

Methoden moderner Röntgenphysik I + II: Struktur und Dynamik kondensierter Materie

Vorlesung zum Haupt/Masterstudiengang Physik SS 2009
M. v. Zimmermann

martin.v.zimmermann@desy.de

HASYLAB at DESY

building 25b, room 222, phone 8998 2698

Materials Science

7. 5. Martin v. Zimmermann	correlated electron materials – structural properties
12. 5. Hermann Franz	glasses I
14. 5. Martin v. Zimmermann	correlated electron materials – magnetic properties
19. 5. Hermann Franz	glasses II
26.5. Hermann Franz	exercises

correlated electron materials: overview

- phase transitions
 - structural phase transition of SrTiO_3
 - x-ray diffraction to investigate phase transitions
 - structural aspects of transition metal oxides
 - orbital and charge order in $\text{La}_{1-x}\text{Ca}_x\text{MnO}_3$
 - resonant scattering to study orbital/charge order
-
- magnetic interactions in transition metal oxides
 - Mott insulator
 - colossal magneto resistance (CMR) effect
 - resonant / non-resonant magnetic scattering

exchange interactions

combination of Coulomb interaction and Pauli principle

$$J \sim -\int \Psi_x^*(\mathbf{r}_1)\Psi_y(\mathbf{r}_1) (e^2/r_{12}) \Psi_y^*(\mathbf{r}_2)\Psi_x(\mathbf{r}_2)$$

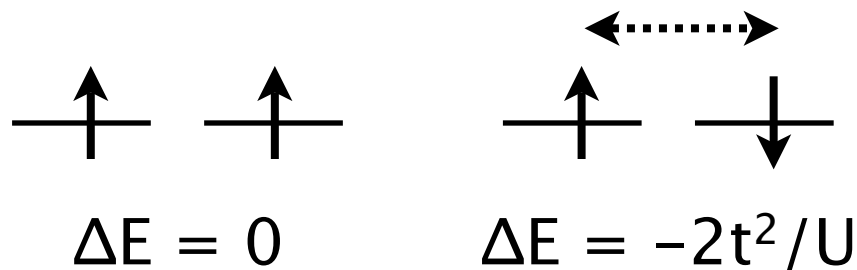
one-band Hubbard model:

$$H = -\sum t_{ij} (c_{i\sigma}^\dagger c_{j\sigma}) + U \sum n_{i\uparrow}n_{i\downarrow}$$
$$= H_{\text{kin}} + H_U$$

t_{ij} hopping amplitude between nn sites $\langle ij \rangle$
 $c_{i\sigma}^\dagger$ creates an electron with spin σ at lattice site i
 U Coulomb repulsion
 $n_{i\sigma}$ number of electrons at site i with spin σ

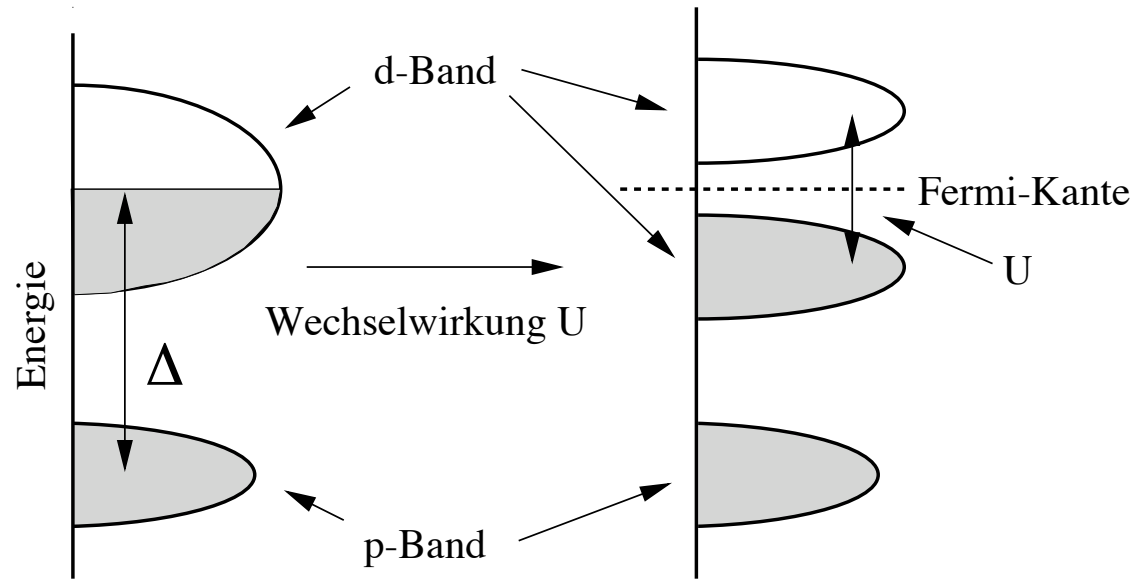
$t \gg U$: metallic system

$t \ll U$: insulator with one electron per site



superexchange:
antiferromagnetic


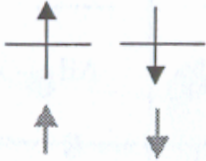

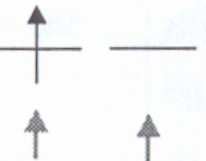
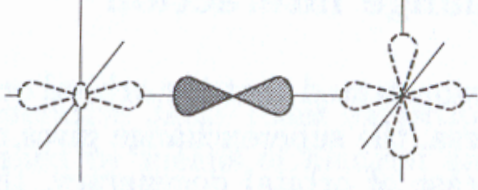
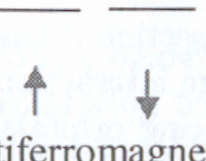
Mott insulator



strongly correlated electron systems: transition metal oxides
high- T_c superconductors
CMR-manganites ...

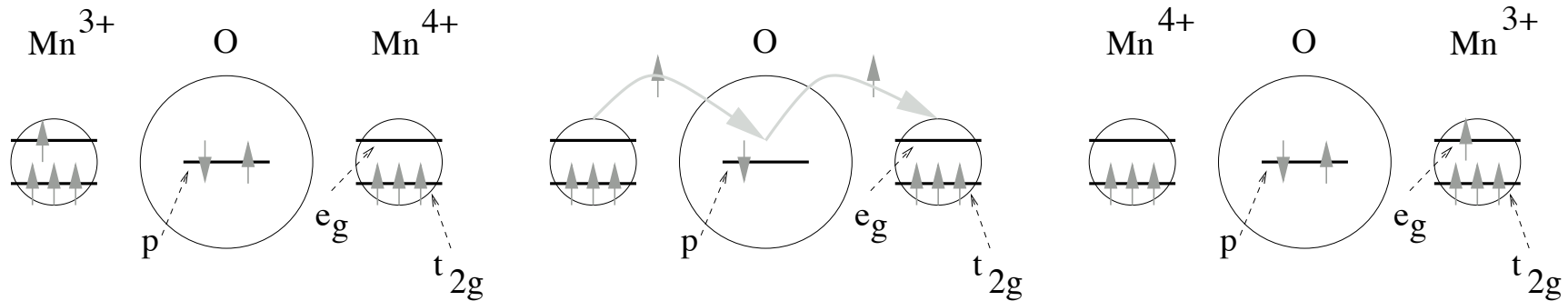
GKA-rules (Goodenough-Kanamori-Andersen)

orbital dependent exchange interaction

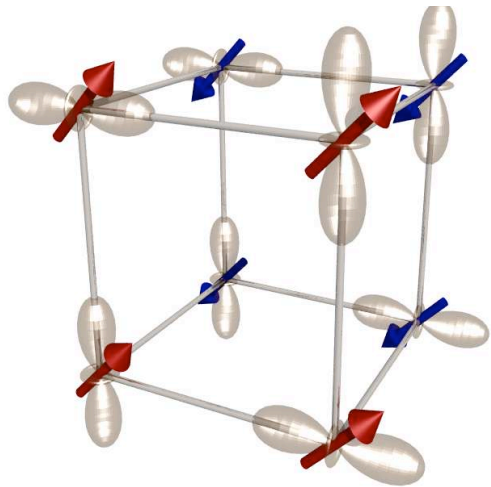
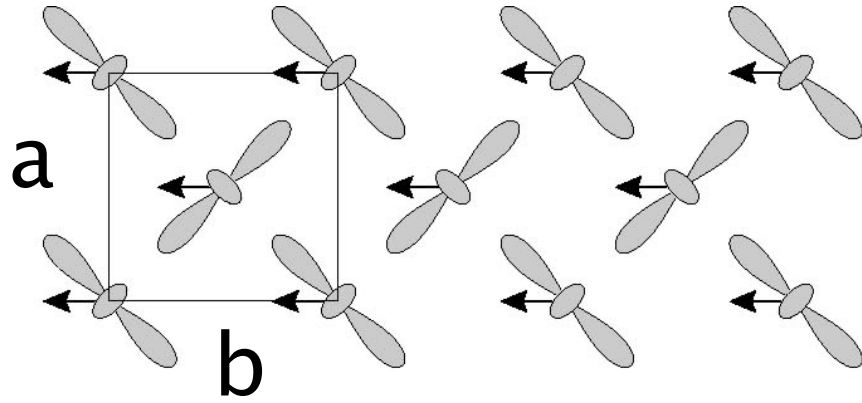
Configuration example	Exchange coupling
<p>(1) </p> <p>occupied occupied</p>	 <p>antiferromagnetic</p>
<p>(2) </p> <p>occupied unoccupied</p>	 <p>ferromagnetic</p>
<p>(3) </p> <p>unoccupied unoccupied</p>	 <p>antiferromagnetic</p>

double exchange interaction

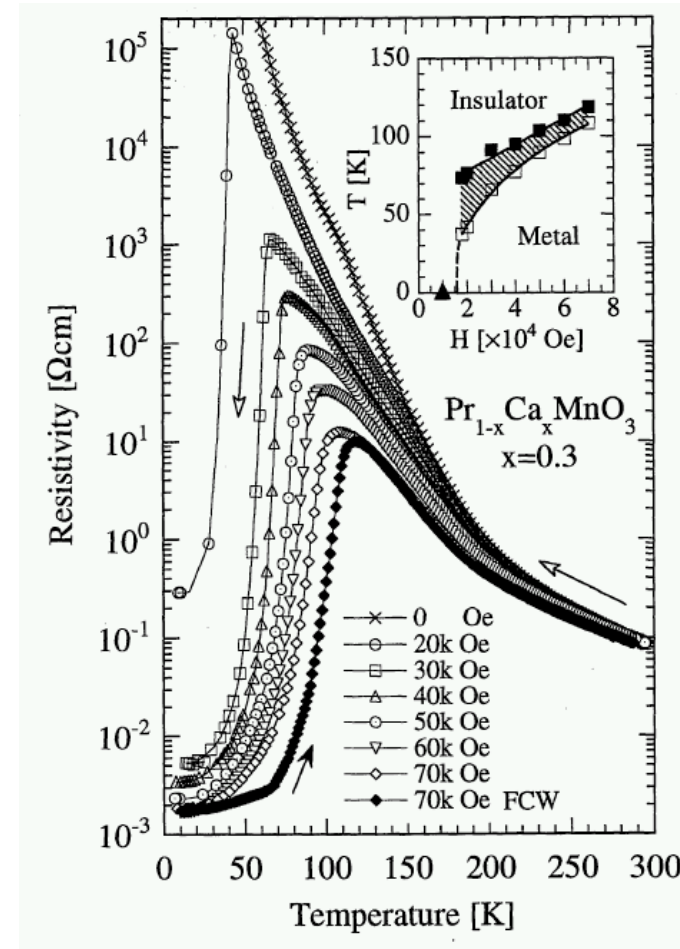
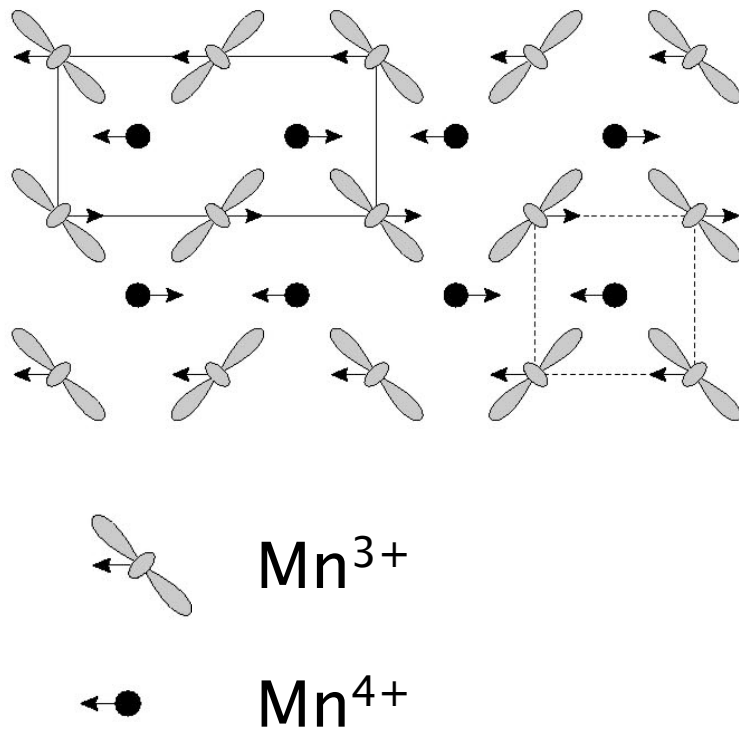
ferromagnetic interaction between different ions due to Hund's coupling



magnetism of LaMnO_3



magnetism of $\text{La}_{0.5}\text{Ca}_{0.5}\text{MnO}_3$ and colossal magneto resistance (CMR) effect

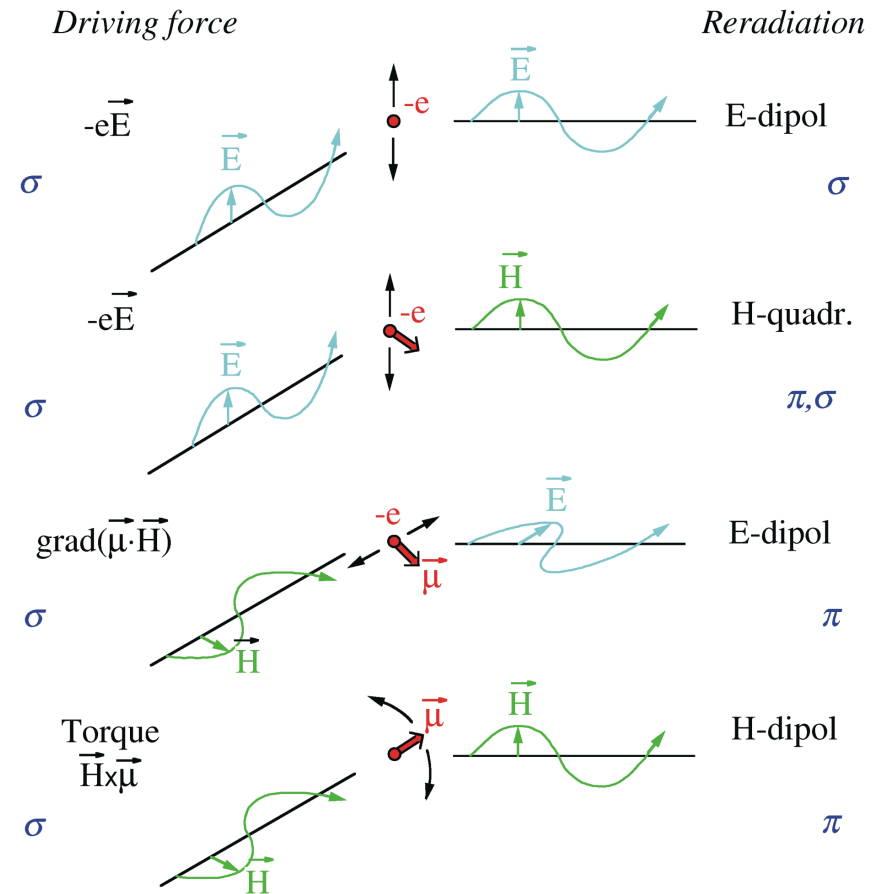
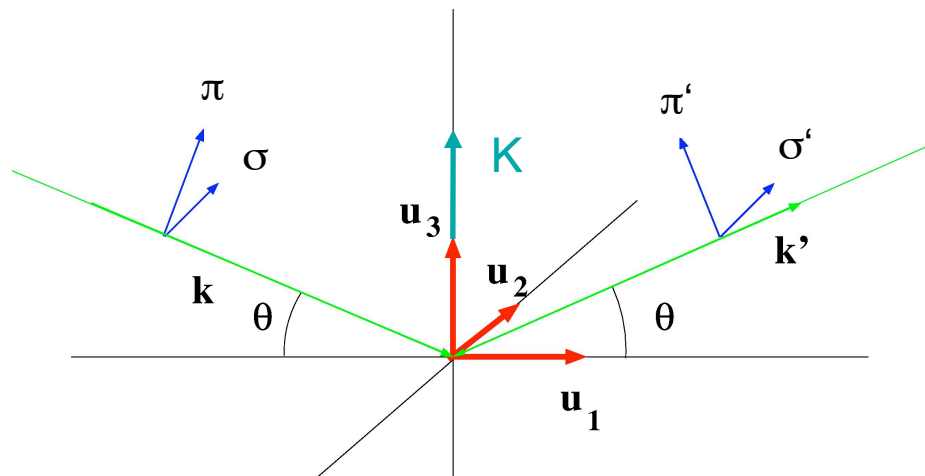


magnetic x-ray scattering

Synchrotronstrahlung linear polarisiert in Ringebene

Streugeometrie vertikal

→ σ – polarisierte einfallende Strahlung



resonant magnetic x-ray scattering

Röntgenstreuung an periodischen Strukturen

(Blume, J. Appl. Phys. **57**, 3615 (1985); Blume and Gibbs, PRB **37**, 1779 (1988)):

$$\frac{d\sigma}{d\Omega} = r_o^2 \left| \sum_n e^{i\vec{Q}\cdot\vec{r}_n} f_n(\vec{k}, \vec{k}', \hbar\omega) \right|^2$$

Streuamplitude:

$$f(\vec{k}, \vec{k}', \omega) = f^{charge}(\vec{Q}) + f^{spin}(\vec{k}, \vec{k}', \omega) + f'(\vec{k}, \vec{k}', \omega) + if''(\vec{k}, \vec{k}', \omega)$$

f^{charge} → Thomsonstreuung

f' und f'' → Energieabhängige Beiträge

f^{spin} → Streuung an Spins

Bei 10 keV:

$$\frac{f^{spin}}{f^{charge}} = 0.02$$

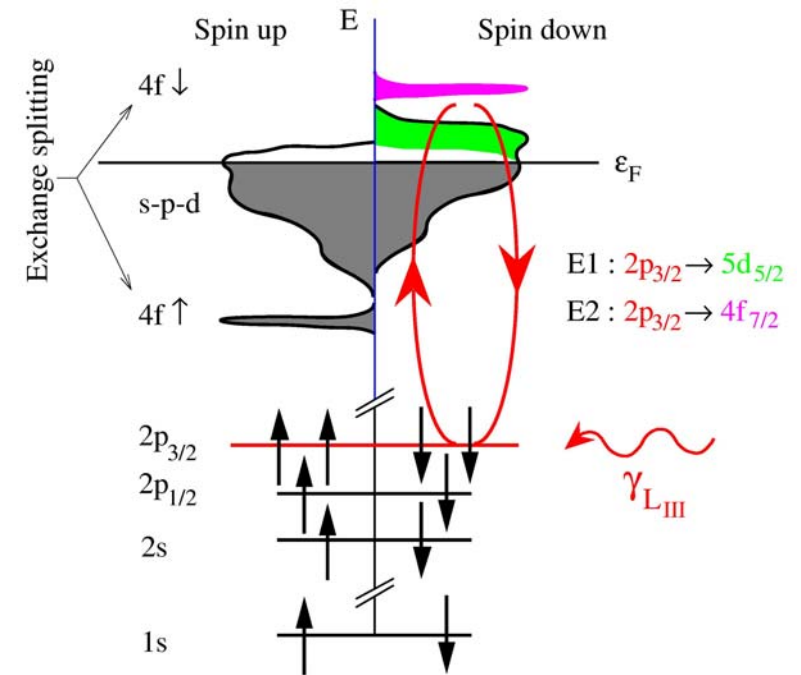
resonant magnetic x-ray scattering

Störungstheorie 1. und 2. Ordnung

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)^2 = r_0^2 \left\langle b \left| \sum_j e^{i\vec{k}\vec{r}_j} \right| a \right\rangle \hat{\epsilon} \cdot \hat{\epsilon}' - i \frac{\hbar\omega}{m c^2} \left\langle b \left| \sum_j e^{i\vec{k}\vec{r}_j} \vec{s}_j \right| a \right\rangle \hat{\epsilon} \times \hat{\epsilon}'$$

$$+ \frac{\hbar^2}{m} \sum_c \sum_{ij} \left(\frac{\left\langle b \left(\frac{\vec{\epsilon} \cdot \vec{P}_i}{\hbar} - i(\vec{k} \times \vec{\epsilon}') \cdot \vec{s}_i \right) e^{-i\vec{k}\vec{r}_i} \right| c \right\rangle \left\langle c \left(\frac{\vec{\epsilon} \cdot \vec{P}_j}{\hbar} + i(\vec{k} \times \vec{\epsilon}) \cdot \vec{s}_j \right) e^{i\vec{k}\vec{r}_j} \right| a \right\rangle}{E_a - E_c + \hbar\omega_k - i\Gamma_c/2}$$

$$+ \left(\frac{\left\langle b \left(\frac{\vec{\epsilon} \cdot \vec{P}_j}{\hbar} + i(\vec{k} \times \vec{\epsilon}) \cdot \vec{s}_j \right) e^{-i\vec{k}\vec{r}_j} \right| c \right\rangle \left\langle c \left(\frac{\vec{\epsilon} \cdot \vec{P}_i}{\hbar} - i(\vec{k} \times \vec{\epsilon}') \cdot \vec{s}_i \right) e^{i\vec{k}\vec{r}_i} \right| a \right\rangle}{E_a - E_c - \hbar\omega_k} \right)^2 \delta(E_a - E_b + \hbar\omega_k - \hbar\omega_k')$$



- K-Kante: geringe Verstärkung
- L-Kante: Faktor 50-1000
- M-Kante: mehrere Größenordnungen bei Actiniden

resonant magnetic x-ray scattering

$$f(\vec{k}, \vec{k}', \omega) = f^{\text{charge}}(\vec{Q}) + f'(\vec{k}, \vec{k}', \omega) + if''(\vec{k}, \vec{k}', \omega) + f^{\text{spin}}(\vec{k}, \vec{k}', \omega)$$

abseits der Resonanzen

$$\hbar\omega \gg E_c - E_a$$

2. Ordnung

1. Ordnung

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)^2 = r_0^2 \left| \langle b | \sum_j e^{i\vec{k}\vec{r}_j} | a \rangle \hat{\epsilon} \cdot \hat{\epsilon}' - i \frac{\hbar\omega}{mc^2} \langle b | \sum_j e^{i\vec{k}\vec{r}_j} \left(\frac{i}{\hbar K} \vec{K} \times \vec{P}_j \cdot \vec{B}_L + \vec{s}_j \cdot \vec{B}_S \right) | a \rangle \right|^2 \delta(E_a - E_b + \hbar\omega_k - \hbar\omega_{k'})$$

$$\rightarrow \rho(K) \cdot B_\rho$$

$$\rightarrow -\frac{\hbar\omega}{mc^2} \left(\frac{1}{2} L(K) \cdot B_L + S(K) \cdot B_S \right)$$

polarisationsabhängige Faktoren

Neutronenstreuung

$$\left(\frac{d^2\sigma}{d\Omega dE}\right)^2 = (\gamma_0)^2 \frac{k'}{k} \left| \langle b | \sum_j e^{i\vec{k}\vec{r}_j} \left(-\frac{i}{\hbar k} \vec{K} \times \vec{P}_j + \vec{K} \times (\vec{s}_j \times \vec{K}) \right) | a \rangle \right|^2 \delta\left(E_a - E_b + \frac{\hbar^2 k^2}{2m_0} - \frac{\hbar^2 k'^2}{2m_0}\right)$$

$$\rightarrow \left(\frac{1}{2} L(K) + S(K) \right) \cdot (K \times (\sigma \times K))$$

resonant magnetic x-ray scattering

$$\langle f_M \rangle = -\frac{\hbar\omega}{mc^2} \begin{vmatrix} \sigma & \pi \\ (k \times k') \cdot S(Q) & \frac{Q^2}{2k^2} \left(\left(\frac{1}{2} L(Q) + S(Q) \right) \cdot k' + \frac{1}{2} L(Q) \cdot k \right) \\ \frac{Q^2}{2k^2} \left(\left(\frac{1}{2} L(Q) + S(Q) \right) \cdot k + \frac{1}{2} L(Q) \cdot k' \right) & \left(\frac{Q^2}{2k^2} L(Q) + S(Q) \right) \cdot (k \times k') \end{vmatrix} \begin{matrix} \sigma' \\ \pi' \end{matrix}$$

Linear polarisierter einfallender Strahl

$$\langle f_M \rangle = -i \frac{\lambda_C}{d} \begin{vmatrix} S_2 \cos \theta & [(L_1 + S_1) \cos \theta + S_3 \sin \theta] \sin \theta \\ -[(L_1 + S_1) \cos \theta + S_3 \sin \theta] \sin \theta & [2L_2 \sin^2 \theta + S_2] \cos \theta \end{vmatrix}$$

Ladungsstreuung

$$\langle f_c \rangle = \begin{vmatrix} 1 & 0 \\ 0 & \cos 2\theta \end{vmatrix}$$

Hohe Energien (kleine Winkel)

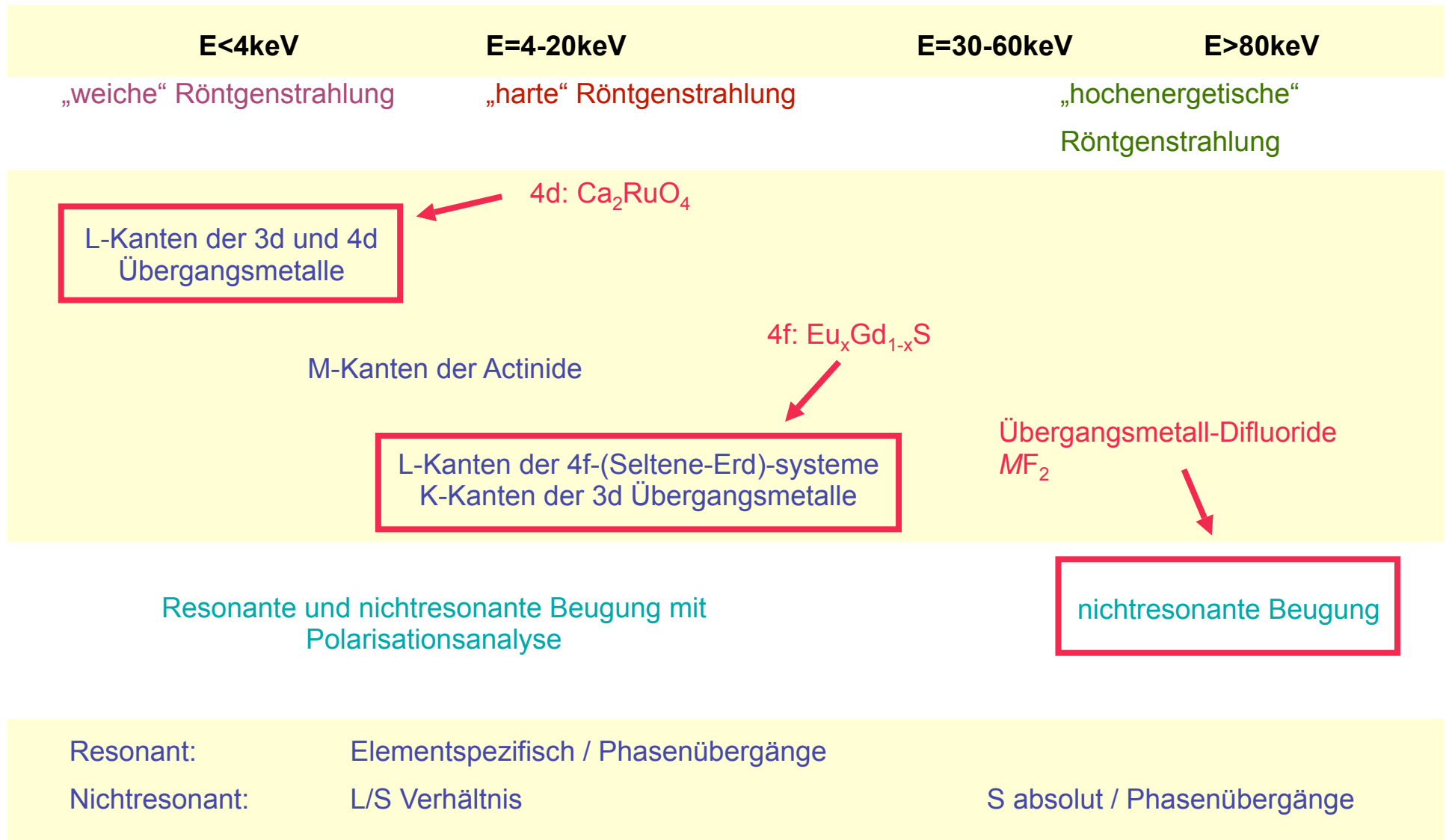
$$\langle f_M \rangle = -i \frac{\lambda_C}{d} \begin{vmatrix} S_2 & 0 \\ 0 & S_2 \end{vmatrix}$$

← polarisationsunabhängig

Magnetischer Streuquerschnitt (E > 80 keV)

$$\frac{d\sigma}{d\Omega} = r_0^2 \left(\frac{\lambda_C}{d} \right)^2 |S_2|^2$$

resonant magnetic x-ray scattering



Scattering scheme with polarization analysis

Non-resonant magnetic scattering amplitude [Blume & Gibbs]

$$f^{mag} = -i \frac{\hbar \omega}{mc^2} \begin{pmatrix} f^{\sigma\sigma'} & f^{\sigma\pi'} \\ f^{\pi\sigma'} & f^{\pi\pi'} \end{pmatrix}$$

$$= -i \frac{\hbar \omega}{mc^2} \begin{pmatrix} S_2 \sin 2\theta & -2 \sin^2 \theta [\cos \theta (L_1 + S_1) - S_3 \sin \theta] \\ 2 \sin^2 \theta [\cos \theta (L_1 + S_1) + S_3 \sin \theta] & \sin 2\theta [2L_2 \sin^2 \theta + S_2] \end{pmatrix}$$

Determination of L/S ratio
Magnetic structure determination

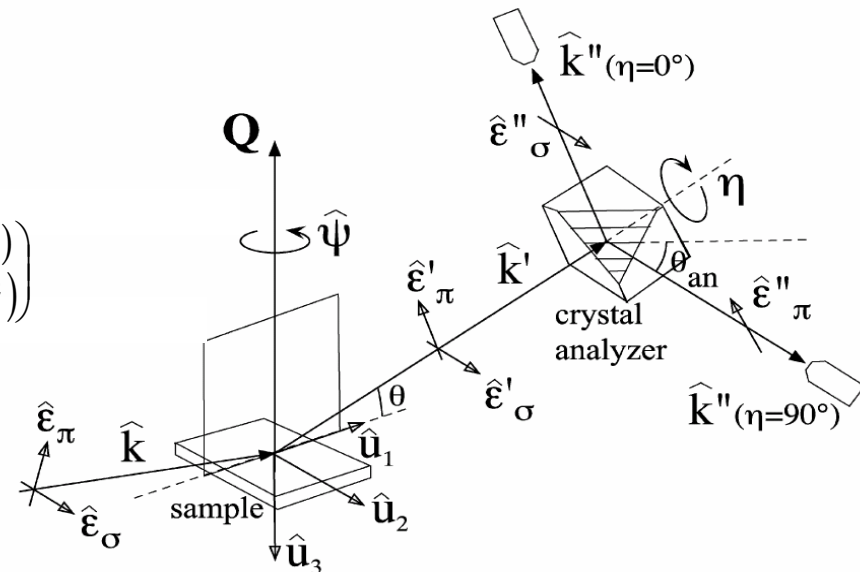
Resonant magnetic scattering amplitude (dipole transitions) [Hill & McMorow]

$$f_{E1}^{res-mag} = \begin{pmatrix} f^{\sigma\sigma'} & f^{\sigma\pi'} \\ f^{\pi\sigma'} & f^{\pi\pi'} \end{pmatrix}$$

$$= F^0 - iF^1 \begin{pmatrix} 0 & m_1 \cos \theta + m_3 \sin \theta \\ m_3 \sin \theta - m_1 \cos \theta & -m_2 \sin 2\theta \end{pmatrix}$$

$$+ F^2 \begin{pmatrix} m_2^2 & m_2(m_1 \sin \theta - m_3 \cos \theta) \\ m_2(m_1 \sin \theta + m_3 \cos \theta) & -\cos^2 \theta (m_1^2 \tan \theta + m_3^2) \end{pmatrix}$$

Strong intensities due to resonance enhancement
Element sensitivity at absorption edges
Magnetic structure determination



L/S determination

Non-resonant magnetic diffraction from KCuF_3
 (Ciaruffo et al., PRB 65, 174455 (2002))

Magnetic moment in basal-plane ($S_3=0$)

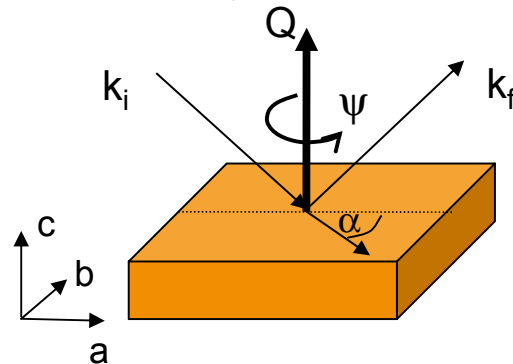
L and S collinear: $S_1 = S \cos \alpha$

$S_2 = S \sin \alpha$

Scattering amplitude:

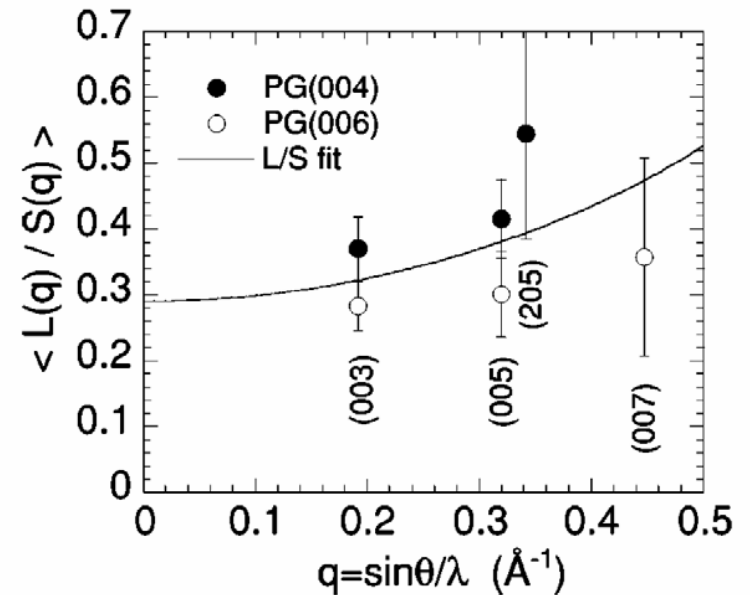
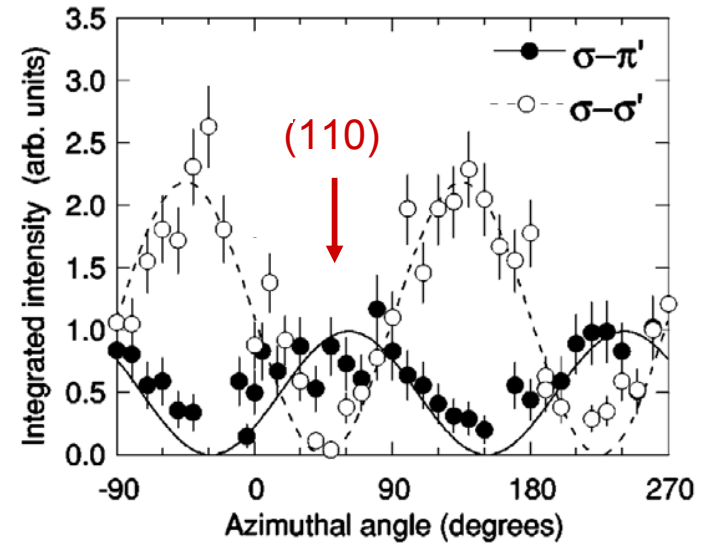
$$\begin{pmatrix} f_{\sigma\sigma'} \\ f_{\sigma\pi'} \end{pmatrix} = \begin{pmatrix} S \sin \alpha \sin 2\theta \\ -2(L+S) \cos \alpha \cos \theta \sin^2 \theta \end{pmatrix}$$

$$\frac{L(Q)}{S(Q)} = \frac{\tan \alpha}{\sin \theta} \sqrt{\frac{I_{\sigma\pi'}}{I_{\sigma\sigma'}}$$



Magnetic moment || (110)

$\rightarrow L/S=0.29(5)$



exercises

Is it possible to observe resonant scattering from orbital order (magnetic order) in LaMnO_3 (lattice parameter 5.4 Angstrom) at the Mn L-edge?

At which position of (h,k,l) can magnetic scattering and scattering from orbital order be measured in LaMnO_3 and $\text{La}_{0.5}\text{Ca}_{0.5}\text{MnO}_3$?

Explain the principle of a polarization analyzer.