Excercises „Methoden Moderner Röntgenphysik II“

— X-Ray Reflectivity —

In Born-Approximation, the x-ray intensity reflected by a surface is given by

$$I(q_z) \propto \frac{1}{q_z^4} \left| \int_{-\infty}^{\infty} \frac{d\rho(z)}{dz} \exp(iq_z z) dz \right|^2$$

with the wave vector transfer q_z along z and the electron density profile $\rho(z)$ along z.

For a smooth single surface of a substrate with refractive index $n=1-\delta$ (no absorption) the exact Fresnel reflectivity $I(\alpha)$ is given by

$$I(\alpha) = \left| \frac{k_{z1} - k_{z2}}{k_{z1} + k_{z2}} \right|^2 \quad \text{with} \quad k_{z1} = \frac{2\pi}{\lambda} \sin\alpha \quad \text{and} \quad k_{z2} = \frac{2\pi}{\lambda} \sqrt{n^2 - \cos^2\alpha}$$

and the incident angle α.

1) **Show that the Fresnel reflectivity is proportional to the Born approximation for an ideally smooth surface and for** $\sin\alpha >> 2\delta$ **.**

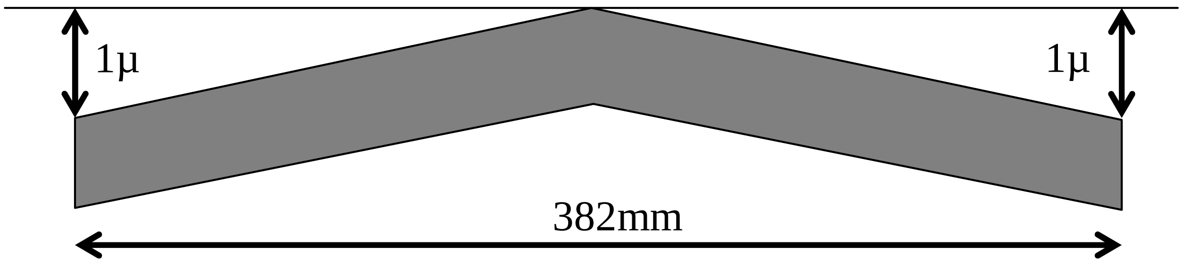
At a synchrotron radiation source such as PETRA at DESY the emitted x-radiation exhibit a photon energy spectrum $I_{PETRA}(E)$. At some user-specified photon energy E_0 with maximum photon flux this function also has maxima at $E_0/3$, E_0 /5, and so on. These so-called harmonics have to be suppressed before the experiment. This can be done by x-ray mirrors

2) **Design a “harmonic suppressor” for x-ray beams. Draw a principle sketch and explain how it works. Use the fact that matter has a refractive index *n=1-δ*.**

3) **Estimate the suppression factor for the 3rd harmonic** $E_0/3$ **using that** $\delta \propto \lambda^2$ **and that the incident angle is** $\alpha << 1$. **Compare the suppression with the typical flux of** 10^8 **photons per second for the 3rd harmonic.**

4) **Most x-ray mirrors deflect in the vertical. Calculate the minimum length of a mirror for a typical vertical beam size of 1mm and a typical incident angle of** $\alpha = 0.15$ **deg.**

At PETRA III a typical distance between the harmonic suppression mirror and the sample would be 40m. The average incident angle should be $\alpha = 0.15\,\text{deg}$. The length of the equally illuminated mirror should be 382mm. Let us assume that the mirror is not perfectly flat but has a kink in the middle and is hanging at each end by 1µm:



5) **With a perfectly flat mirror the beam shape at the sample position would be rectangular with a height of approx. 1mm. Estimate the shape of the beam at the sample position using the slightly bend mirror.**