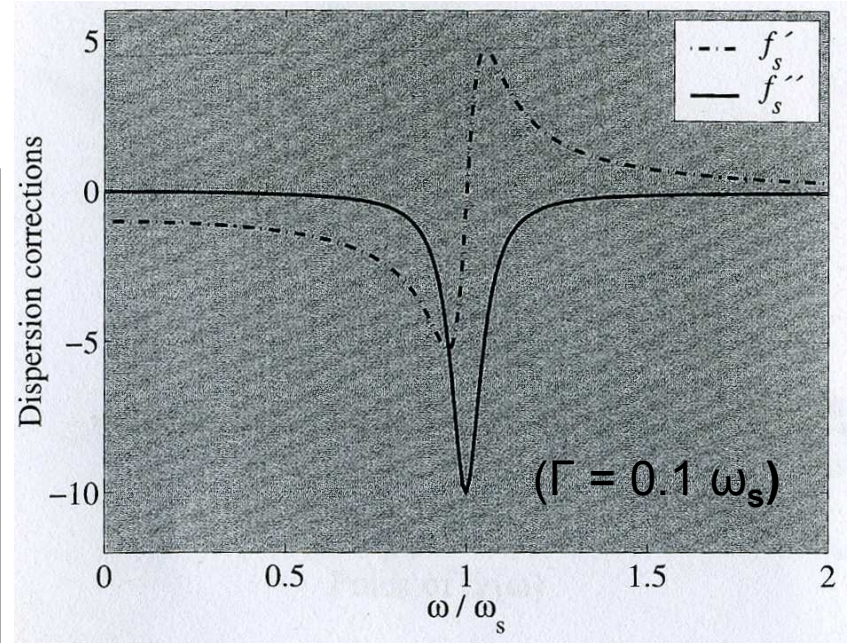
$$f_s = \frac{\omega^2 - \omega_s^2 + i\omega\Gamma + \omega_s^2 - i\omega\Gamma}{(\omega^2 - \omega_s^2 + i\omega\Gamma)}$$

$$= 1 + \frac{\omega_s^2 - i\omega\Gamma}{(\omega^2 - \omega_s^2 + i\omega\Gamma)}$$

$$\approx 1 + \frac{\omega_s^2}{(\omega^2 - \omega_s^2 + i\omega\Gamma)}$$

dispersion correction $\chi(\omega)$

$$\chi(\omega) = f'_s + i f''_s = \frac{\omega_s^2}{(\omega^2 - \omega_s^2 + i\omega\Gamma)}$$



with:

$$f'_s = \frac{\omega_s^2 (\omega^2 - \omega_s^2)}{(\omega^2 - \omega_s^2)^2 + (\omega\Gamma)^2}$$

$$f''_s = \frac{\omega_s^2 \omega \Gamma}{(\omega^2 - \omega_s^2)^2 + (\omega\Gamma)^2}$$

Resonant scattering

Note: since $f'' = -(k/4\pi) \sigma_a(E)$ (see J. A-N. & D. McM. p. 70) it follows that the absorption cross-section for a single oscillator model is:

$$\sigma_{a,s}(\omega) = 4 \pi r_0 c \frac{\omega_s^2 \Gamma}{(\omega - \omega_s)^2 + (\omega \Gamma)^2}$$

this function has:

- sharp peak at $\omega = \omega_s$
- $\Delta\omega_{\text{FWHM}} \approx \Gamma$

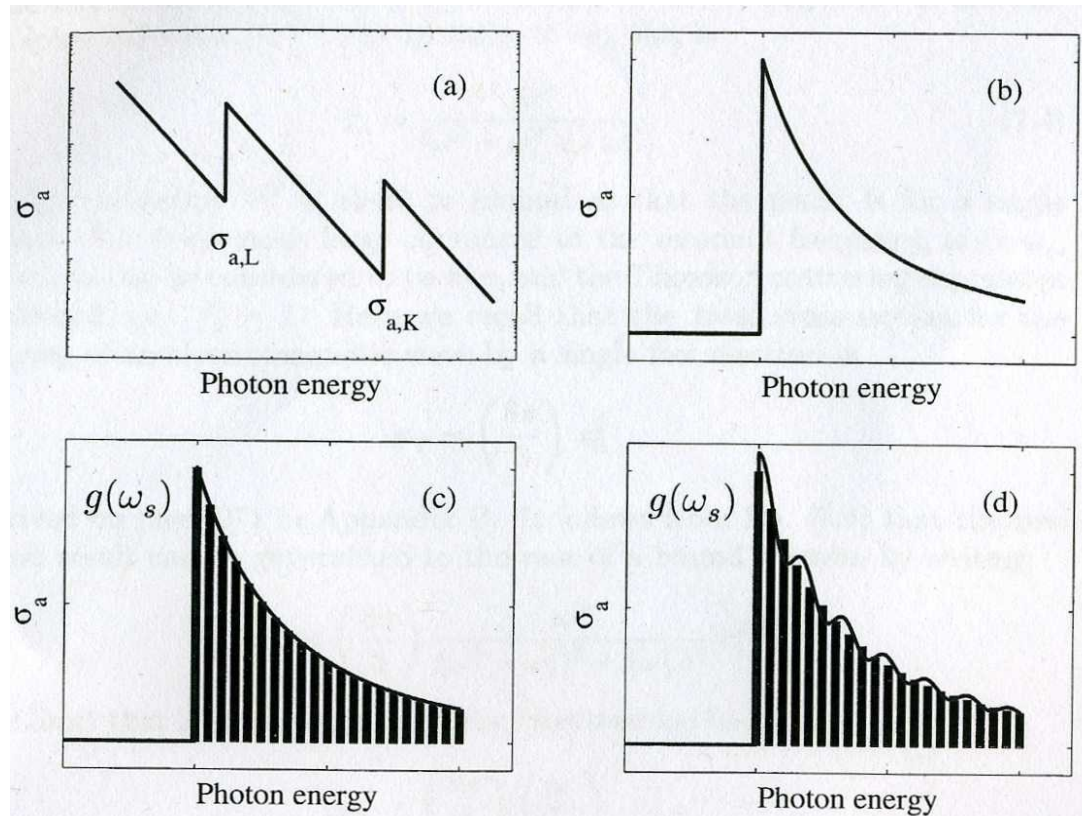
thus $\sigma_a(E)$ may be written with help of a delta function:

$$\sigma_{a,s}(\omega) = 4 \pi r_0 c \frac{\pi}{2} \delta(\omega - \omega_s) \quad (\text{D})$$

Resonant scattering

The experimentally observed absorption cross-section is NOT a single line spectrum as suggested by (D).

There is a continuum of free states above an absorption edge that the electron can be excited into. This implies a series of different ω_s :



Resonant scattering

Absorption cross section for multiple harmonic oscillators:

$$\sigma_a(\omega) = 2 \pi^2 r_0 c \sum_s g(\omega_s) \delta(\omega - \omega_s)$$

where $g(\omega_s)$ is the relative weight of each transition

The real part of the dispersion becomes:

$$f''(\omega) = \sum_s g(\omega_s) f'_s(\omega, \omega_s) \quad (\text{F})$$

(F) does not describe e.g. “white lines” or “EXAFS” oscillations (see figure) in the absorption cross section arising from the particular environment of the resonantly scattering atom.

Resonant scattering

measure absorption cross-section and use (E) to obtain f'' :

$$f''(\omega) = - \left(\frac{\omega}{4 \pi r_0 c} \right) \sigma_a(\omega)$$

use [Kramers-Kronig relations](#) to obtain f' :

$$f'(\omega) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{f''(\omega')}{(\omega' - \omega)} d\omega' = \frac{2}{\pi} P \int_0^{+\infty} \frac{\omega' f''(\omega')}{(\omega'^2 - \omega^2)} d\omega'$$

$$f''(\omega) = - \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{f'(\omega')}{(\omega' - \omega)} d\omega' = - \frac{2\omega}{\pi} P \int_0^{+\infty} \frac{f'(\omega')}{(\omega'^2 - \omega^2)} d\omega'$$

P stands for “principal value” (see also comments J. A-N & D. McM p. 242)

- # Resonant scattering

Friedel's law and Bijvoet pairs

The phase problem in crystallography

The MAD method

(Resonant) Magnetic Scattering