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Formulas for synchrotron radiation from bending magnets and storage rings

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Radiated power of a particle with charge q:

$$P = \frac{c \cdot q^2 \cdot E^4}{6\pi \cdot \epsilon_0 \cdot E_0^4 \cdot r^2}$$

$$P[W] = 6.762567 \cdot 10^{-7} \frac{E[GeV]^4}{r[m]^2} \;\; for\ electrons$$

$(Example:\ P = 0.35mW;\ E = 50GeV;\ r = 110m)$

Radiated energy of an electron (per turn):

$$E_{ph} = \frac{e^2 \cdot E^4}{3\epsilon_0 \cdot E_0^4 \cdot r} = \frac{4\pi}{3} \cdot \frac{r_e}{E_0^3} \cdot \frac{E^4}{r}$$

$$E_{ph}[eV] = 88462.70 \cdot \frac{E[GeV]^4}{r[m]} \;\; for\ electrons$$

$(Example:\ E_{ph} = 2.977MeV;\ E = 4.5GeV;\ r = 12.1849m)$
$(Example:\ E_{ph} = 9.567MeV;\ E = 12GeV;\ r = 191.7295m)$
$(Example:\ E_{ph} = 5.03GeV;\ E = 50GeV;\ r = 110m)$

Radiated synchrotron radiation power in a storage ring:

$$P = \frac{E_{ph}}{e} \cdot I; \quad I = \frac{N_e \cdot e \cdot c}{U}$$

$$P = \frac{e}{3\epsilon_0 \cdot E_0^4} \cdot \frac{E^4}{r} \cdot I = \frac{4\pi \cdot r_e}{3e \cdot E_0^3} \cdot \frac{E^4}{r} \cdot I$$

$$P[W] = 88462.70 \cdot \frac{E[GeV]^4}{r[m]} \cdot I[A] \;\; for\ electrons$$

$(Example:\ P = 446.6kW;\ E = 4.5GeV;\ r = 12.1849m;\ I = 0.15A)$
$(Example:\ P = 574.0kW;\ E = 12GeV;\ r = 191.7295m;\ I = 0.06A)$

Radiated power of a bending magnet:

$$P = \frac{2 \cdot r_e}{3e \cdot E_0^3} \cdot L \cdot I \cdot \frac{E^4}{r^2} = \frac{2 \cdot c^2 \cdot r_e \cdot e}{3E_0^3} \cdot L \cdot I \cdot E^2 \cdot B^2$$

$$P[W] = 14079.28 \cdot L[m] \cdot I[A] \cdot \frac{E[GeV]^4}{r[m]^2}$$

$$= 1265.382 \cdot L[m] \cdot I[A] \cdot E[GeV]^2 \cdot B[T]^2 \;\; for\ electrons$$

$(Example: \;\; P = 18.61kW; \; L = 3.19m; \; I = 0.15A;$
$E = 4.5GeV; \; r = 12.1849m)$
$(Example: \;\; P = 2.563kW; \; L = 5.378m; \; I = 0.06A;$
$E = 12GeV; \; r = 191.7295m)$

Central power density of a bending magnet for $\epsilon = 0$:

$$P = \frac{1}{0.608} \cdot \frac{2 \cdot r_e}{3 \cdot \sqrt{2\pi} \cdot e \cdot E_0^4} \cdot \frac{E^5}{r} \cdot I$$

$$P[W/mrad^2] = 18.1 \cdot \frac{E[GeV]^5}{r[m]} \cdot I[A] \;\; for\ electrons$$

$(Example: \;\; P = 441W/mrad^2; \; E = 4.5GeV; \; r = 12.1849m;$
$I = 0.15A)$

Critical wavelength of synchrotron radiation from a bending magnet:

$$\lambda_c = \frac{4\pi}{3} \cdot \frac{E_0^3}{c \cdot B \cdot E^2} = \frac{4\pi}{3} \cdot \frac{r \cdot E_0^3}{E^3}$$

$$\lambda_c[nm] = \frac{1.864353}{B[T] \cdot E[GeV]^2} = 0.5589191 \cdot \frac{r[m]}{E[GeV]^3} \;\; for\ electrons$$

$(Example: \;\; \lambda_c = 0.0747nm; \; r = 12.1849m; \; E = 4.5GeV)$
$(Example: \;\; \lambda_c = 0.0620nm; \; r = 191.7295m; \; E = 12GeV)$
$(Example: \;\; \lambda_c = 0.01616nm; \; r = 608m; \; E = 27.6GeV)$

Critical energy of synchrotron radiation from a bending magnet:

$$E_c = \frac{3}{2} \cdot \frac{\hbar \cdot c}{E_0^3} \cdot \frac{E^3}{r} = \frac{2\pi \cdot \hbar \cdot c}{\lambda_c}$$

$$E_c = \frac{3}{2} \cdot \frac{\hbar \cdot c^2}{E_0^3} \cdot B \cdot E^2$$

$$E_c[eV] = 665.0255 \cdot B[T] \cdot E[GeV]^2 \ \ for\ electrons$$

$$E_c[eV] = 2218.286 \cdot \frac{E[GeV]^3}{r[m]} = \frac{1239.842}{\lambda_c[nm]} \ \ for\ electrons$$

$(Example:\ E_c = 16.589keV;\ E = 4.5GeV;\ r = 12.1849m)$
$(Example:\ E_c = 19.993keV;\ E = 12GeV;\ r = 191.7295m)$
$(Example:\ E_c = 76.708keV;\ E = 27.6GeV;\ r = 608m)$

Photon beam emittance for zero electron emittance:

$$\epsilon_{ph} = \frac{\lambda_{ph}}{4\pi}$$

$(Example:\ \epsilon_{ph} = 0.5nmrad;\ \lambda_{ph} = 6.28nm)$

Photon beam coherence angle:

$$\sigma'_{coh} = \frac{\lambda_{ph}}{4\pi \cdot \sigma_{total}}; \quad \sigma_{total} = \sqrt{\sigma_e^2 + \sigma_{ph}^2}$$

$(Example:\ \sigma'_{coh} = 362nrad;\ \lambda_{ph} = 0.15nm;\ \sigma_{total} = 33\mu m)$

Divergence of synchrotron radiation from a bending magnet (vertical plane):

$$\theta = 0.608 \cdot \frac{E_0}{E} = \frac{0.608}{\gamma}$$

$$\theta = \frac{E_0}{E} \cdot (\frac{\lambda}{\lambda_c})^{1/3}\ for\ \lambda >> \lambda_c; \quad \lambda > 507 \cdot \lambda_c$$

$$\theta = 0.565 \cdot \frac{E_0}{E} \cdot (\frac{\lambda}{\lambda_c})^{0.425}\ for\ \lambda > 0.750 \cdot \lambda_c$$

$$\theta = \frac{E_0}{E} \cdot (\frac{\lambda}{3\lambda_c})^{1/2}\ for\ \lambda < 0.750 \cdot \lambda_c$$

$(Example:\ \theta = 0.069mrad;\ E = 4.5GeV)$
$(Example:\ \theta = 0.514mrad;\ E = 4.5GeV;\ \lambda = 10nm;\ \lambda_c = 0.0747nm)$
$(Example:\ \theta = 2.209mrad;\ E = 4.5GeV;\ \lambda = 550nm;\ \lambda_c = 0.0747nm)$

Divergence of synchrotron radiation from a bending magnet (horizontal plane):

$$\theta = \frac{7}{12} \cdot \frac{0.608}{\gamma}$$

Spectral flux (bending magnet):

$$I[phot./(sec\,mrad\,0.1\%\,bandw.)] = 2.458{\cdot}10^{10}{\cdot}I_e[mA]{\cdot}E_e[GeV]{\cdot}\frac{E}{E_c}{\cdot}\int_{E/E_c}^{\infty} K_{5/3}(\eta)d\eta$$

$$\int_1^{\infty} K_{5/3}(\eta)d\eta = 0.6522$$

$(Example:\ I = 1.082 \cdot 10^{13} phot./(sec\,mrad\ 0.1\%\ bandw.);$
$I_e = 150mA;\ E_e = 4.5GeV;\ E = E_c)$

Spectral central brightness for $\epsilon = 0$ (bending magnet):

$$I[phot./(sec\,mrad^2\ 0.1\%\ bandw.)] = 1.325{\cdot}10^{10}{\cdot}I_e[mA]{\cdot}E_e[GeV]^2{\cdot}(\frac{E}{E_c})^2{\cdot}K_{2/3}(\frac{E}{2E_c})$$

$$K_{2/3}(\frac{1}{2}) \approx 1.45$$

$(Example:\ I \approx 5.8 \cdot 10^{13} phot./(sec\ mrad^2\ 0.1\%\ bandw.);$
$I_e = 150mA;\ E_e = 4.5GeV;\ E = E_c)$