Synchrotron Radiation Production and Properties

Part II: Insertion Devices

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Different quantities to describe photon intensity

Total Flux *F*
number of photons
per time and energy interval

$$[F_{tot}] = \frac{Number\ of\ photons}{s}$$

Spectral Flux
number of photons
per time, energy, and solid angle

$$[F] = \frac{Number\ of\ photons}{s \cdot 0.1\% BW}$$

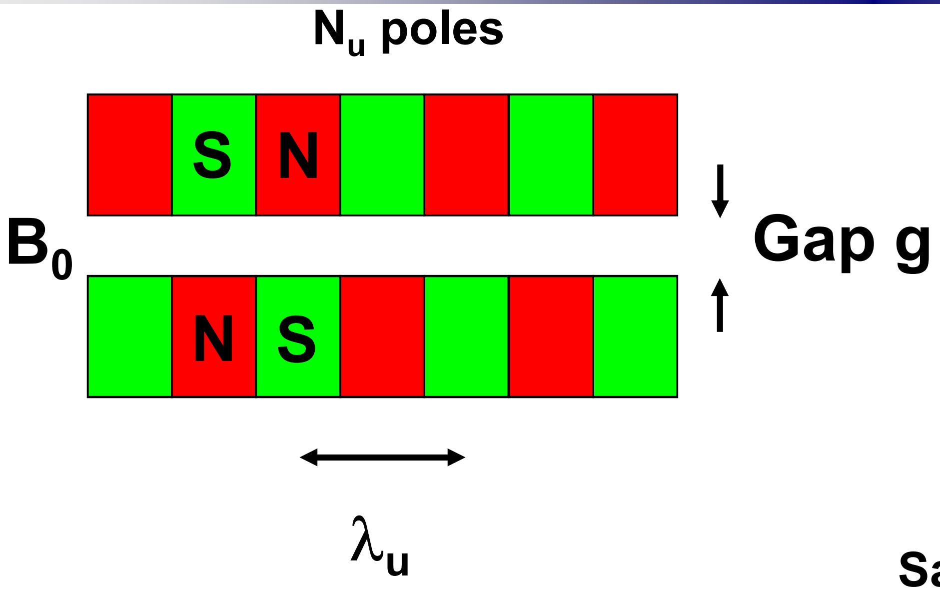
Brilliance *B*
number of photons
per time, energy, solid angle
and source area

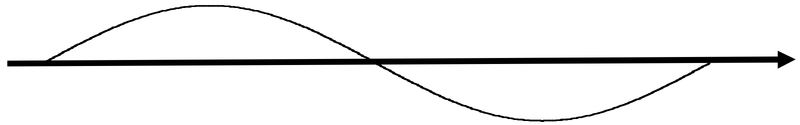
$$[B] = \frac{Number\ of\ photons}{s \cdot mm^2 \cdot mrad^2 \cdot 0.1\% BW}$$

Peak brilliance B^{peak}
brilliance scaled to total pulse duration

$$B^{peak} = \frac{B}{\tau \times f}$$

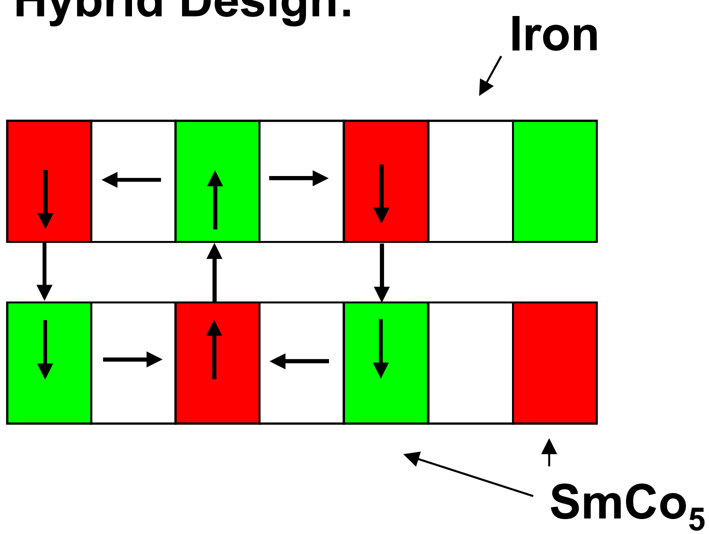
τ - pulse duration
f - pulse frequency





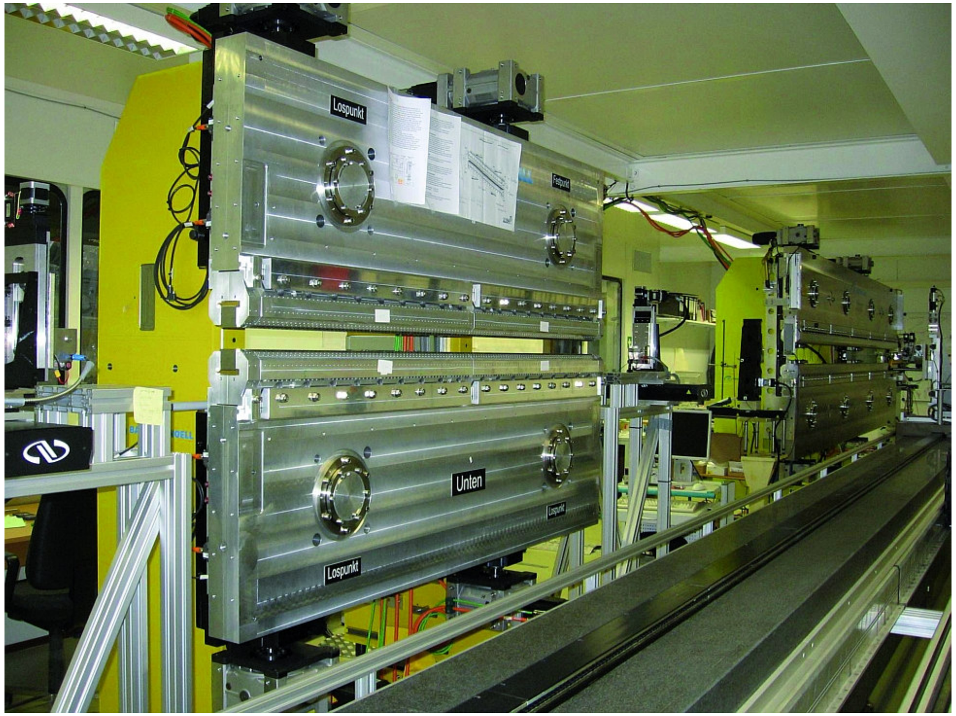
Samarium-Cobalt, $B_{remanent}$ = 0.9 T

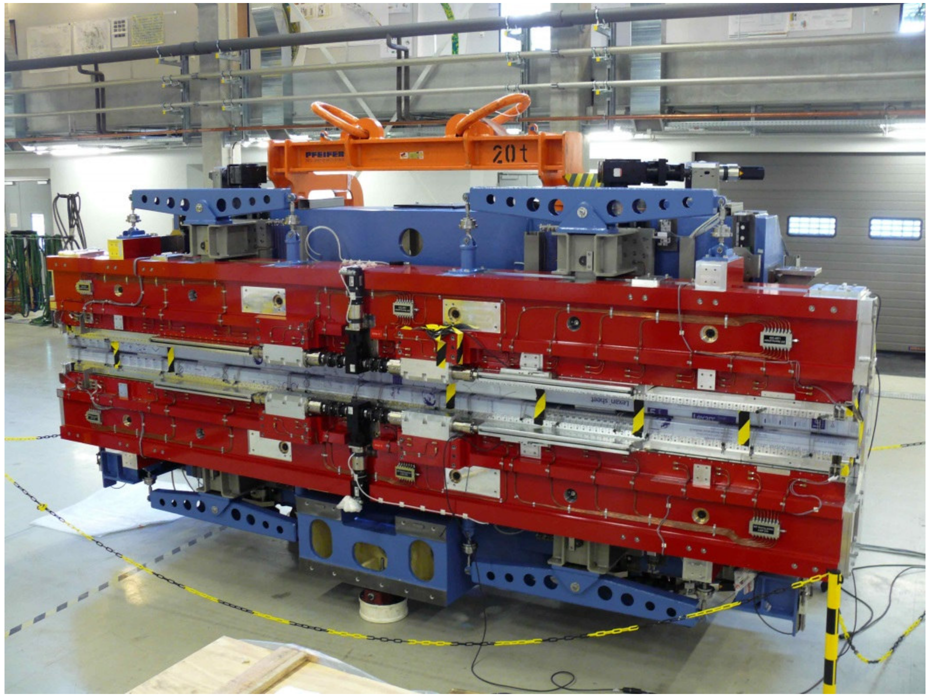
Hybrid Design:



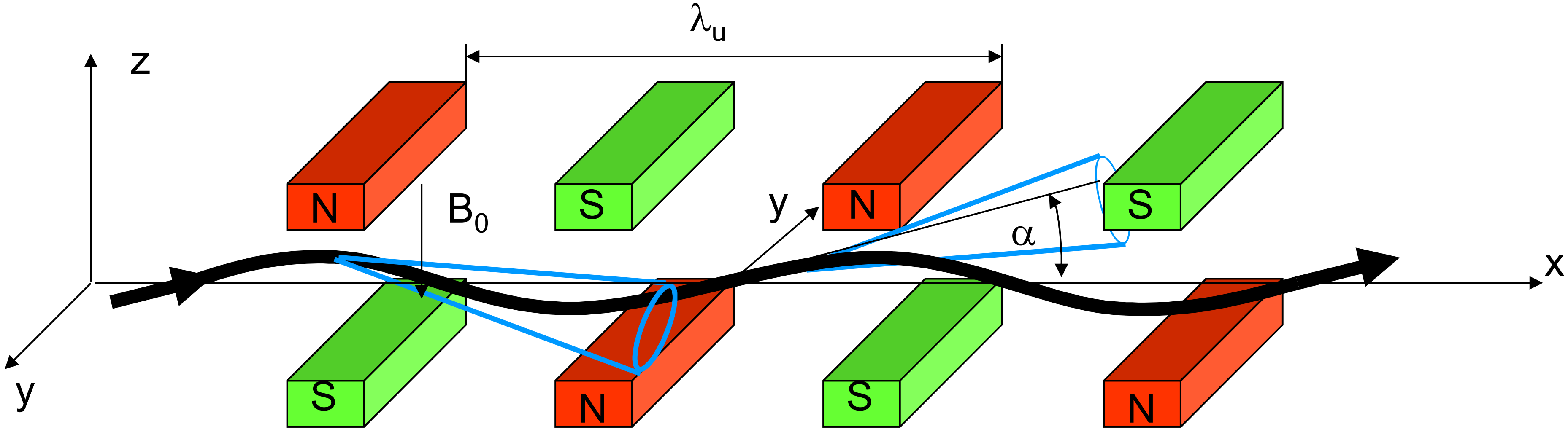
$$B_0[T] = 3.33 \cdot \exp\left\{ -\frac{g}{\lambda_u}\left(5.47 - 1.8\frac{g}{\lambda_u} \right) \right\}$$

Undulators for PETRA III





Equation of motion in a wiggler/undulator



Equation of motion:

$$\vec{F} = e\vec{v} \times \vec{B} = \frac{d\vec{p}}{dt} = m_0\gamma \frac{d\vec{v}}{dt}$$ with $$\vec{v} = \begin{pmatrix} v_y \\ 0 \\ v_x \end{pmatrix}$$ and $$\vec{B} = \begin{pmatrix} 0 \\ B_z \\ B_x \end{pmatrix}$$

$$\frac{d\vec{v}}{dt} = \frac{e}{m_0\gamma} \begin{pmatrix} -\dot{x}B_z \\ -\dot{y}B_x \\ \dot{y}B_z \end{pmatrix} = \begin{pmatrix} \ddot{y} \\ \ddot{z} \\ \ddot{x} \end{pmatrix}$$

$$\dot{y} << c$$
$$\dot{x} = \beta c = const$$
$$\dot{z} \approx 0$$

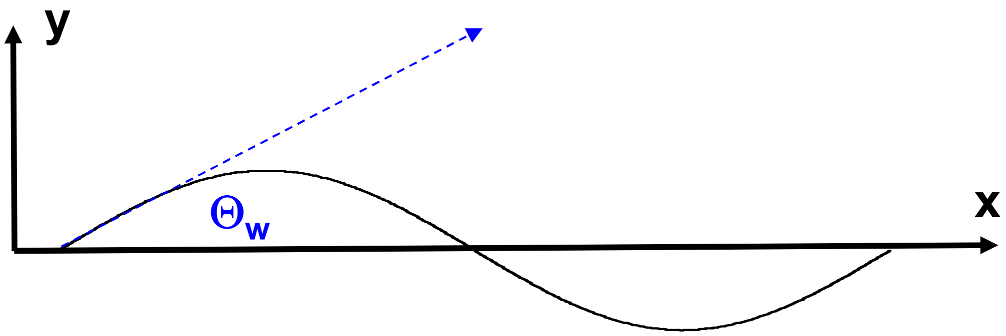
$$\ddot{y} = \frac{-\beta c e B_0}{m_0\gamma} \cos\left(\frac{2\pi\, x}{\lambda_u}\right)$$

Integration after changing from temporal to spatial derivation:

$$\ddot{y} = \frac{-\beta c e B_0}{m_0 \gamma} \cos\left(\frac{2\pi\, x}{\lambda_u}\right) \quad \textit{with} \quad \dot{y} = \frac{dy}{dx}\frac{dx}{dt} = y'\beta c \quad \textit{and} \quad \ddot{y} = y''\beta^2 c^2$$

$$\textit{and} \quad \beta \approx 1$$

$$y(x) = \frac{\lambda_u^2 e B_0}{4\pi^2 m_0 \gamma\, c} \cos\left(\frac{2\pi\, x}{\lambda_u}\right)$$

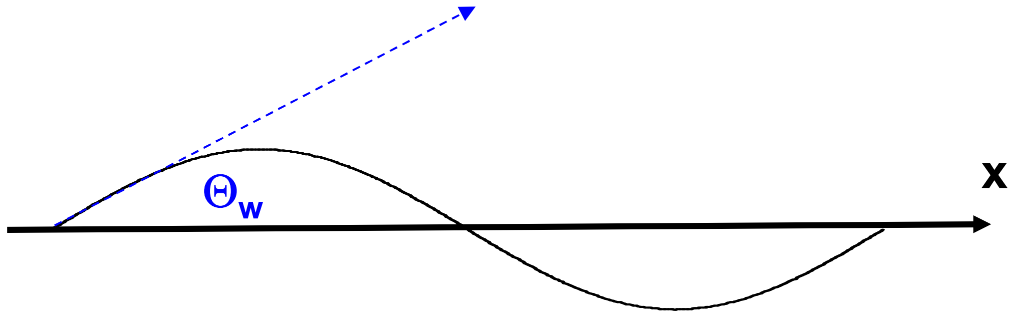


$$\Theta_w = y'_{\max} = \frac{1}{\gamma} \cdot \frac{\lambda_u e B_0}{2\pi\, m_0 c^2}$$

↑

K

Difference between wigglers and undulators



$$\theta_w = \frac{K}{\gamma} \quad \text{with} \quad K = \frac{\lambda_u e B}{2\pi m_e c}$$

(K-parameter)

K = 1 → Θ_w = radiation cone opening

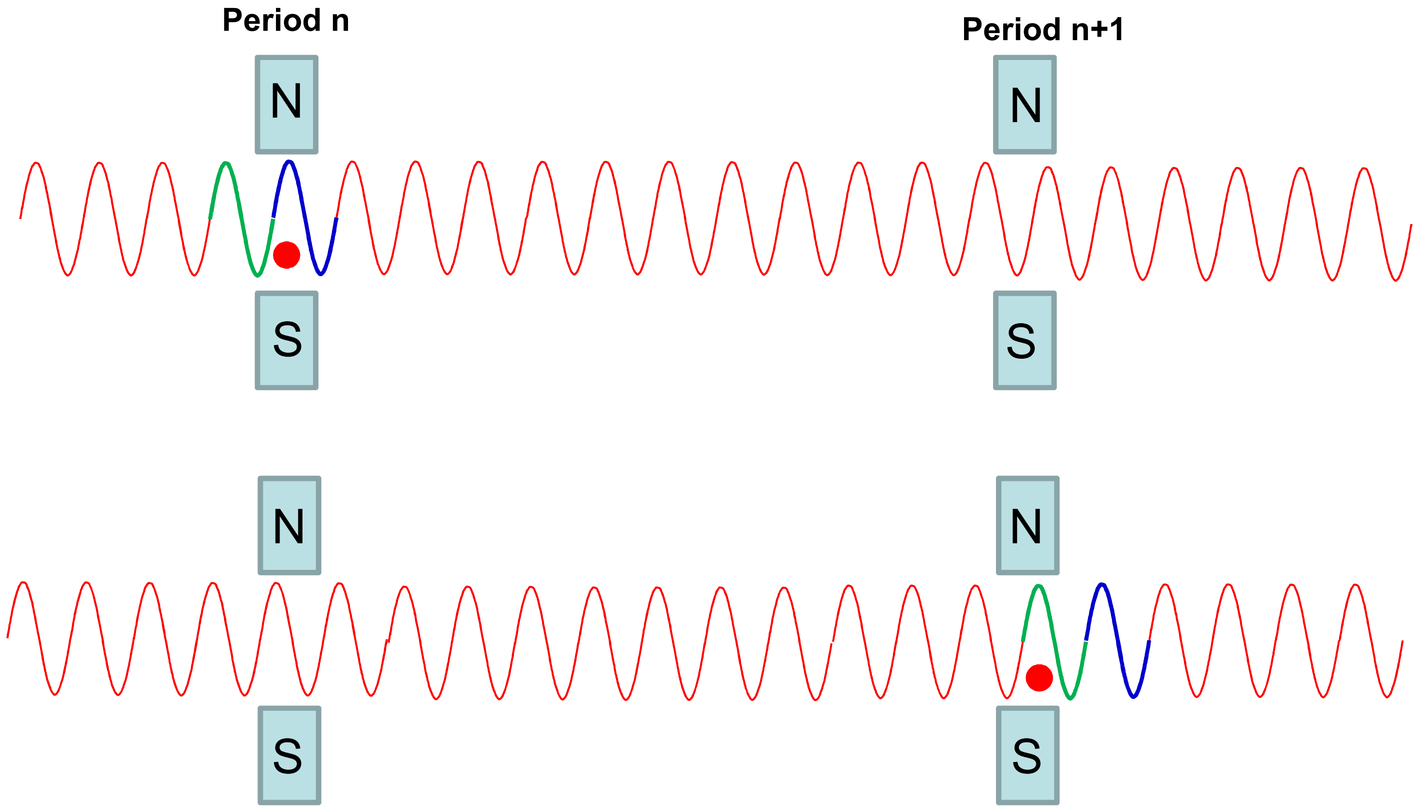
K > 1 → Wiggler

K ≤ 1 → Undulator

Wigglers

Incoherent superposition of bending magnet radiation.
Total intensity increases proportional to number of poles (Flux $\propto$ N)

Undulator Interference Condition



Undulators

For special wavelength: Coherent superposition of amplitudes

$A_{tot} = N \cdot A \rightarrow I_{tot} = N^2 \cdot I$

Total intensity increases with number of poles squared (Flux $\propto N^2$)

Particle velocity along Undulator/Wiggler Axis

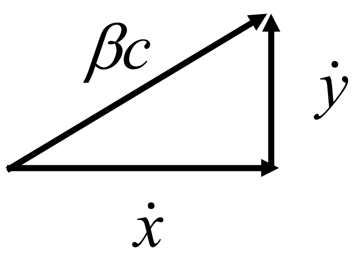
$$y'(x) = \frac{K}{\gamma} \sin\left(\frac{2\pi x}{\lambda_u}\right)$$

$$\dot{y} = y'\beta c \quad and \quad x = \beta c t \quad and \quad \omega_u = \frac{2\pi}{\lambda_u} \cdot \beta c \quad leads\ to$$

$$\dot{y}(t) = \beta c \frac{K}{\gamma} \sin(\omega_u t)$$

$$\dot{x}(t) = \langle \dot{x} \rangle + \Delta\dot{x}(t)$$

$$\Delta\dot{x}(t) = \frac{c\beta^2 K^2}{4\gamma^2} \cos(2\omega_u t)$$



$$\dot{x}^2 = (\beta c)^2 - \dot{y}^2$$

Mean velocity $$\langle \dot{x} \rangle = c\left(1 - \frac{1}{2\gamma^2}\left[1 + \frac{\beta^2 K^2}{2}\right]\right) \equiv \beta^* c$$

Frequency in the Laboratory frame:

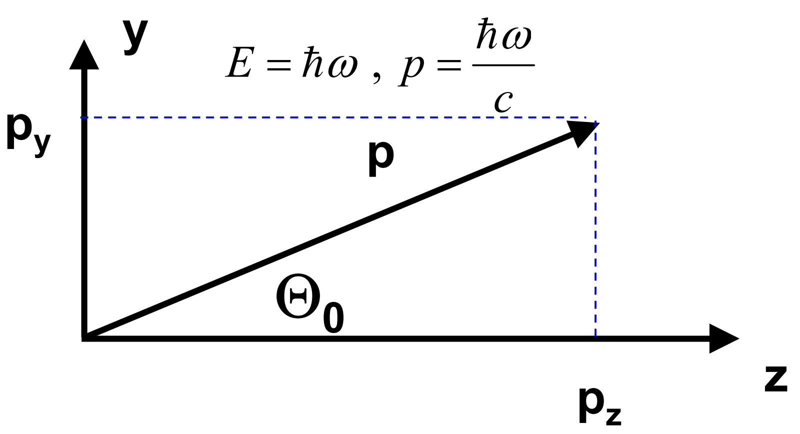
$$\omega_u = \frac{2\pi\beta^* c}{\lambda_u}$$

Frequency in the electron-frame:
(moving with $\beta^* c$)

$$\omega_u^* = \gamma\omega_u \qquad \left(\lambda_u \rightarrow \frac{\lambda_u}{\gamma} \right)$$

Photon in the Laboratory frame:

$$E = \hbar\omega \; , \; p = \frac{\hbar\omega}{c}$$



Transformation to electron-frame:

electron | laboratory

$$\begin{pmatrix} p_x^* \\ p_y^* \\ p_z^* \\ E^*/c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -\beta^*\gamma \\ 0 & 0 & -\beta^*\gamma & \gamma \end{pmatrix} \cdot \begin{pmatrix} 0 \\ p\sin\Theta_0 \\ p\cos\Theta_0 \\ E/c \end{pmatrix}$$

$$\frac{E^*}{c} = \gamma \frac{E}{c}\left(1 - \beta^* \cdot \boxed{\frac{p \cdot c}{E}} \cdot \cos\Theta_0\right)$$

$\frac{p \cdot c}{E}$ =1

$$E = \hbar\omega$$

$$E^* = \hbar\omega_u^* = \hbar\gamma\omega_u$$

$$\frac{\omega}{\omega_u} = \frac{\lambda_u}{\lambda} = \frac{1}{1-\beta^*\cos\Theta_0}$$

Coherence relation for undulator radiation

Insert Expression for mean velocity β*c

$$\langle \dot{x} \rangle = c\left(1 - \frac{1}{2\gamma^2}\left[1 + \frac{\beta^2 K^2}{2}\right]\right)$$

$$\textit{with} \quad \beta = 1 \quad \textit{and} \quad \cos\Theta_0 \approx 1 - \frac{\Theta_0^2}{2} \quad \left(\textit{because} \quad \Theta_0 \approx \frac{1}{\gamma} << 1\right)$$

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\Theta_0^2\right)$$

Note: $\gamma = \dfrac{E}{m_e c^2} = 1957 \cdot E[GeV]$

Spectral width of an undulator

Number of undulator periods = N_u →

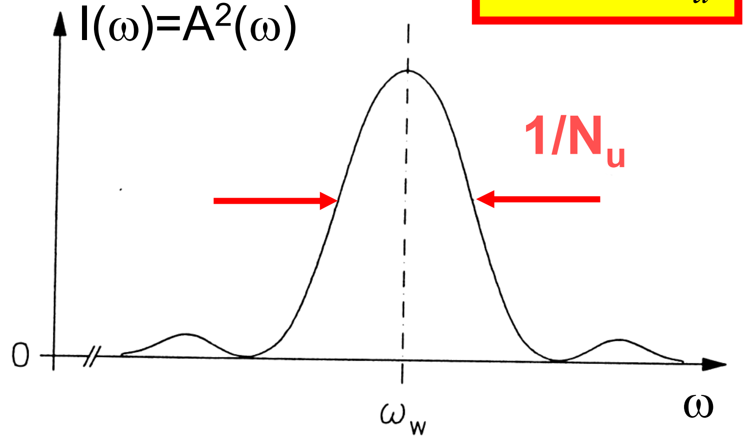
Wavetrain $u(t) = a \cdot \exp(i\omega_u t)$ **with N_u oscillations, duration:** $T = \frac{N_u \lambda_u}{c}$

Fourier Decomposition of this finite wavetrain:

$$A(\omega) = \frac{a}{\sqrt{2\pi T}} \int_{-T/2}^{T/2} \exp(-i\omega t) \cdot \exp(i\omega_u t) \cdot dt =$$

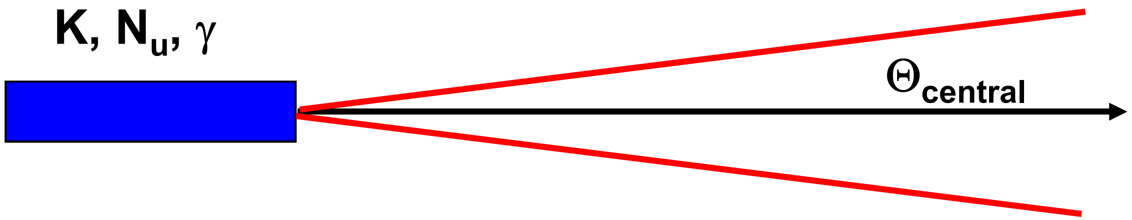
$$= \frac{a}{\sqrt{2\pi}} \cdot \frac{\sin\left(\pi N_u \frac{\omega - \omega_u}{\omega_u}\right)}{\pi N_u \frac{\omega - \omega_u}{\omega_u}}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{N_u}$$



Opening of central radiation cone (angular spread):

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\Theta_0^2\right)$$ and $$\frac{\Delta\lambda}{\lambda} = \frac{1}{N_u}$$

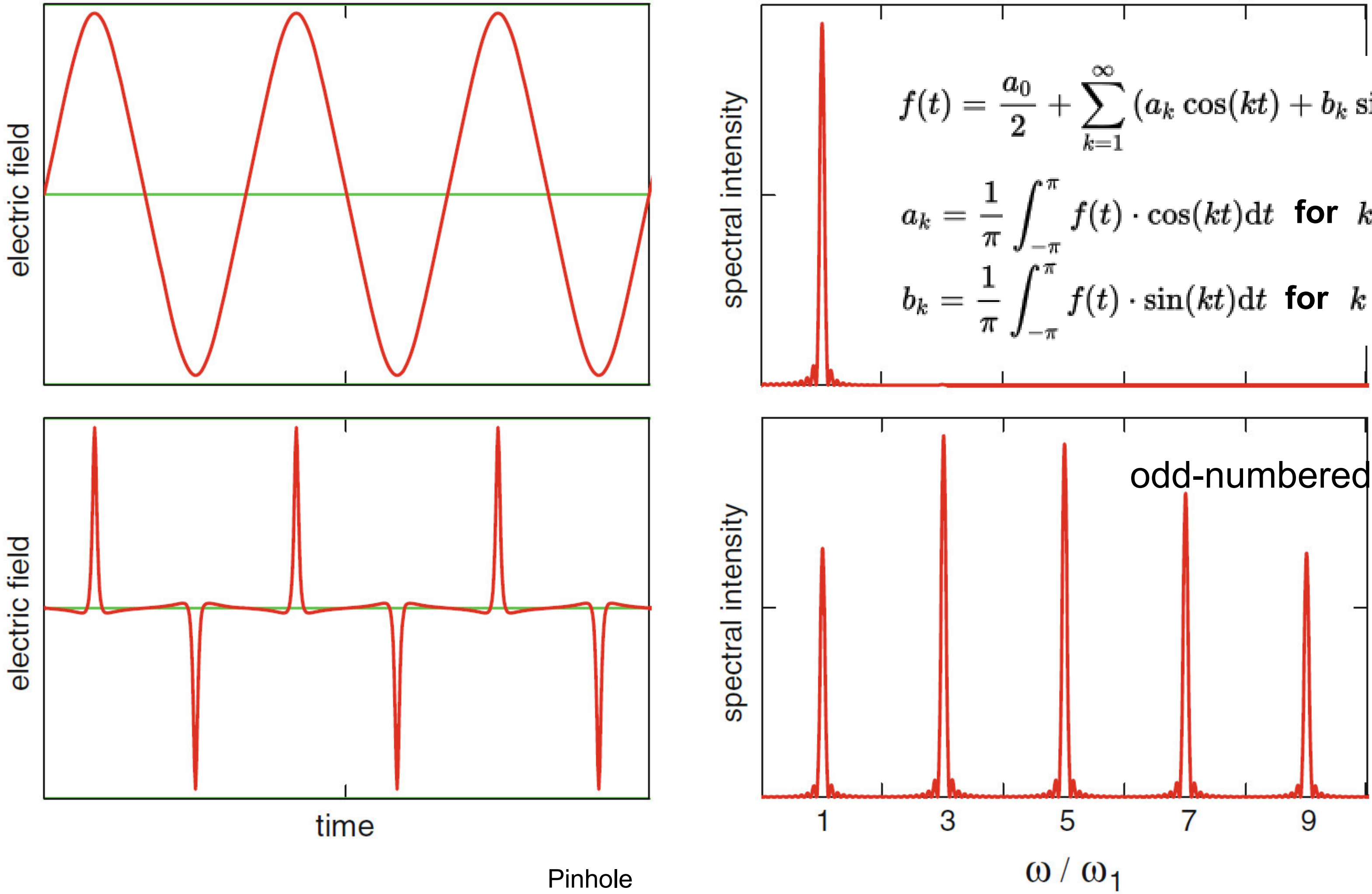


Caused by the fact that energy spread cannot exceed the natural bandwith

$$\theta_{central} = \frac{\sqrt{1 + \frac{K^2}{2}}}{\gamma\sqrt{2N_u}}$$

Higher Harmonics

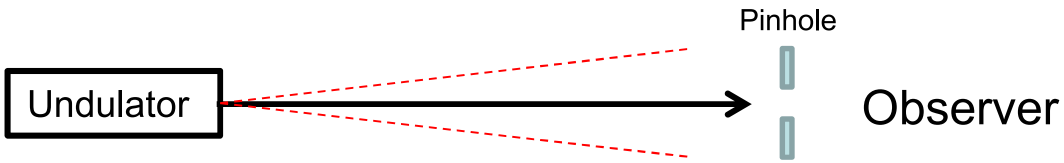
Observed E-field — Corresponding spectrum



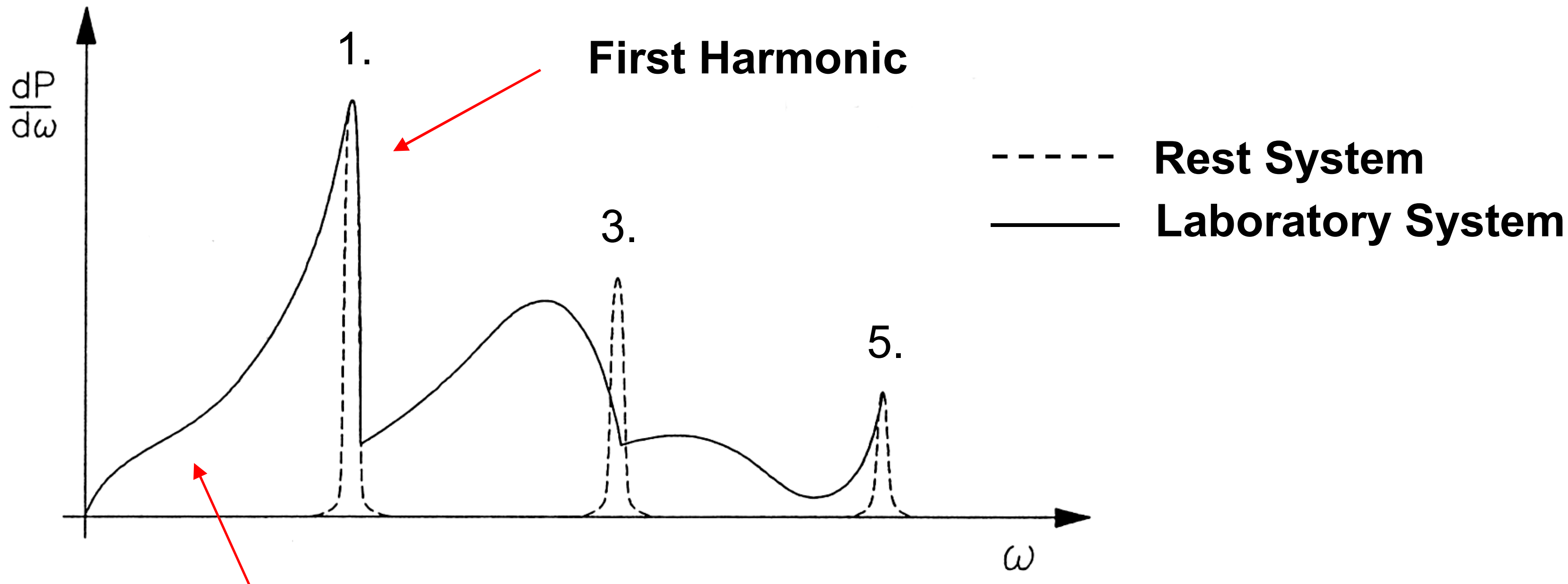
$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(kt) + b_k \sin(kt)\right)$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \cdot \cos(kt)\mathrm{d}t \quad \textbf{for} \quad k \geq 0$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t) \cdot \sin(kt)\mathrm{d}t \quad \textbf{for} \quad k \geq 1$$



For zero emittance

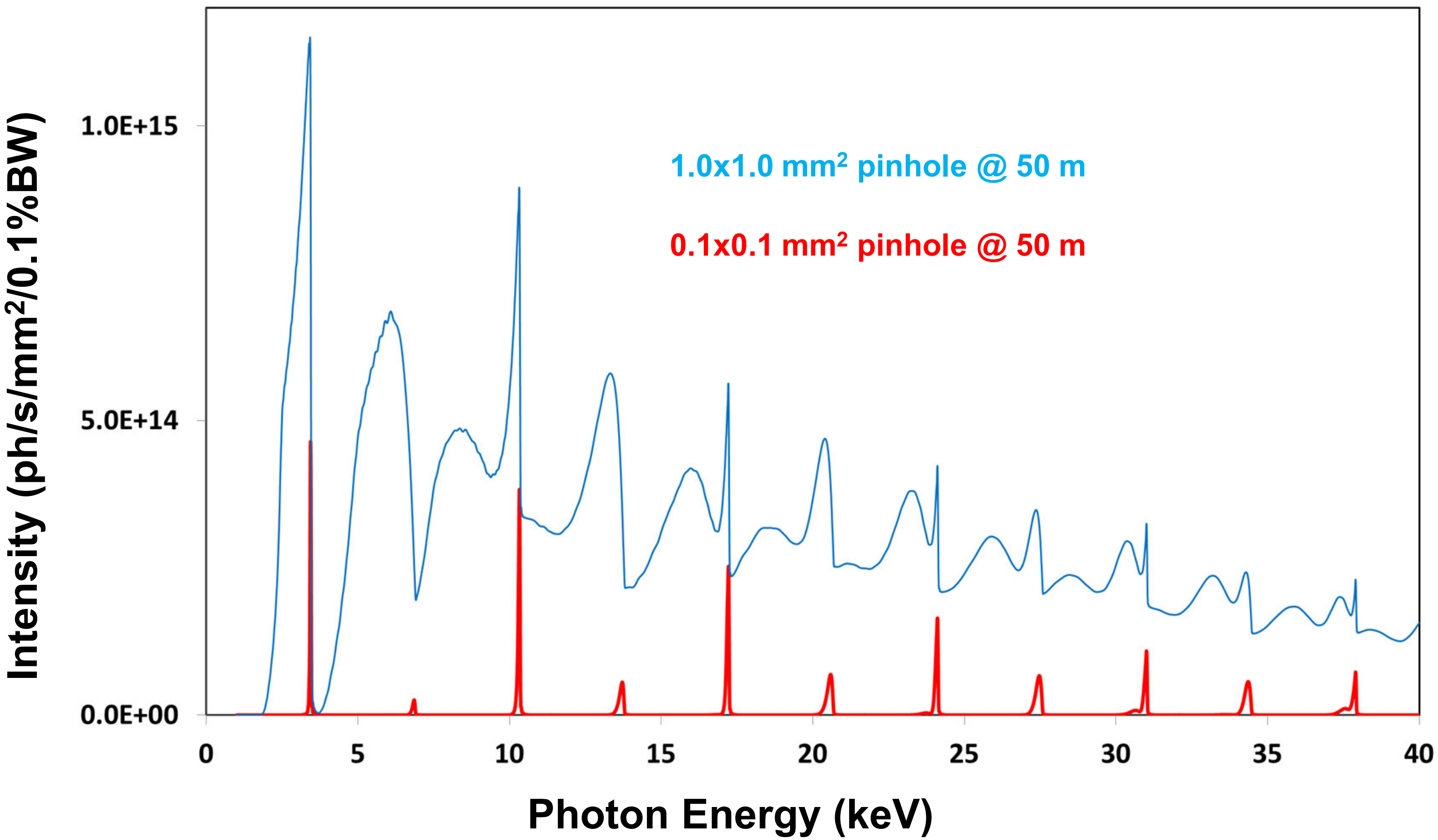


Deviation from sinoidal path ➔ higher harmonics

Line Broadening due to relativistic Doppler Effect for Photons emitted with off-axis direction

Total photon flux in harmonic peak $\propto N_u^2$

Undulator Spectrum Through Pinhole



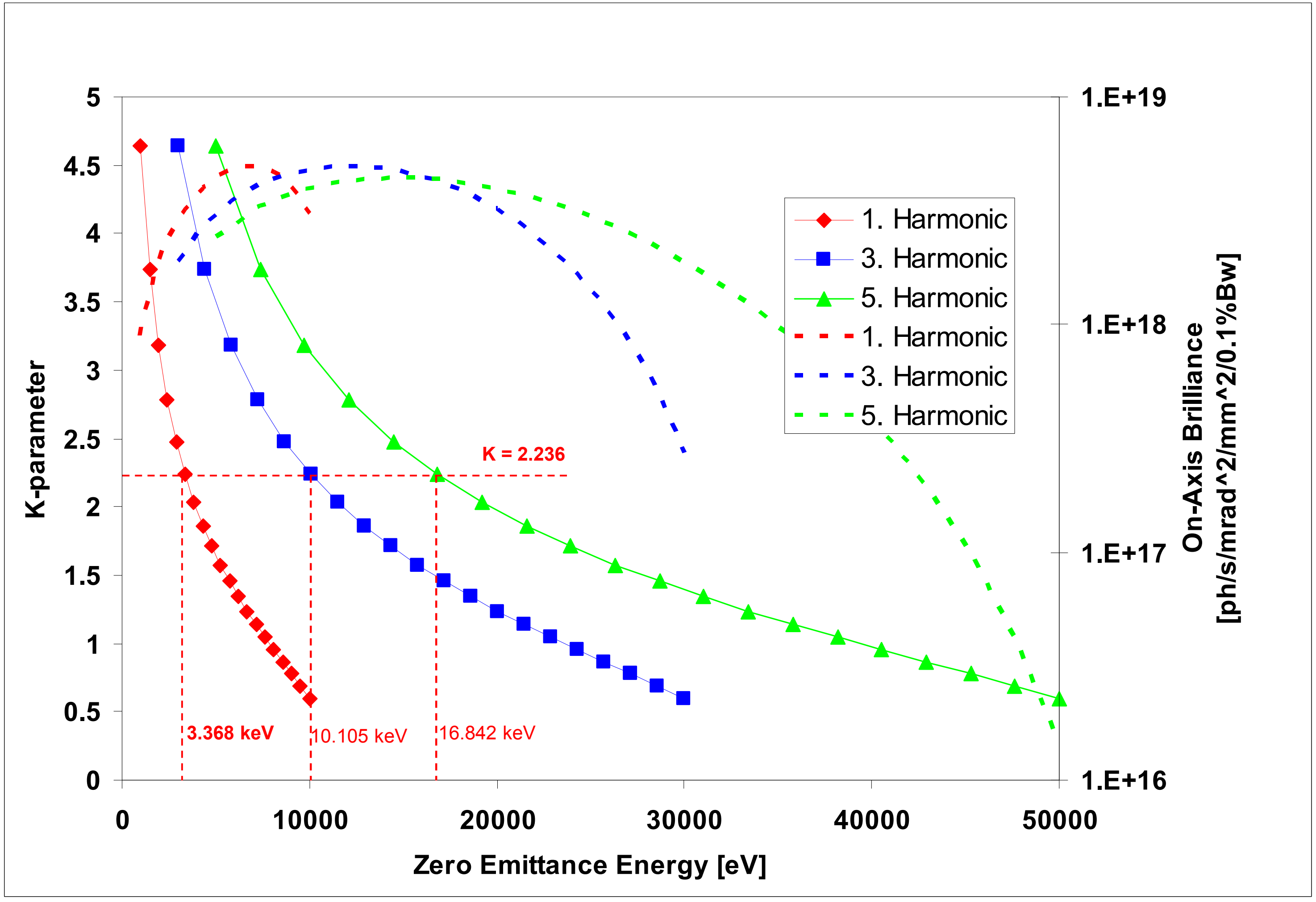
Machine: E = 6.0 GeV, I = 100 mA

e-Beam: σ_x = 0.141mm, σ_y = 0.005 mm, σ'_x = 0.0071 mrad, σ'_y = 0.0018 mrad

Undulator: λ_u = 2.9 cm, N = 100, K = 2.2

Undulator Tuning

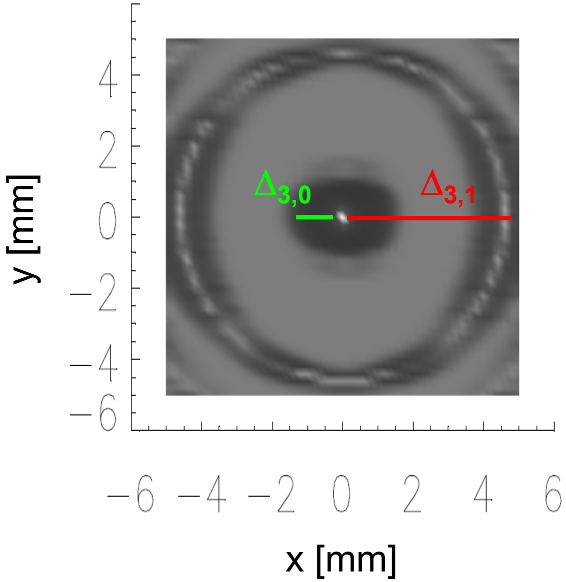
$$K = \sqrt{2 \cdot \left(\frac{0.95 \cdot n \cdot E^2[GeV]}{\lambda_u[cm] \cdot E_n[keV]} - 1 \right)}$$

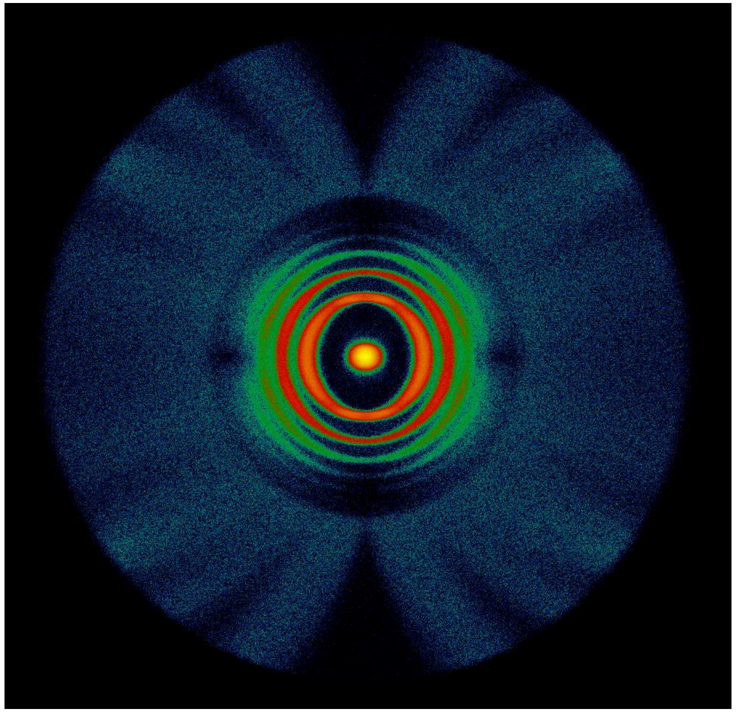


Undulator Central Cone

γ = 11741.708 (E = 6 GeV)
I = 0.1 A
K = 2.236
n = 3
N = 17
→
$\theta_{3,0}$ = 15.7 μrad
$\theta_{3,1}$ = 92.0 μrad

→ 50 m distance
$\Delta_{3,0}$ = 0.8 mm
$\Delta_{3,1}$ = 4.6 mm

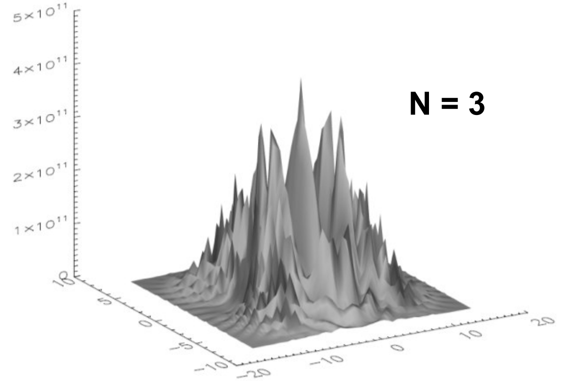


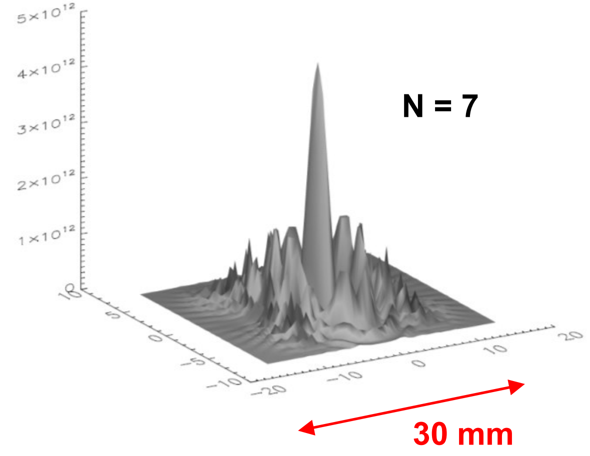


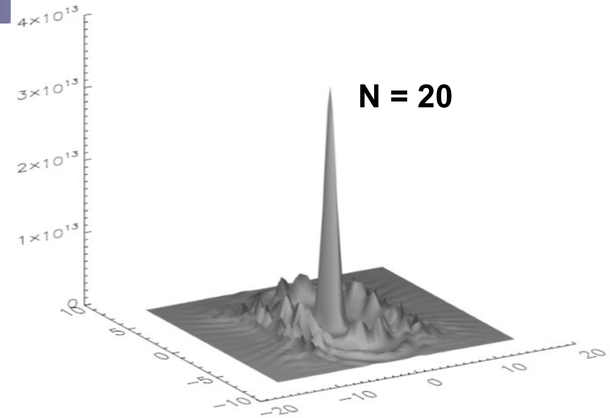
$$\theta_{n,0}(K)=\frac{1}{\gamma}\sqrt{\frac{1+\frac{K^2}{2}}{2\cdot N\cdot n}}$$

$$\theta_{n,m}(K)=\frac{1}{\gamma}\sqrt{\frac{1}{n}\left(m+\frac{K^2}{2}\right)}$$

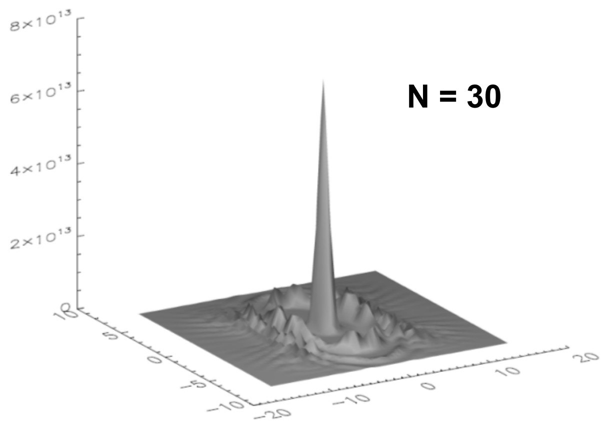
Dependence of Central Cone on N

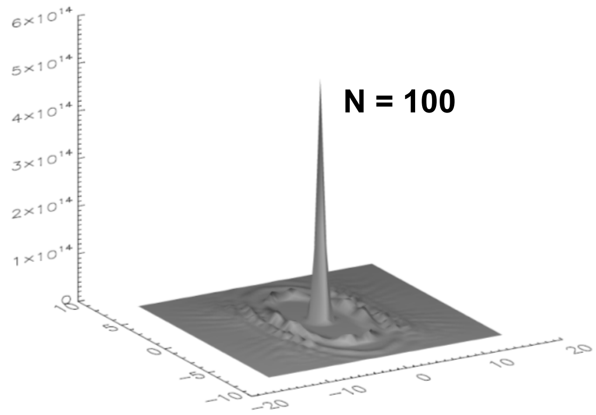






$$\theta_{n,0}(K) = \frac{1}{\gamma} \cdot \sqrt{\frac{1+\frac{K^2}{2}}{2 \cdot N \cdot n}}$$





$$\theta_{n,m}(K) = \frac{1}{\gamma} \cdot \sqrt{\frac{1}{n}\left(m + \frac{K^2}{2}\right)}$$

γ = 11741.708

Spatial flux density (ph/s/mm^2/0.1%Bw) of radiation from structure tuned to K = 2.236
at the corresponding third harmonic zero emittance energy E_0=10105.263 eV 50 m behind source

λ_u = 2.9 cm

Summary: Insertion Devices

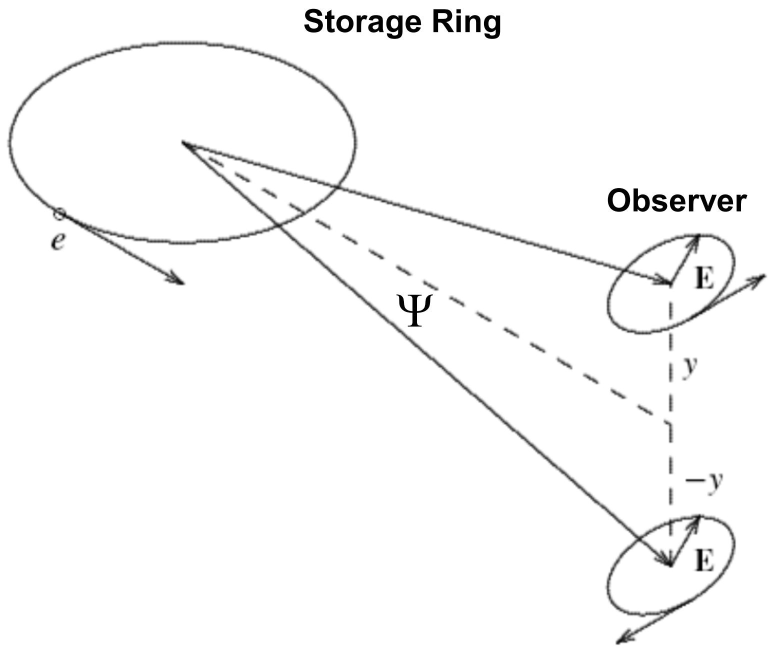
1. Characterized by K-parameter $$K = \frac{\lambda_u eB}{2\pi m_e c}$$ K < 1 →Undulator

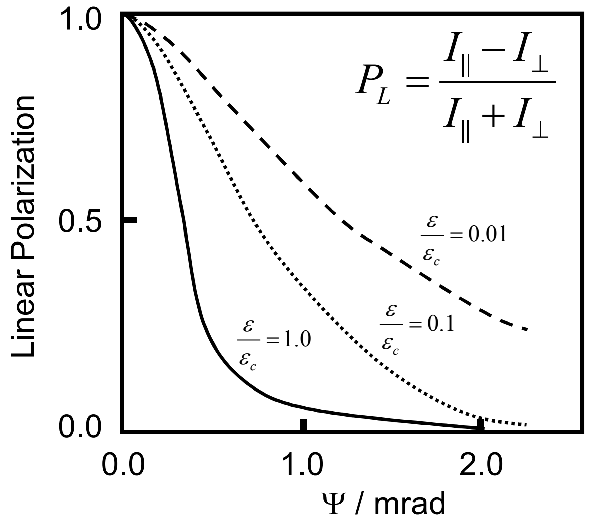
2. Coherence relation $$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\Theta_0^2\right)$$

3. Bandwidth $$\frac{\Delta\lambda}{\lambda} = \frac{1}{N_u}$$

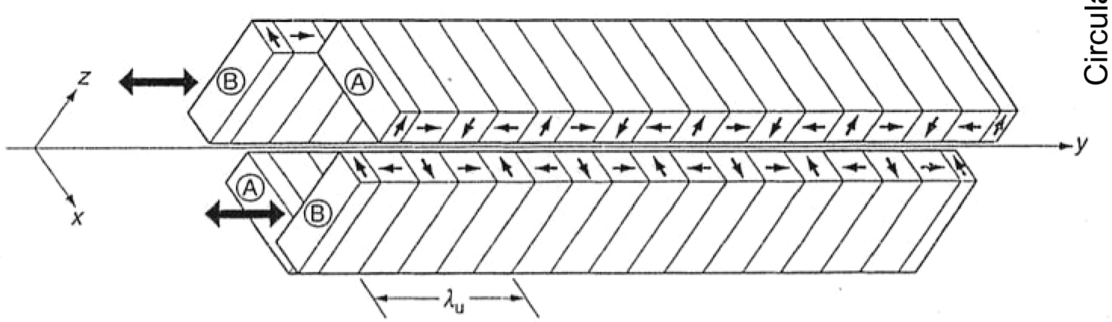
4. Central cone opening $$\theta_{central} = \frac{\sqrt{1+\frac{K^2}{2}}}{\gamma\sqrt{2N_u}}$$

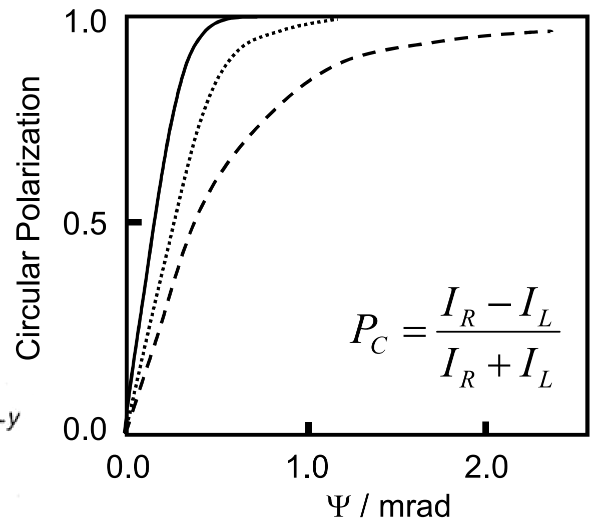
Polarization



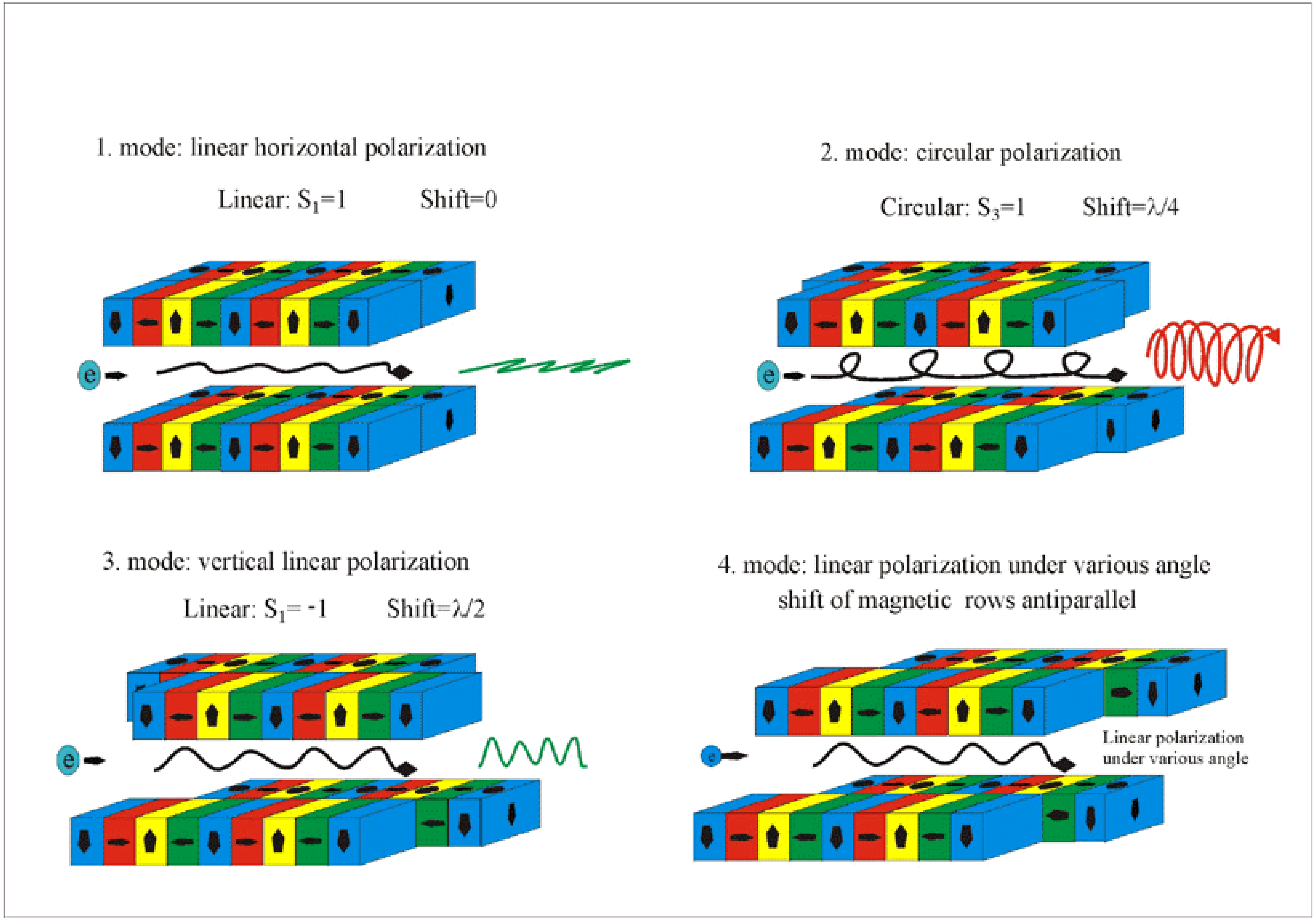


Helical Undulator

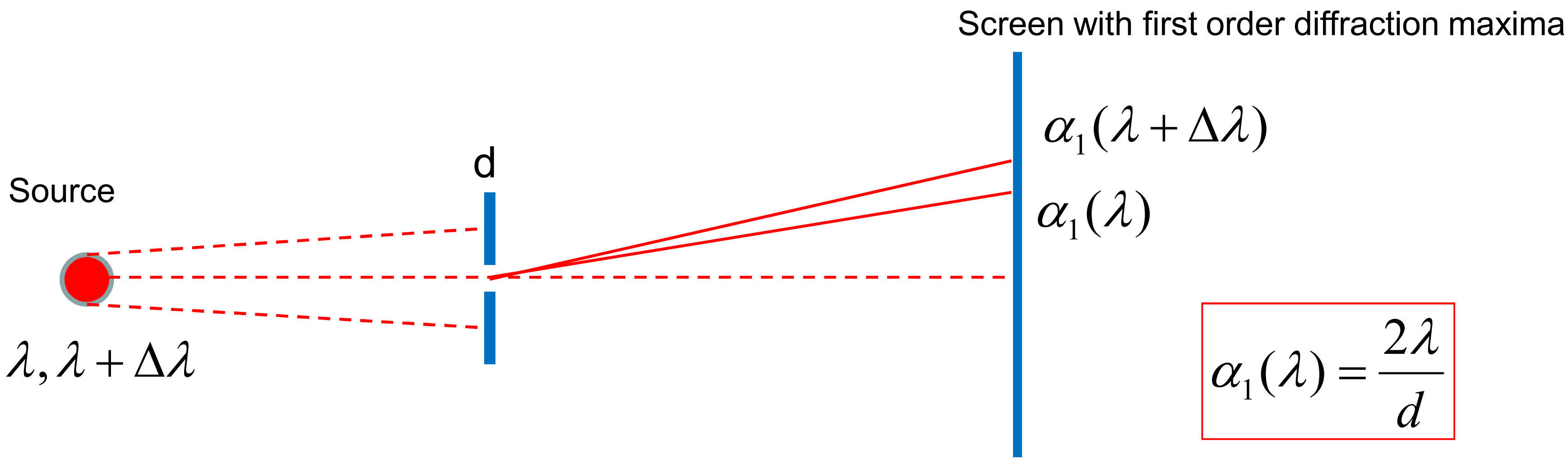




Advanced Polarized Photon Light Emitter (*APPLE*)



Longitudinal (temporal) coherence



$$\alpha_1(\lambda) = \frac{2\lambda}{d}$$

Rough criterion for visibility of pattern

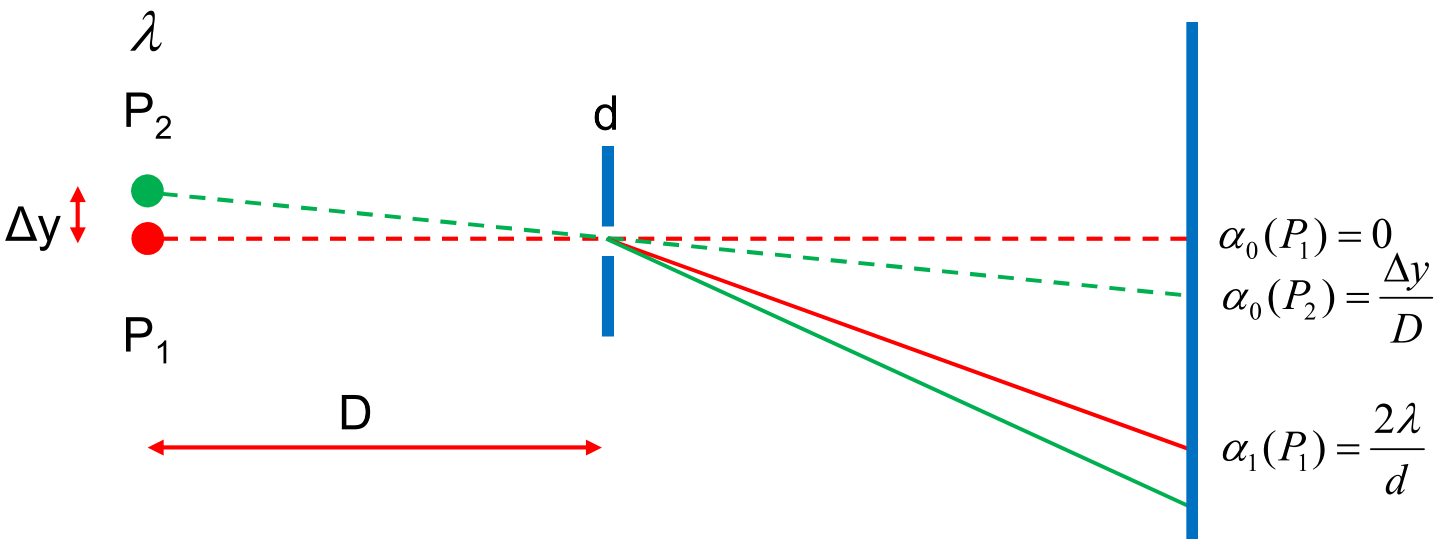
$$\frac{\alpha_1(\lambda+\Delta\lambda)-\alpha_1(\lambda)}{\alpha_1(\lambda)} < 1$$

➔

$$\frac{\Delta\lambda}{\lambda} < 1$$

Good longitudinal coherence needs monochromatisation

Lateral coherence



Rough criterion for visibility of pattern $\alpha_0(P_2) < \alpha_1(P_1)$

➔ $\Delta y \cdot \frac{d}{D} < 2\lambda$ ➔ $\Delta y \cdot \Delta\theta_y < 2\lambda$

Pinhole illumination angle

Definition of „Coherent Power“
(Fraction of coherent light)

$$\frac{2\lambda}{\Delta y \Delta\theta_y} \cdot \frac{2\lambda}{\Delta z \Delta\theta_z} = \frac{4\lambda^2}{(\Delta y \Delta\theta_y)(\Delta z \Delta\theta_z)}$$

Denominator: Brilliance !!

Diffraction Limit

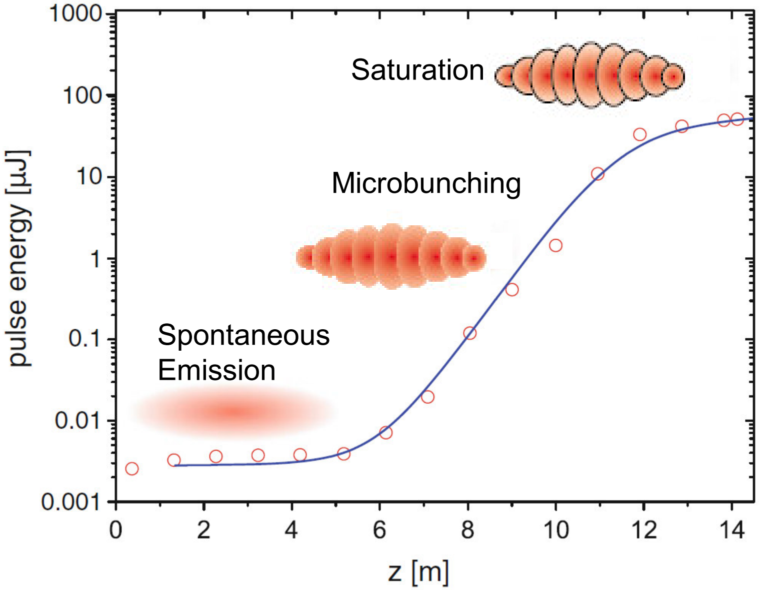
Diffraction at a pinhole leads to $\Delta\theta_y \approx \Delta\theta_z \approx \frac{2\lambda}{d}$

➔ Minimum possible brilliance is given by

$$\Delta y \cdot \Delta\theta \approx 2\lambda$$

Note: With this brilliance the lateral coherence becomes 100%

High-Gain SASE Free-Electron Lasers



Self **A**mplified **S**pontaneous **E**mission (SASE)

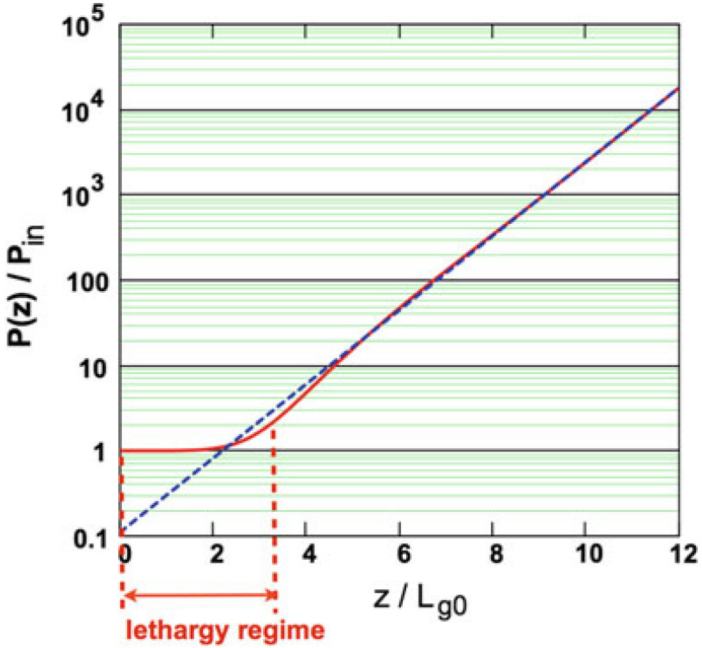
$$P(z) = A \cdot P_{in} \cdot \exp\left(\frac{2z}{L_g}\right)$$

$A = 1/9$ Input coupling factor

P_{in} Effective input power

$L_g \cong \frac{\lambda_u}{4\pi\rho}$ Field gain length

ρ: FEL parameter (typical 10^{-4} – 10^{-5})



red curve: analytic solution

blue line: approximation

$$P(z) \cong \frac{P_{in}}{9}\exp(z/L_{g0})$$

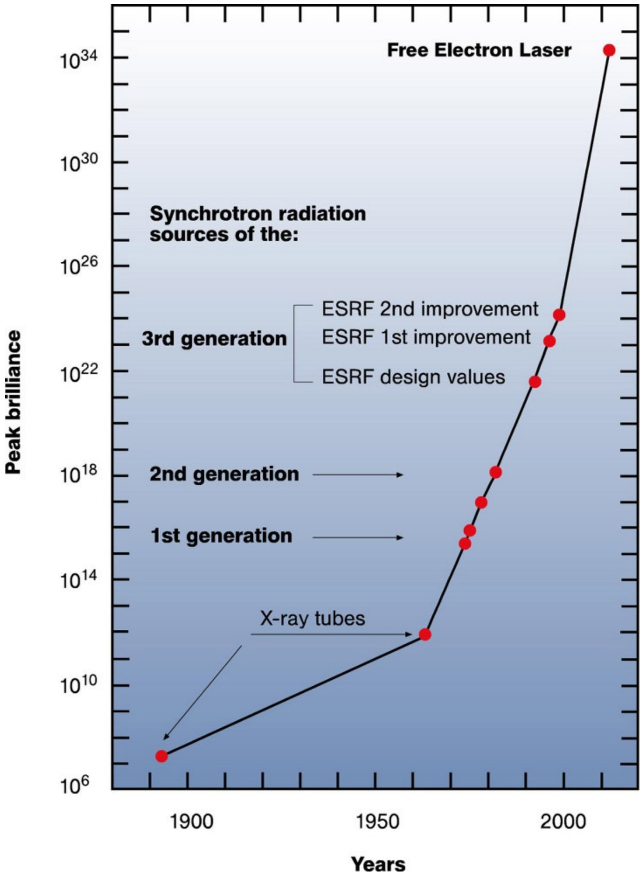
Intensities at the various generations of SR-sources

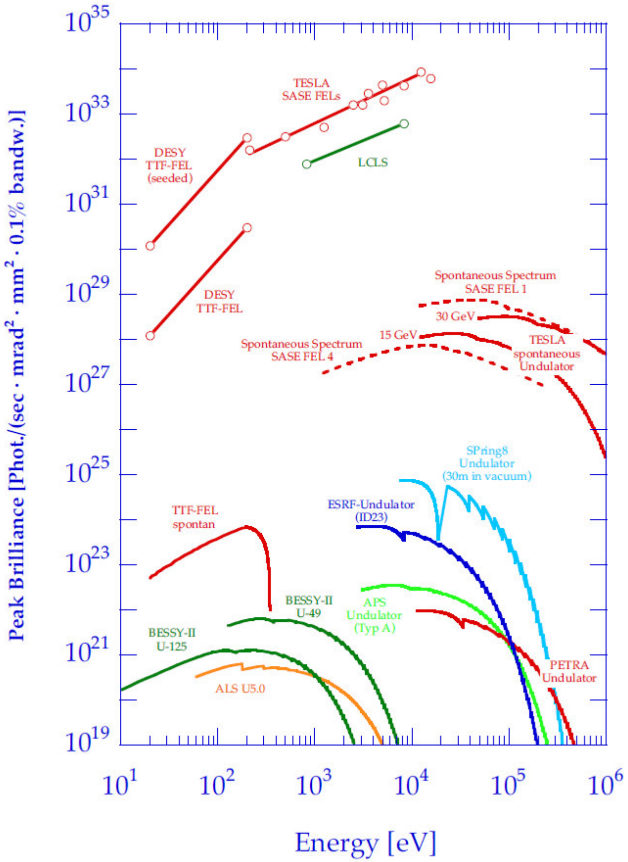
Wiggler: Intensity proportional to the beam current → $I \sim n_e$

Wiggler: Incoherent addition of „bending magnet" radiation → $I \sim N \times n_e$

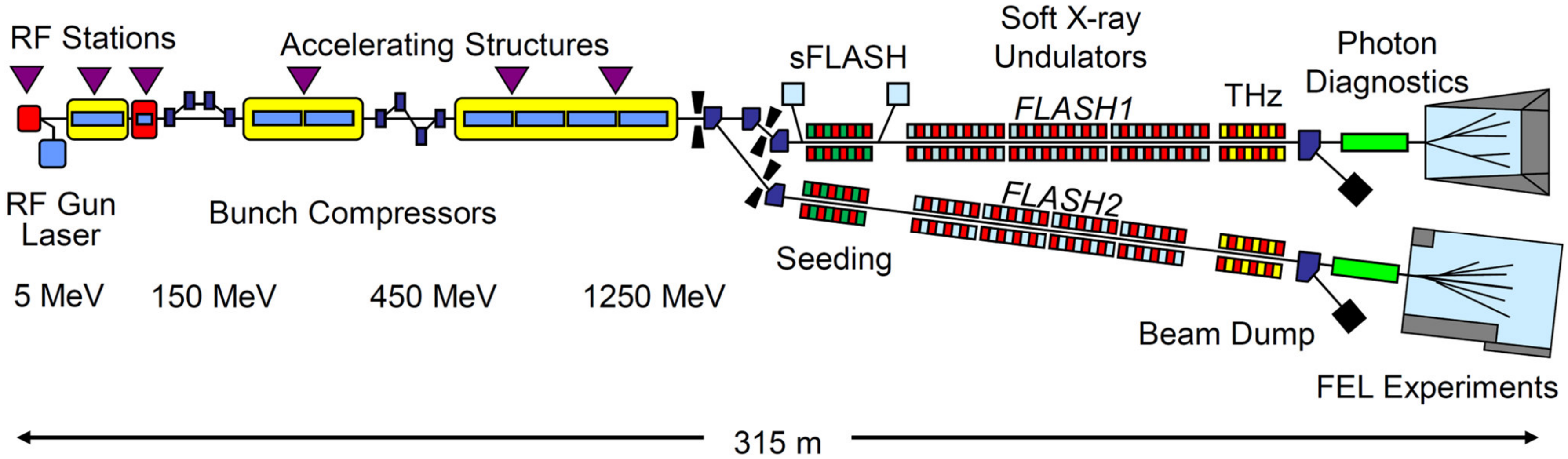
Undulator: Coherent addition of amplitudes from each period → $I \sim N^2 \times n_e$

FEL: Coherent addition of amplitudes from each microbunch → $I \sim N^2 \times n_e^2$

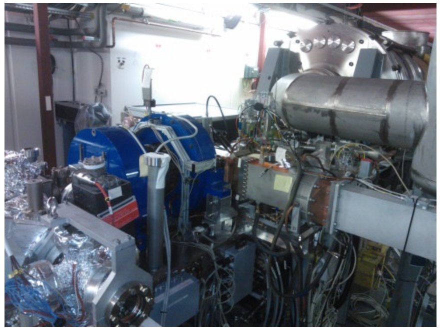




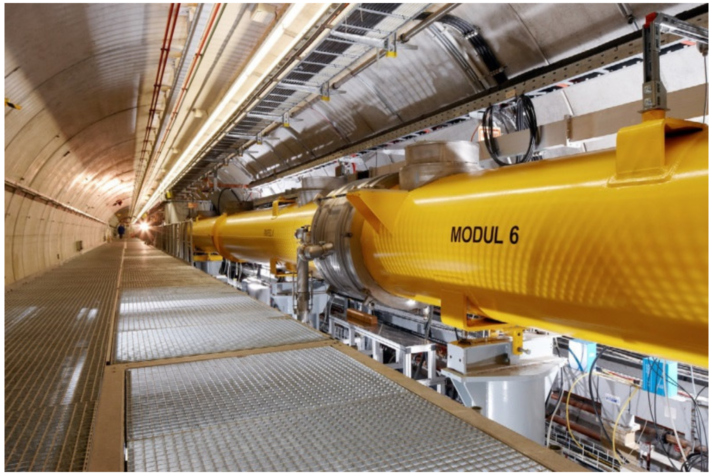
FLASH



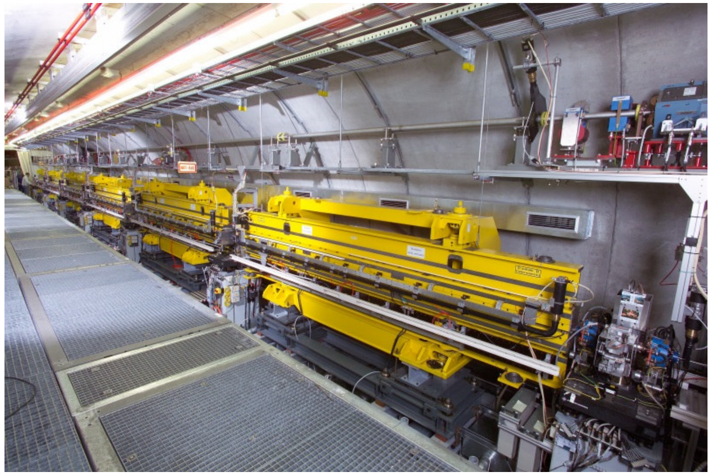
RF Gun



Accelerator Structures **(Supercond., 15 MV/m)**

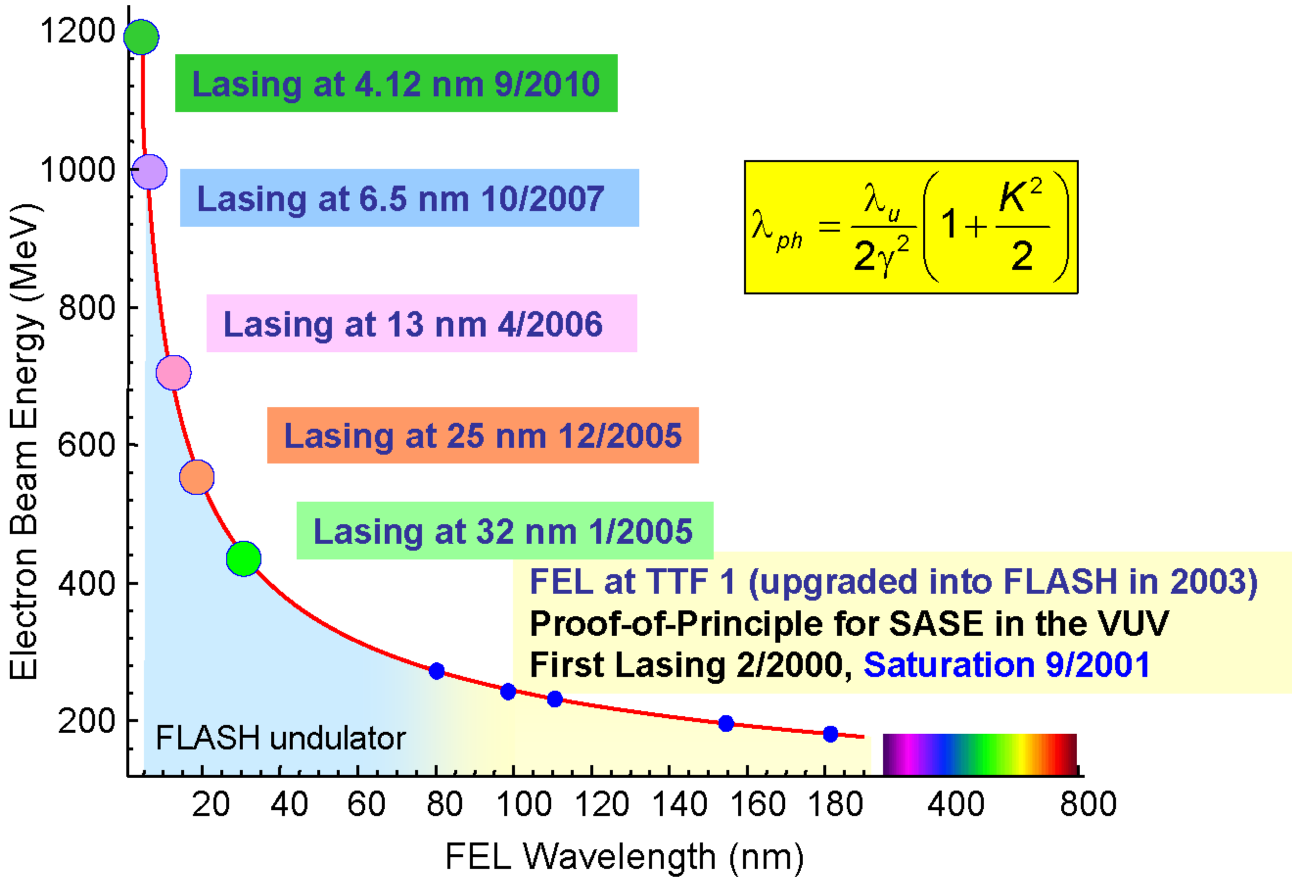


Undulators
(6 x 4.5 m, Period 27mm, Gap 12mm)



Bunch Compressors **(Magnetic Chicane, Current increase 50A → 1000 A)**

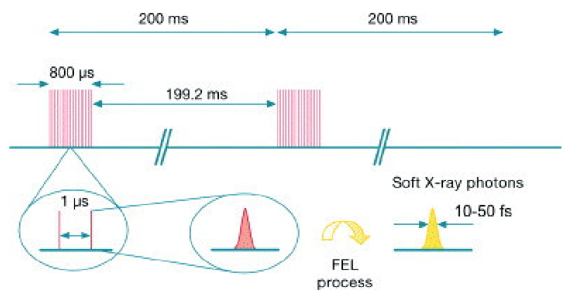
FLASH Parameters



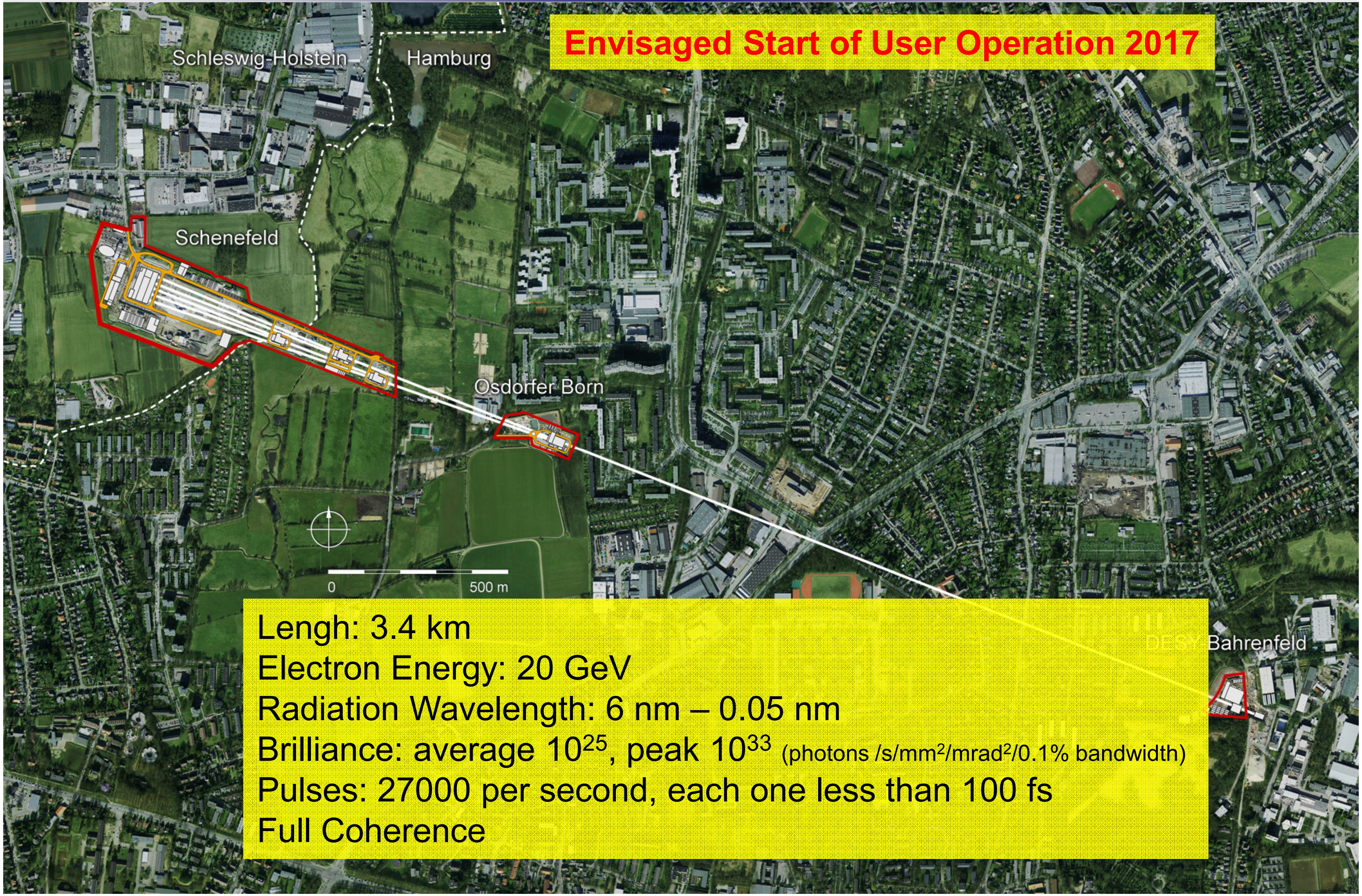
$$\lambda_{ph} = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

FLASH I Parameters

Electron Beam		
Energy	0.38 - 1.25	GeV
Number of Bunches per second	1 - 8000 (delivered up to 5000 to users)	
Pulse Repetition Rates (within pulse train)	40, 50, 100, 200, 250, 500, 1000	kHz
Photon Beam		
Wavelength	4.2 - 45	nm
Photon Energy	28 - 295	eV
Pulse Duration (FWHM)	30 - 300	fs
Average Pulse Energy (single bunch)	1 - 500	µJ
Average Pulse Energy (pulse trains)	1 - 200	µJ
3rd Harmonic Wavelength	down to 1.7 nm	
3rd Harmonic Pulse Energy	typically 0.5 % of fundamental	



X-Ray Free Electron Laser (European XFEL)



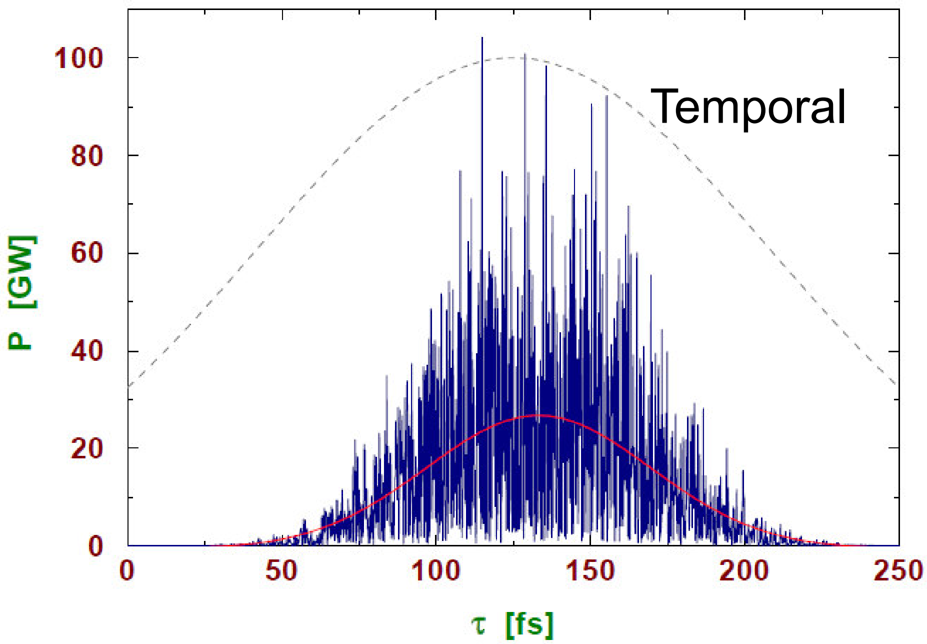
Envisaged Start of User Operation 2017

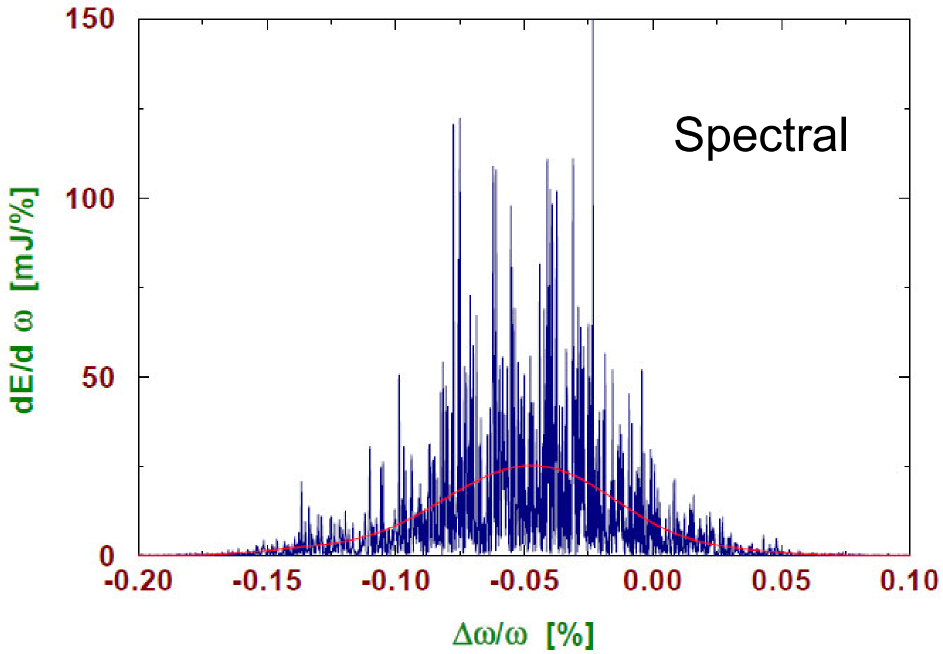
Lengh: 3.4 km
Electron Energy: 20 GeV
Radiation Wavelength: 6 nm – 0.05 nm
Brilliance: average 10^{25}, peak 10^{33} (photons /s/mm²/mrad²/0.1% bandwidth)
Pulses: 27000 per second, each one less than 100 fs
Full Coherence

XFEL – SASE 1 Characteristics

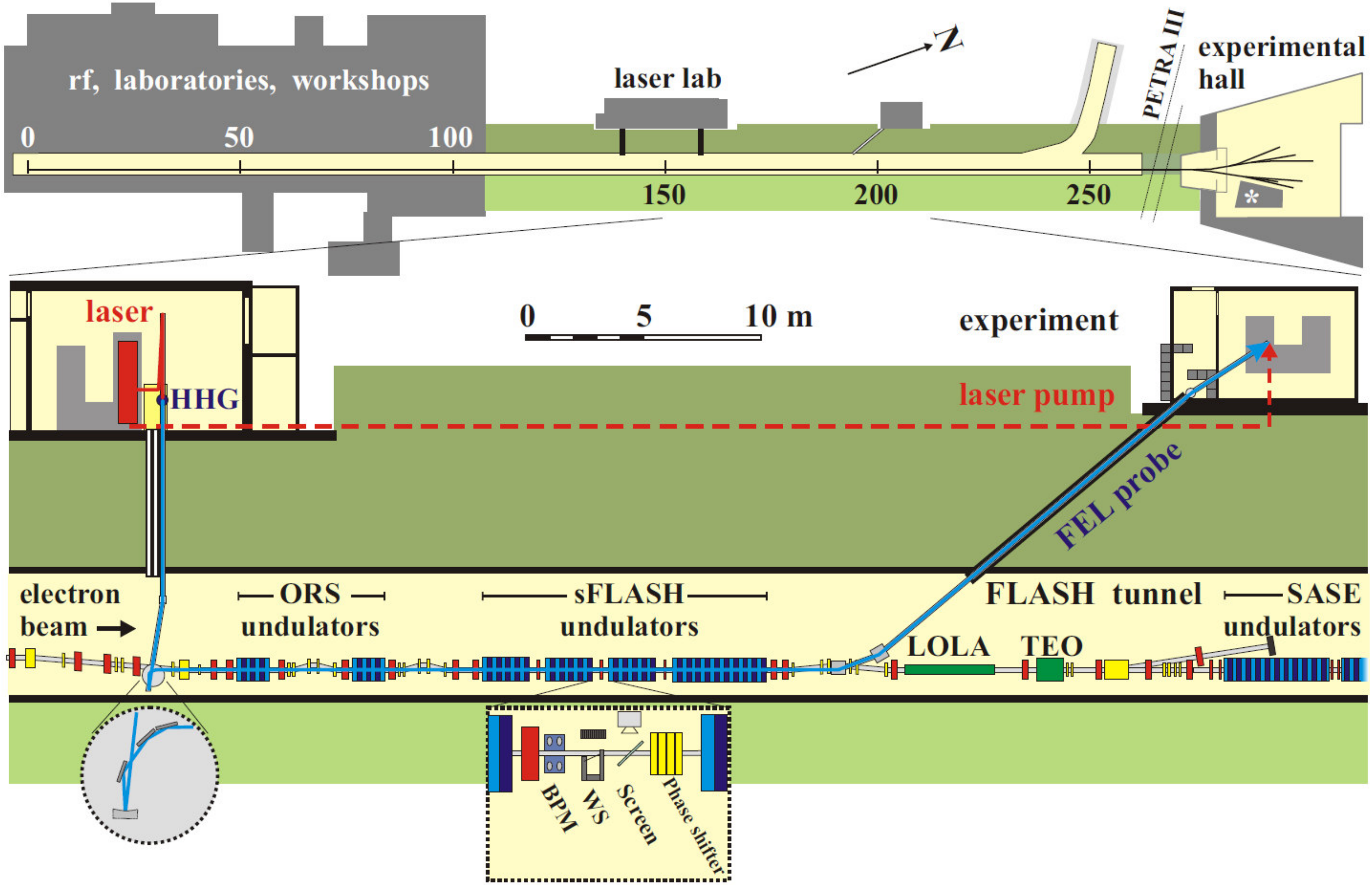
Wavelength	*0.1 – 0.5*	*nm*
Peak power	37	GW
Average power	210	W
Photon beam size (FWHM)	100	µm
Photon beam divergence (FWHM)	0.8	µrad
Bandwidth (FWHM)	0.08	%
Coherence time	0.3	fs
Pulse duration (FWHM)	100	fs
Min. pulse separation	93	ns
Max. number of pulses per train	11500	
Repetition rate	*5*	*Hz*
Number of photons per pulse	$1.8 \cdot 10^{12}$	
Average flux of photons	$1.0 \cdot 10^{17}$	1/s
Peak brilliance	$8.7 \cdot 10^{33}$	1/(s mrad² · mm² · 0.1% BW)
Average brilliance	$4.9 \cdot 10^{25}$	1/(s mrad² · mm² · 0.1% BW)

Parameters
XFEL-SASE 1

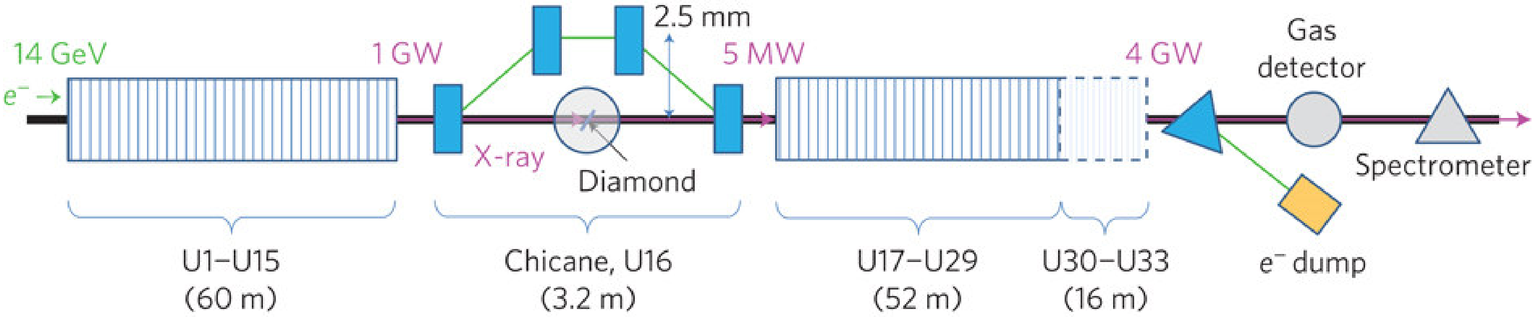


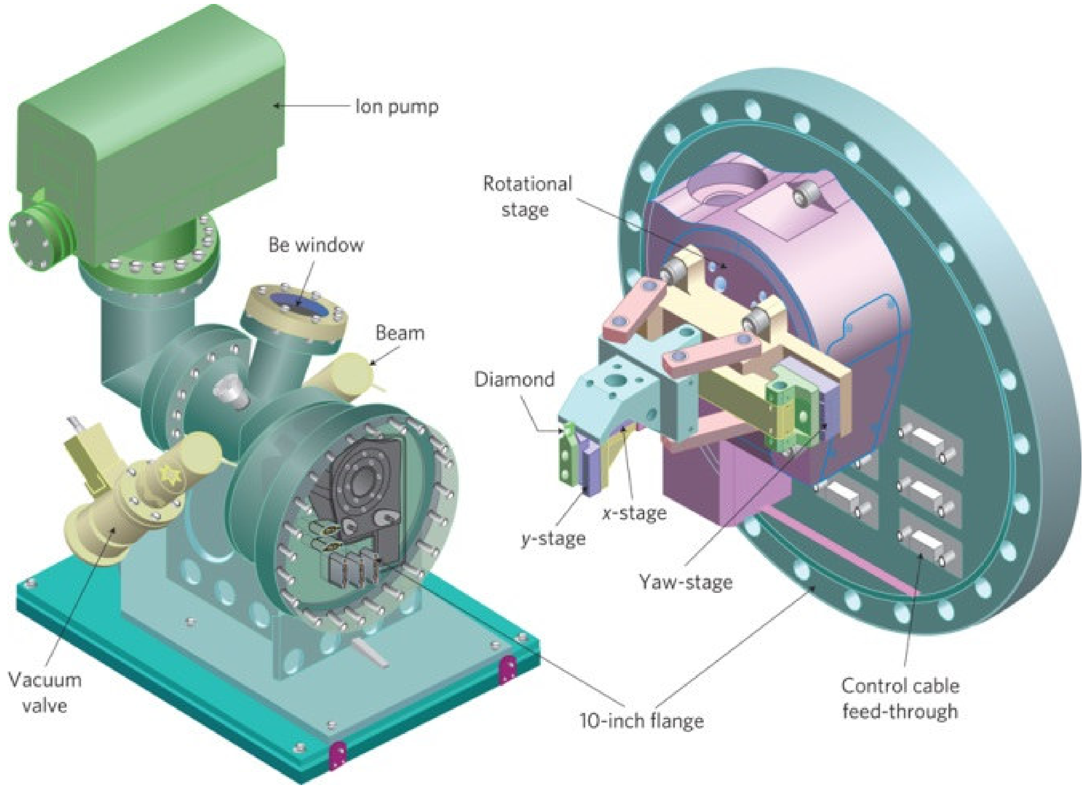


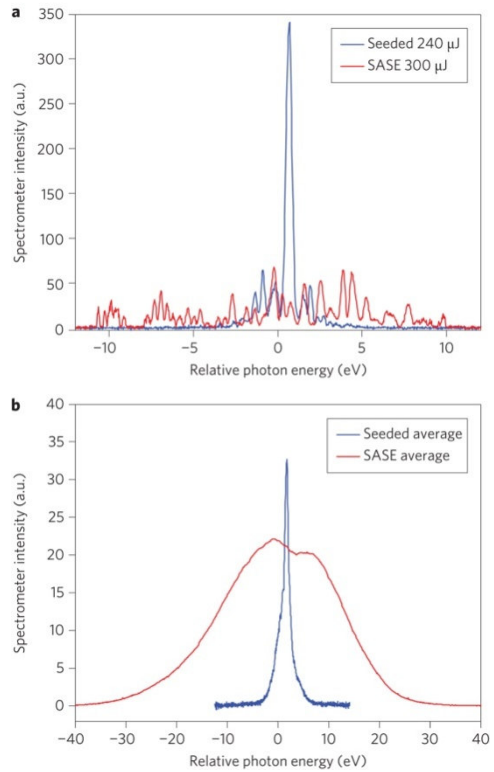
Direct Seeding at FLASH



Self-Seeding at LCLS







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Springer Tracts in Modern Physics, vol. 258 (2014)

Software XOP 2.3
Program package to model SR-sources and more (optics, raytracing…)
http://www.esrf.eu/Instrumentation/software/data-analysis/xop2.3